3. Hybrid-Vlasov code

The normalized equations used in our model (see Eq. (6) for characteristic quantities) are the following:

(1) The *Vlasov equation* for the ion distribution function $f(\mathbf{x}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \tag{14}$$

(2) The *Ohm's law* for the electric field

$$\mathbf{E} - d_e^2 \Delta \mathbf{E} = -(\mathbf{u} \times \mathbf{B}) + \frac{1}{n}(\mathbf{j} \times \mathbf{B}) + \frac{1}{n} d_e^2 \nabla \cdot \boldsymbol{\Pi} - \frac{1}{n} \nabla P_e + \frac{d_e^2}{n} \nabla \cdot [\mathbf{uj} + \mathbf{ju}] - \frac{1}{n} d_e^2 \nabla \cdot \left(\frac{\mathbf{jj}}{n}\right) \tag{15}$$

Here, the ion density n, the ion bulk velocity $\mathbf{u}$ and the ion pressure tensor $\boldsymbol{\Pi}$ are obtained as the moments of the ion distribution function f:

$$n(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}, \tag{16}$$

$$n\mathbf{u}(\mathbf{x}, t) = \int \mathbf{v} f(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}, \tag{17}$$

$$\boldsymbol{\Pi}(\mathbf{x}, t) = \int (\mathbf{v} - \mathbf{u})(\mathbf{v} - \mathbf{u}) f(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} \tag{18}$$

The electron pressure P_e is considered a function of the density $n = n_e = n_i$, as, for example, in the isothermal approximation $P_e = nk_B T_e$, where T_e is the electron temperature. The choice of the appropriate Ohm's law depends on the problem we intend to solve (cf. previous section); however in all cases the electric field is evaluated via the Ohm's law at a given point of the simulation loop.

(3) Maxwell equations, for the magnetic field $\mathbf{B}$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \mathbf{j} \tag{19}$$

We use the following characteristic quantities:

$$\bar{u} = u_A; \quad \bar{\omega} = \Omega_{ci}; \quad \bar{l} = u_A/\Omega_{ci} = c/\omega_{pi} = d_i; \quad \bar{n}; \quad \omega_{pi} = 4\pi \bar{n} e^2/m_i; \quad \omega_{pe} = 4\pi \bar{n} e^2/m_e;$$
$$\overline{P}_{p/e} = \bar{n} m_i u_A^2; \quad \overline{E} = m_i u_A \Omega_{ci}/e; \quad \overline{B} = m_i c \Omega_{ci}/e \tag{6}$$

Here, u_A is the Alfvén velocity, Ω_{ci} and ω_{pi} the ion cyclotron and the ion plasma frequencies, respectively and d_i the ion skin depth. The electron skin depth, in units of d_i (i.e. in dimensionless units) is given by the square root of the mass ratio, $d_e = \sqrt{(m_e/m_i)}$.