Particle-laden turbulent flows: small scale clustering and two-way coupling effects

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in collaboration with

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HPC methods for Computational Fluid Dynamics and Astrophysics

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Why particle laden flows?

Technological devices

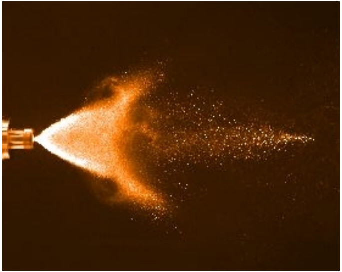
- Liquid and solid fuels

  [Abramzon, Int. JHMT (1989)]
- Cyclonic separators

  [Kilstrom, Patent No. 5,935,279. (1999)]
- Sprays

  [Jenny et *al.* Prog. Comb. Sci. (2012)]

ICE injector



Vacuum cleaner

ICE combustion chamber

Why particle laden flows?

Natural phenomena

- Particle dispersion

  [Biferale et *al.* PoF (2005)]

- Rain formation in clouds

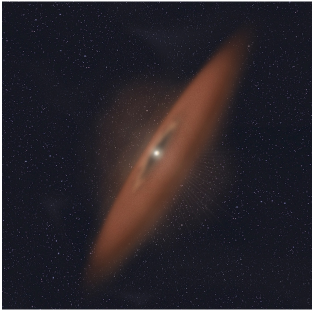
  [Falkovich et *al.* Nat. (2002)]

- Planetesimal formation

  [Johansen et *al.* Nat. (2007)]

Etna Volcano, Sicily

Circumstellar disk HD 141943

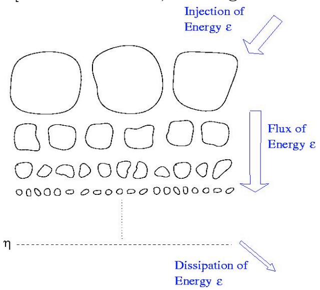


Storm on mount Amiata, Tuscany

A (possible) paradigm of Turbulence

- cahotic behavior [space/time]
- wide range of scales
  - integral scale L_0
  - power input $W \sim \bar{\epsilon}$
  - Kolmogorov scale $\eta = (\nu^3/\bar{\epsilon})^{1/4}$ [viscous dissipation]
- energy cascade

[Richardson 1920, Kolmogorov 1941]



Leonardo da Vinci, vortici d'acqua

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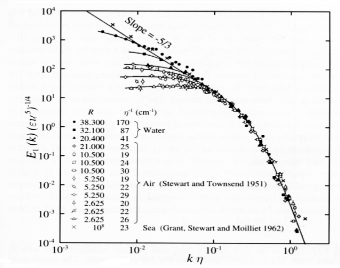
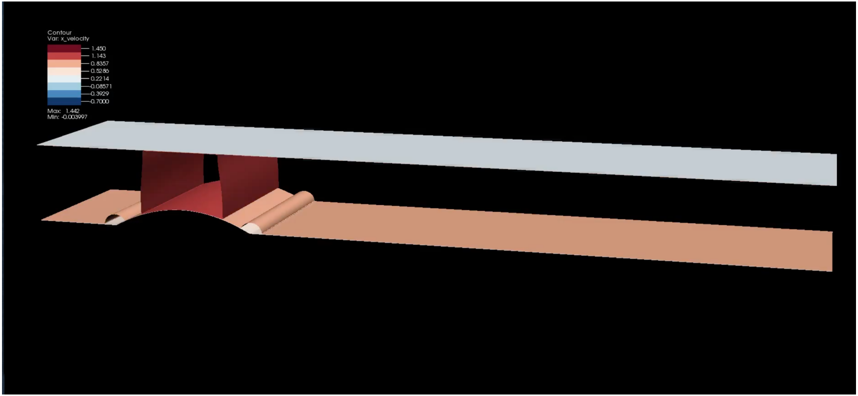


Leonardo da Vinci, vortici d'acqua

Direct Numerical Simulation of Turbulence

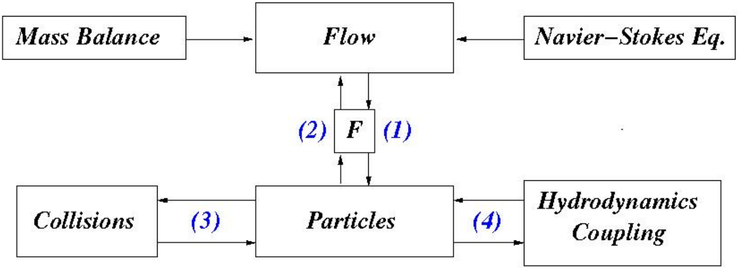
DNS by Nek5000 @ Re=10000 on Fermi Tier0



Turbulent flow past a curved wall

[Mollicone et *al.* JFM (2017)]

Fluid/disperse-phase interaction



[Elghobashi (1994); Balachandar & Eaton (2010)]

- (1) one-way coupling
  [point-like particles and dilute suspension $\Phi_V \ll 1$]
- (1) + (2) two-way coupling
  [as in (1) but $\rho_p/\rho_f \gg 1 \Rightarrow$ finite mass loading $\Phi = \rho_p/\rho_f\, \Phi_V$]
- (1) + (2) + (3) + (4) four-way coupling
  [finite size effects]

Inertial particles

- Point-particles $d_p/\eta \ll 1$ in a diluted suspensions $\Phi_V \ll 1$

  [interactions and collisions neglected]
- Inter-phase momentum exchange negligible $\Phi = \frac{\rho_p}{\rho_f}\Phi_V \ll 1$
- Stokes regime $Re_p = |\mathbf{v}_p| d_p/\nu \to 0$
- Newton's equations

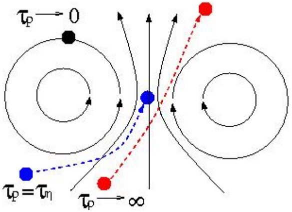
$$\frac{d\mathbf{x}_p}{dt} = \mathbf{v}_p; \quad \frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u}|_{\mathbf{x}_p} - \mathbf{v}_p}{\tau_p}$$

Stokes time: $\tau_p = \frac{\rho_p}{\rho_f}\frac{d_p^2}{18\nu}$ Stokes number: $St_\eta = \frac{\tau_p}{\tau_\eta}$

- Two different limits
  - *Lagrangian particles*

    $\tau_p \to 0; \quad \mathbf{v}_p = \mathbf{u}|_{\mathbf{x}_p}$
  - *Ballistic particles*

    $\tau_p \to \infty; \quad d\mathbf{v}_p/dt = 0$



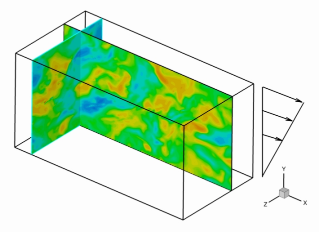
Inertial particles: Hydrogen Disk

- Physical conditions
  - speed of sound $c \sim 1000$ m/s
  - mean free path $\lambda \sim 1$ mm
  - Kolmogorov scale $\eta \sim 10$ m
  - gas density $\rho_f \sim 10^{-6}$ kg/m^3
  - particle density $\rho_p \sim 10^3$ kg/m^3
  - particle (dust) diameter $d_p \sim \mu\text{m} \div \text{m}$

- Remarks
  - incompressible flow
  - $\eta \gg \lambda \Rightarrow$ continuum approach, e.g. Navier-Stokes eq.s
  - large (very!) particle-to-fluid density ratio
  - Stokes numbers $St_\eta \sim 0.1 \div 100$
  - mass loading $\Phi \sim 0.1 \div 10$

    [Hogan & Cuzzi, PRE (2007)]

The Homogeneous Shear Flow

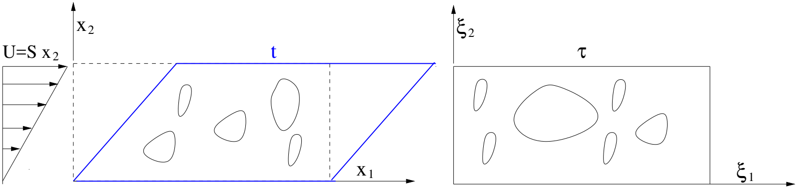


$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla\pi+\nu\nabla^2\mathbf{v}$$

$$\nabla\cdot\mathbf{v} = 0; \;\; \mathbf{v} = Sx_2\,\mathbf{e}_1 + \mathbf{u}$$
$$S = \text{shear rate } [T^{-1}]$$

Rogallo's transformation [Rogallo, TM 81315 (1981)]

$$\xi_1 = x_1 - U(x_2)t;\; \xi_2 = x_2;\; \xi_3 = x_3;\; \tau = t;\; \Rightarrow \nabla_\xi = (\partial_{\xi_1}, \partial_{\xi_2} - S\tau\partial_{\xi_1}, \partial_{\xi_3})$$



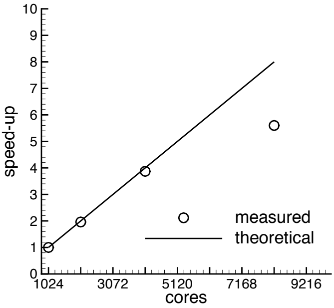
Physical space

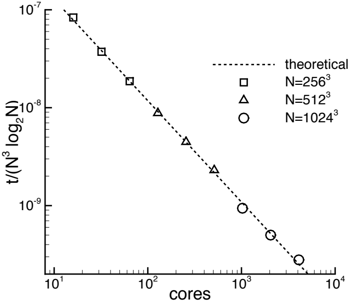
Computational space

$$\frac{\partial \mathbf{u}}{\partial \tau} = -\mathbf{u}\cdot\nabla\mathbf{u} - \nabla_\xi\pi + \nu\nabla_\xi^2\mathbf{u} - Sv\mathbf{e}_1\,; \quad \nabla_\xi^2\pi = \nabla_\xi\cdot N[\mathbf{u}]$$

The Homogeneous Shear Flow

- In house developed code
  - Pseudo-Spectral + 3/2 dealiasing rule
  - IV^{th} order Low-Storage Runge-Kutta
  - OpenMP version, Caspur (Giorgio Amati) [NCAR and ESSL fft libraries]
  - MPI version, CINECA (Francesco Salvadore) [P3DFFT + FFTW readapted for dealiasing]





Scaling test on Fermi Tier0 @ CINECA

The Homogeneous Shear Flow

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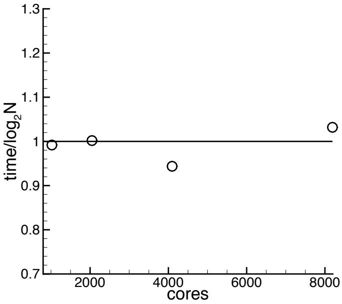
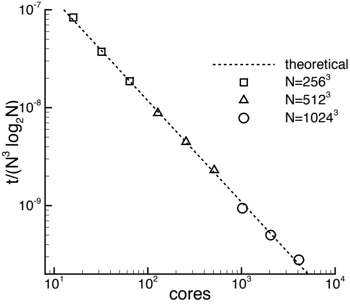


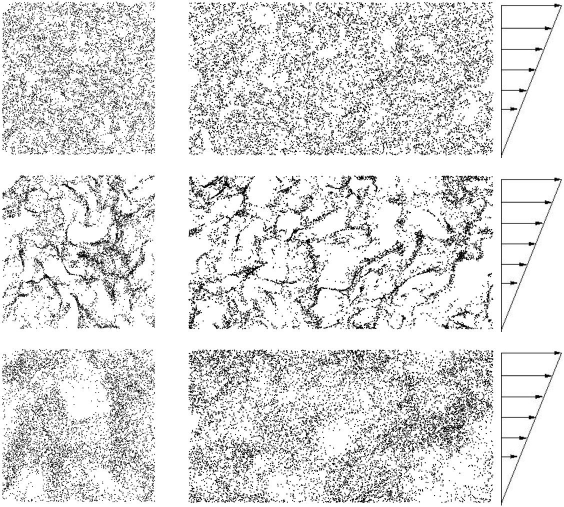


Scaling test on Fermi Tier0 @ CINECA

Instantaneous particle distribution: clustering

Homogeneous shear flow ⇒ effect of St_η

- $St_\eta = 0.1$
- $St_\eta = 1$
- $St_\eta = 10$



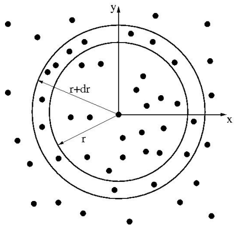
carrier fluid $Re_\lambda = 100$ $S^* = (L_0/L_S)^{2/3} = 7$

Radial Distribution Function $g(r)$: clustering

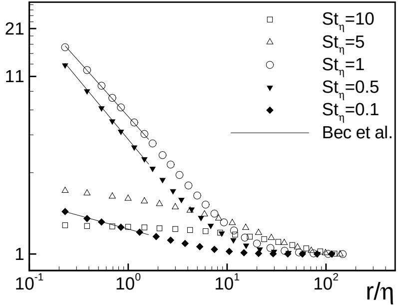
- RDF: probability to find particle pairs at distance $|\mathbf{r}|$

  [Sundaram & Collins *JFM* (1997)]

- Uniform distribution $g(r) = 1$
- for $r \to 0$ $\; g(r) \propto r^{-\alpha}$, $\alpha(St_\eta) > 0$ denotes small scale clustering

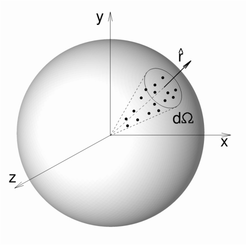


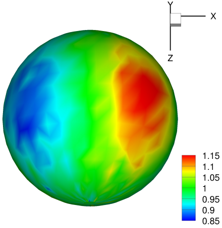
- *warning*: clustering is not isotropic



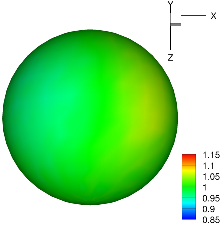
Angular Distribution Function $g(r, \widehat{\mathbf{r}})$

In shear flows clustering needs a more complete description





$r = 4\eta$



$r = 35\eta$

- Pairs $d\mu_r = \nu_r(r, \hat{\mathbf{r}}) d\Omega$ in a cone of radius r, direction $\hat{\mathbf{r}}$ and solid angle $d\Omega$

$$g(r, \hat{\mathbf{r}}) = \frac{1}{r^2} \frac{d\nu_r}{dr} \frac{1}{n_0} \,,$$

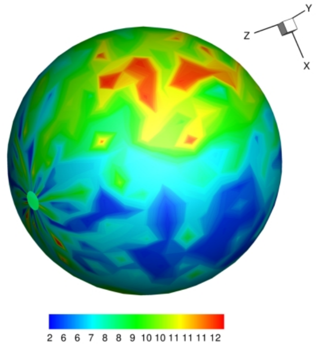
[P.G. Picano & Casciola *JFM* 2009]

- Non uniform distribution on the solid angle: *anisotropic clustering*

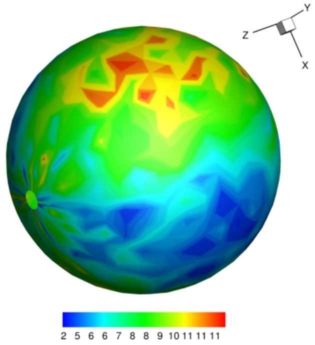
Inter-Particle collisions

- Number of collisions $\dot{n}_c(\sigma;\hat{\mathbf{r}})$ per unit time and volume along the direction $\hat{\mathbf{r}}$ on the sphere $\sigma = \eta$ for $St_\eta = 1$

computed



model

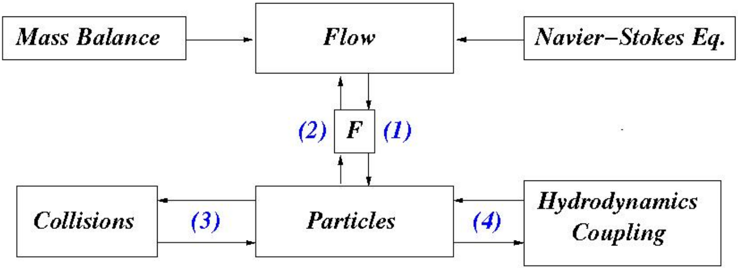


- $\dot{n}_c(\sigma;\hat{\mathbf{r}})$ can be estimated as

$$\dot{n}_c(\sigma;\hat{\mathbf{r}}) = 0.5\, n_0^2\, \sigma^2\, g(\sigma;\hat{\mathbf{r}})\, Q^{(1)}(\sigma;\hat{\mathbf{r}}|\delta v < 0)$$

[P.G., Picano, Sardina, & Casciola *Phys. D* (2011)]

Fluid/disperse-phase interaction



[Elghobashi (1994); Balachandar & Eaton (2010)]

- (1) one-way coupling
- (1) + (2) two-way coupling
- (1) + (2) + (3) + (4) four-way coupling

⇒ turbulence modulation in the *two-way coupling regime*

Particle In Cell approach (PIC) [Crowe et al. *J. Fluid Eng.* (1977)]

- Eulerian description (fluid) & Lagrangian tracking (particles) [Eaton (2009); Balachandar & Eaton (2010); Elghobashi (1994)]

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \pi + \nu \nabla^2 \mathbf{u} - \frac{1}{\rho_f} \sum_p \mathbf{D_p(t)} \delta \left[\mathbf{x} - \mathbf{x_p(t)} \right]$$

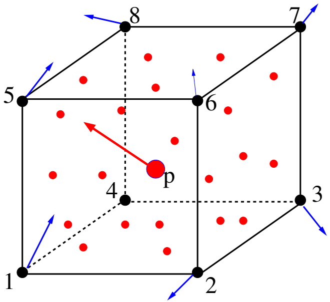
- The *back-reaction* **F** is singular: average on the cell ΔV_{cell}

$$\mathbf{F}(\mathbf{x}_p) = \frac{1}{\Delta V_{cell}} \frac{1}{\rho_f} \mathbf{D}_p$$

equivalent to $\{\mathbf{F}(\mathbf{x}_q)\}_{q=1,8}$

$$\sum_{q=1}^{8} \mathbf{F}(\mathbf{x}_q) = \mathbf{F}(\mathbf{x}_p)$$

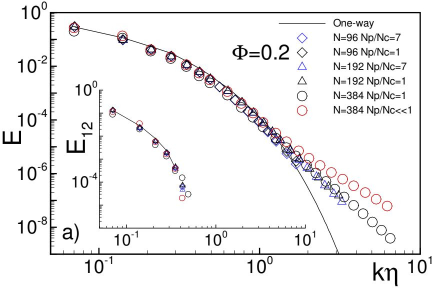
$$\sum_{q=1}^{8} (\mathbf{x}_q - \mathbf{x}_p) \times \mathbf{F}(\mathbf{x}_q) = 0$$

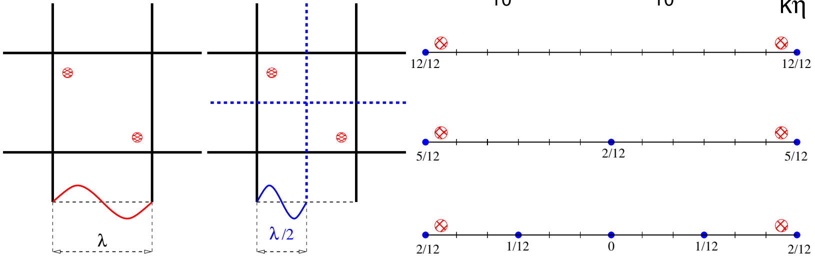


PIC: numerical issues

The tails of the spectra hardly decay when $N_p/N_c \ll 1$

- fine grids require a large number of particles
- grid dependent forcing
- limitations on the mass load





[P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **715** (2013)]

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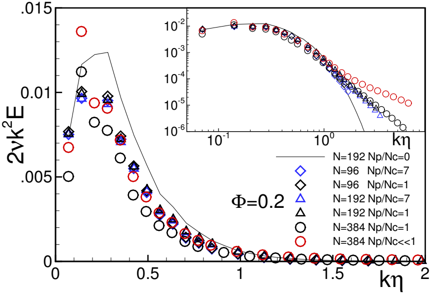
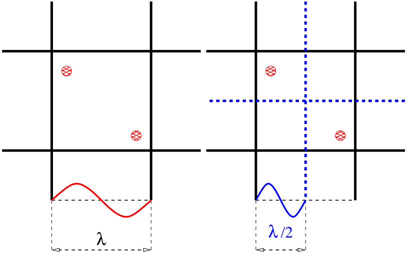
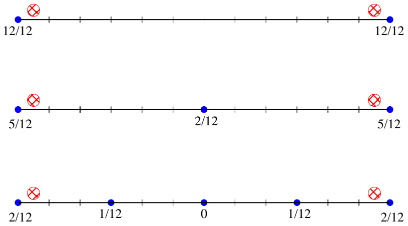
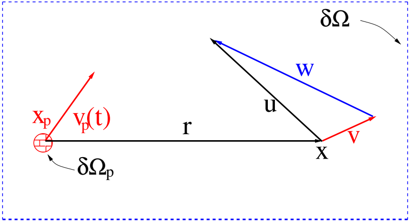






[P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **715** (2013)]

Exact Regularized Point Particle (ERPP) method



$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} \\ \mathbf{u}|_{\partial\Omega_p} = \mathbf{v}_p; \quad \mathbf{u}|_{\partial\Omega} = \mathbf{u}_{wall} \\ \mathbf{u}(\mathbf{x}, t_n) = \mathbf{u}_0(\mathbf{x}) \end{cases}$$

Decompose the (incompressible) fluid velocity $\mathbf{u}$ in a background flow $\mathbf{w}$ and a perturbation $\mathbf{v}$, namely $\mathbf{u} = \mathbf{w} + \mathbf{v}$

$$\begin{cases} \dfrac{\partial \mathbf{w}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla \pi + \nu\nabla^2\mathbf{w} \\ \mathbf{w}|_{\partial\Omega} = \mathbf{u}_{wall} - \mathbf{v}|_{\partial\Omega} \\ \mathbf{w}(\mathbf{x}, t_n) = \mathbf{u}_0(\mathbf{x}) \end{cases} \qquad \begin{cases} \dfrac{\partial \mathbf{v}}{\partial t} = -\nabla \tilde{p} + \nu\nabla^2\mathbf{v} \\ \mathbf{v}|_{\partial\Omega_p} = \mathbf{v}_p - \mathbf{w}|_{\partial\Omega_p} \\ \mathbf{v}(\mathbf{x}, t_n) = 0 \end{cases}$$

Perturbation $\mathbf{v}$ *described in terms of unsteady Stokes equations*

[P.G., F. Picano, G. Sardina, C.M. Casciola, JFM **773** (2015)]

ERPP: perturbation field

Exact solution of the unsteady Stokes problem

$$v_i(\mathbf{x},t) = \int_0^t d\tau \int_{\partial\Omega} t_j(\boldsymbol{\xi},\tau) G_{ij}(\mathbf{x},\boldsymbol{\xi},t,\tau) - v_j(\boldsymbol{\xi},\tau)\mathcal{T}_{ijk}(\mathbf{x},\boldsymbol{\xi},t,\tau) n_k(\boldsymbol{\xi})\, dS$$

G_{ij} the *unsteady Stokeslet*; $\mathcal{T}_{ijk}$ the associated stress tensor

For *small particles* the *far field* disturbance is estimated in terms of multipole expansion [Kim & Karilla, Microfluidics, (2000)]

$$v_i(\mathbf{x},t) \simeq -\int_0^t D_j(\tau) G_{ij}(\mathbf{x},\mathbf{x}_p,t,\tau)\, d\tau$$

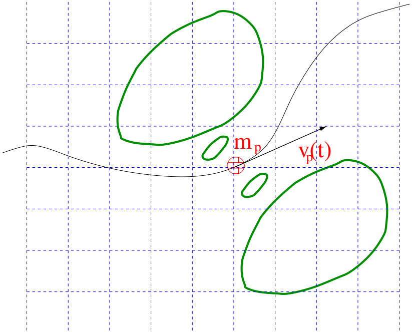
with $\mathbf{D}(\tau)$ hyrodynamic force, i.e. the Stokes Drag ($\cdots$ or more [Maxey & Riley, (1983); Gatignol (1983)])

ERPP: vorticity

Eulerian *far field* disturbance $\mathbf{v}(\mathbf{x}, t)$ described by the unsteady singularly forced Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} - \nu \nabla^2 \mathbf{v} + \nabla \tilde{p} = -\frac{\mathbf{D}(t)}{\rho_f} \, \delta \left[\mathbf{x} - \mathbf{x}_p(t) \right]$$

How to regularize the solution of the disturbance field?



Physics of the coupling

The vorticity, once generated along the particle trajectory, is diffused by viscosity and then injected into the Eulerian grid

vorticity generated by the particle $\Rightarrow$ Eulerian Navier−Stokes solver

ERPP: vorticity diffusion

Why vorticity? $\Rightarrow$ Diffusion equation

$$\partial_t \boldsymbol{\zeta} - \nu \nabla^2 \boldsymbol{\zeta} = \frac{\mathbf{D}(t)}{\rho_f} \times \nabla \delta \left[\mathbf{x} - \mathbf{x}_p(t) \right]$$

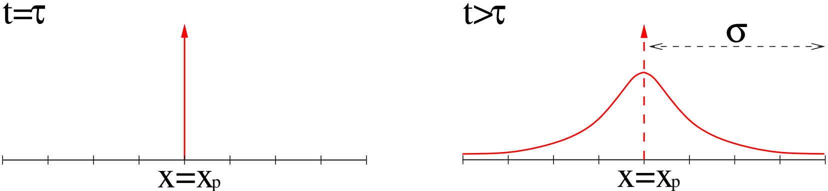
Fundamental solution

$$\partial_t g - \nu \nabla^2 g = \delta \left(\mathbf{x} - \mathbf{x}_p \right) \delta(t - \tau)$$

$$g(\mathbf{x}, \mathbf{x}_p, t, \tau) = \frac{1}{\left(2\pi\sigma^2\right)^{3/2}} \exp\left(\frac{-\|\mathbf{x} - \mathbf{x}_p\|^2}{2\sigma^2} \right), \quad \sigma(t-\tau) = \sqrt{2\nu(t-\tau)}$$

For $t > \tau$ the solution is both

- *regular*, e.g. $g \in C^\infty$
- *local*, i.e decays more than exponentially



ERPP: vorticity regularization

- *Analytical solution* expressed as a convolution with the fundamental solution of the diffusion equation

$$\boldsymbol{\zeta}(\mathbf{x},t) = \frac{1}{\rho_f}\int_0^t \mathbf{D}(\tau)\times\nabla g\left[\mathbf{x}-\mathbf{x}_p(\tau), t-\tau\right] d\tau$$

- For $\tau \simeq t$, $g(\mathbf{x},\mathbf{x}_p,t,\tau)$ tends to behave as badly as the Dirac delta function $\Rightarrow$ split $\boldsymbol{\zeta} = \boldsymbol{\zeta}_{Regular} + \boldsymbol{\zeta}_{Singular}$

- The regularization procedure adopts a temporal cut-off ϵ_R

$$\boldsymbol{\zeta}_R(\mathbf{x},t) = \frac{1}{\rho_f}\int_0^{t-\epsilon_R} \mathbf{D}(\tau)\times\nabla g\left[\mathbf{x}-\mathbf{x}_p(\tau), t-\tau\right] d\tau$$

$\Rightarrow$ Regularized field $\boldsymbol{\zeta}_R$ everywhere smooth and characterized by the smallest spatial scale $\sigma_R = \sqrt{2\nu\epsilon_R}$

ERPP: coupling with the carrier phase

- The regular component of the vorticity field $\boldsymbol{\zeta}_R$ satisfy

$$\frac{\partial \boldsymbol{\zeta}_R}{\partial t} - \nu \nabla^2 \boldsymbol{\zeta}_R = \frac{1}{\rho_f} \nabla \times \mathbf{D}(t - \epsilon_R) g \left[\mathbf{x} - \mathbf{x}_p(t - \epsilon_R), \epsilon_R \right]$$

- The regular (perturbation) velocity field $\mathbf{v}_R$ follows as

$$\frac{\partial \mathbf{v}_R}{\partial t} - \nu \nabla^2 \mathbf{v}_R = -\frac{1}{\rho_f} \mathbf{D}(t - \epsilon_R) g \left[\mathbf{x} - \mathbf{x}_p(t - \epsilon_R), \epsilon_R \right]$$

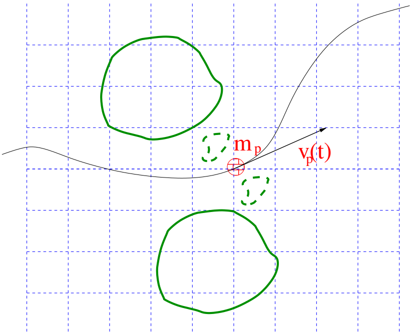
- The fluid velocity $\mathbf{u} = \mathbf{w} + \mathbf{v}_R$ is then given by

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} - \sum_{p=1}^{N_p} \frac{\mathbf{D}_p(t - \epsilon_R)}{\rho_f} g \left[\mathbf{x} - \mathbf{x}_p(t - \epsilon_R), \epsilon_R \right]$$

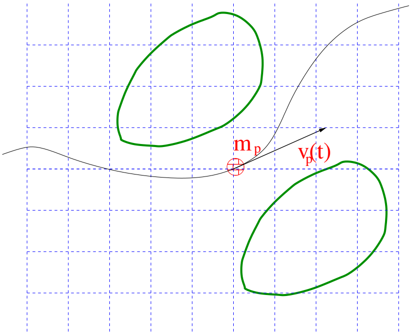
- Remarks
  - simply add an extra term in any $N - S$ solver
  - "anticipated" Green function: diffusion timescale ϵ_R
  - the function g is *local* in space $\Rightarrow$ *computational efficiency*

ERPP: a cartoon

complete field



regularized field

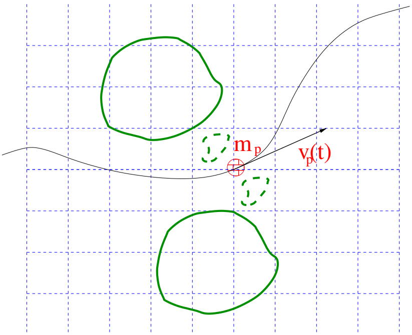


$$\boldsymbol{\zeta}(\mathbf{x}, t) = \frac{1}{\rho_f} \int_0^t \mathbf{D}(\tau) \times \nabla g \, d\tau \qquad \boldsymbol{\zeta}_R(\mathbf{x}, t) = \frac{1}{\rho_f} \int_0^{t-\epsilon_R} \mathbf{D}(\tau) \times \nabla g \, d\tau$$

⇒ Vorticity at scales smaller that $\sigma_R = \sqrt{2\nu\epsilon_R}$ is not neglected but injected at later times

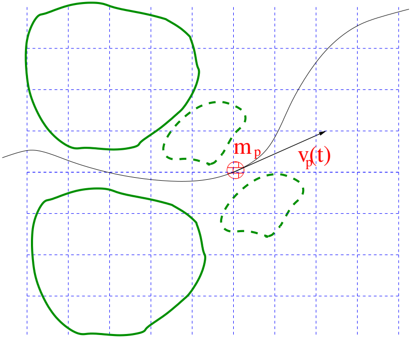
ERPP: a cartoon

complete field



$$\boldsymbol{\zeta}(\mathbf{x},t) = \frac{1}{\rho_f}\int_0^t \mathbf{D}(\tau) \times \nabla g \, d\tau$$

regularized field at later times



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⇒ Vorticity at scales smaller that $\sigma_R = \sqrt{2\nu\epsilon_R}$ is not neglected but injected at later times

Hydrodynamic force in the two-way coupling regime

- Newton's law

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{D}_p(t) = 6\pi\mu a_p \left[\tilde{\mathbf{u}}(\mathbf{x}_p, t) - \mathbf{v}_p(t)\right]$$

⇒ $\tilde{\mathbf{u}}(\mathbf{x}_p, t)$ fluid velocity at $\mathbf{x}_p$ *in absence of the particle*

[Boivin et al. (1998); Jenny et al. (2012), P.G. et al. (2013,2015); Horwitz et al. (2016)]

- Removal of the self-disturbance $\mathbf{v}_{pth}$ from $\mathbf{u}(\mathbf{x}, t)$

$$\tilde{\mathbf{u}}(\mathbf{x}_p, t) = \mathbf{u}(\mathbf{x}_p, t) - \mathbf{v}_{pth}\left[\mathbf{x}_p(t) - \mathbf{x}_p(t_n); Dt\right]$$

- Self-disturbance velocity evaluated in closed form

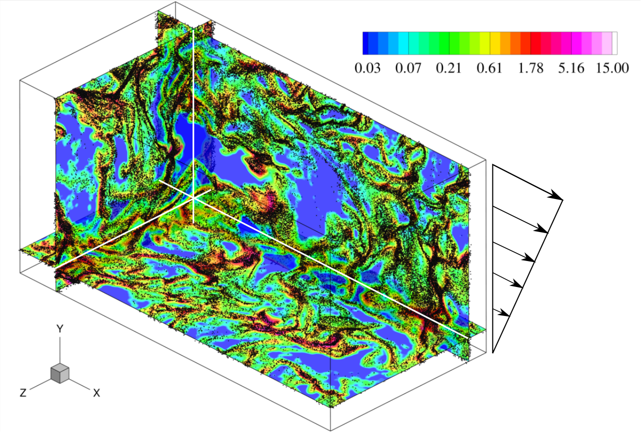
$$\mathbf{v}(\mathbf{r}, Dt) = \frac{1}{(2\pi\sigma^2)^{3/2}} \left\{ \left[e^{-\eta^2} - \frac{f(\eta)}{2\eta^3}\right] \mathbf{D}_p^n - \left(\mathbf{D}_p^n \cdot \hat{\mathbf{r}}\right) \left[e^{-\eta^2} - \frac{3f(\eta)}{2\eta^3}\right] \hat{\mathbf{r}} \right\}$$

where $\mathbf{r} = \mathbf{x}(t) - \mathbf{x}_p(t_n)$; $\quad \eta = r/\sqrt{2}\sigma$; $\quad \sigma = \sqrt{2\nu(\epsilon_R + Dt)}$ and

$$f(\eta) = \frac{\sqrt{\pi}}{2} \mathrm{erf}(\eta) - \eta e^{-\eta^2}$$

Turbulent Flows: Homogeneos shear flow

$$Re_\lambda = 80 \; St_\eta = 1, \; \Phi = 0.4 \; N_p = 2.200.000$$

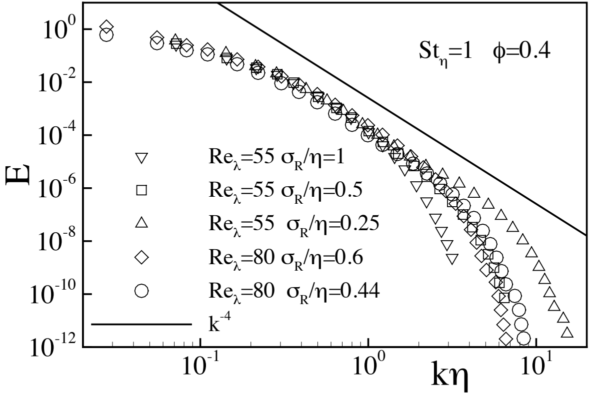


- Remarks
  - regularization scale $\sigma_R = \eta$
  - the feedback field is everywhere *smooth*

Energy spectrum: scaling in the two-way regime

- Enegy balance $T(k) + P(k) - D(k) + \Psi(k) = 0$
- At scales $k\eta \sim 1$ and $k\sigma_R < 1$

feedback $\sim$ dissipation $\Rightarrow \Psi(k) \sim D(k) \Rightarrow E(k) \propto k^{-4}$



$\Rightarrow$ Convergent solutions w.r.t. regularization parameter σ_R/η

[P.G., Battista, C.M. Casciola, Phys. Rev. Fluids (2016)]

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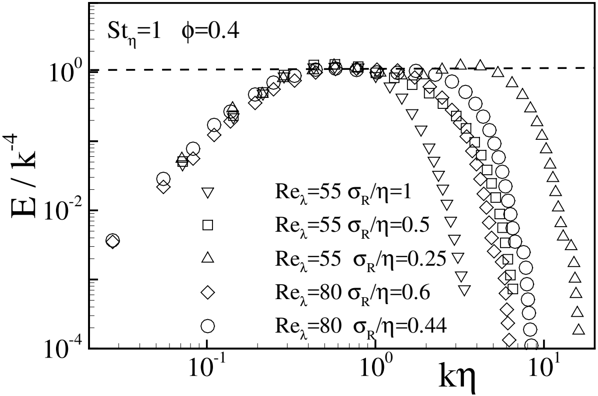

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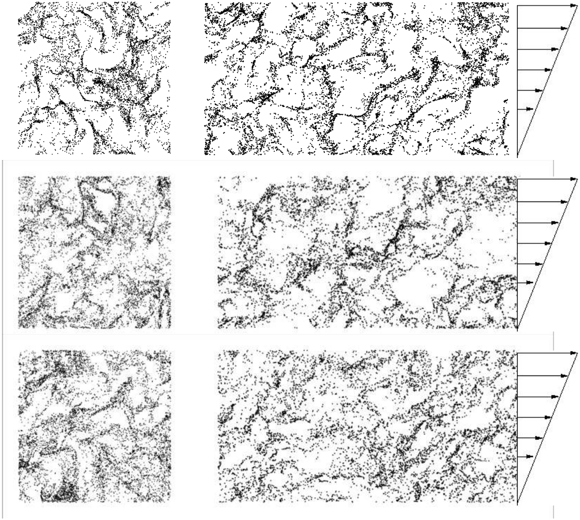
[P.G., Battista, C.M. Casciola, Phys. Rev. Fluids (2016)]

Instantaneous particles distribution: clustering

Two-way coupling $St_\eta = 1 \Rightarrow$ mass load effect Φ

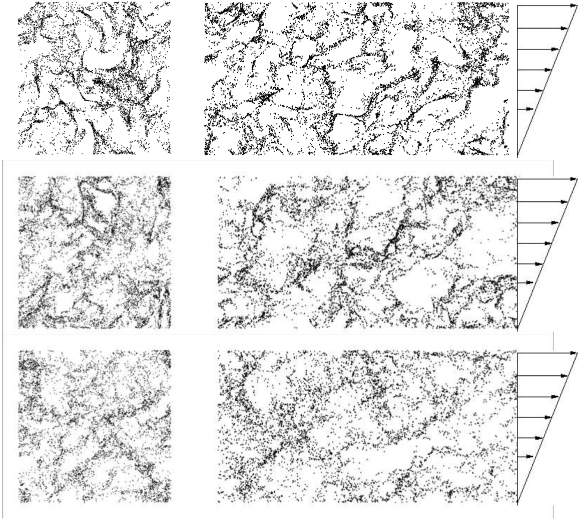
- $\Phi = 0$
- $\Phi = 0.2$
- $\Phi = 0.4$



Instantaneous particles distribution: clustering

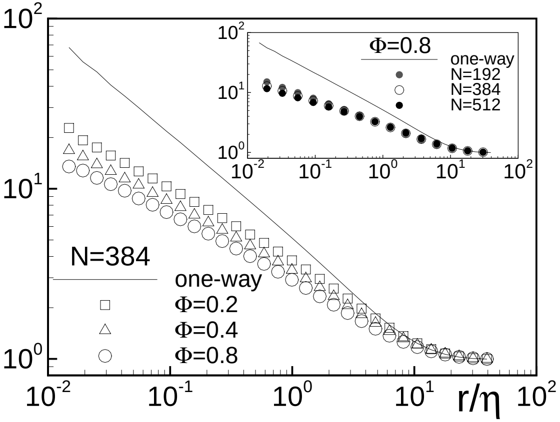
Two-way coupling $St_\eta = 1 \Rightarrow$ mass load effect Φ

- $\Phi = 0$
- $\Phi = 0.2$
- $\Phi = 0.8$



Clustering in the two-way regime

- RDF controlled by the Mass Loadig Φ



⇒ Small scale clustering reduced by two-way coupling effects

Final remarks

- ERPP method
  - based on physical arguments ⇒ Unsteady Stokes Solutions
  - easy to implement and portable
  - computationally efficient
- Turbulence in multiphase flows
  - small scale (anisotropic) clustering controlled by St_η
  - anisotropic forcing at small scales
  - scaling $E(k) \propto k^{-4}$ where feedback $\sim$ dissipation
  - small scale clustering attenuated by two-way coupling effects

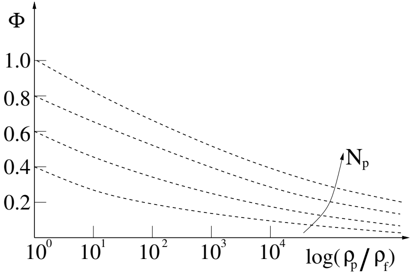
Credits

- PRACE projects FP7 RI-283493 and #2014112647
- ERC grant #339446 **BIC: Bubbles from Inception to Collapse**, P.I. *C.M. Casciola*

Remarks about the mass load Φ

Six dimensionless parameters $\left\{ Re_0;\ St_\eta;\ \frac{\rho_p}{\rho_f};\ \frac{d_p}{\eta};\ \Phi;\ N_p \right\}$

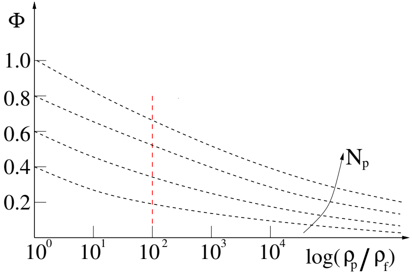
$$St_\eta = \frac{1}{18}\frac{\rho_p}{\rho_f}\left(\frac{d_p}{\eta}\right)^2 \qquad \Phi = 9\sqrt{2}\pi\, N_p\, \frac{St_\eta^{3/2}}{Re_0^{9/4}\ (\rho_p/\rho_f)^{1/2}}$$



Remarks about the mass load Φ

In experiments or numerics fix $\left\{ Re_0;\ St_\eta;\ \frac{\rho_p}{\rho_f};\ \frac{d_p}{\eta};\ \Phi;\ N_p \right\}$

$$St_\eta = \frac{1}{18}\frac{\rho_p}{\rho_f}\left(\frac{d_p}{\eta}\right)^2 \qquad \Phi = 9\sqrt{2}\pi\, N_p\, \frac{St_\eta^{3/2}}{Re_0^{9/4}\,(\rho_p/\rho_f)^{1/2}}$$



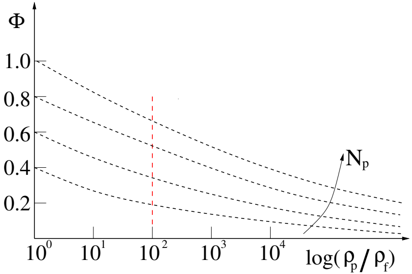
In the ERPP

- smooth perturbation field
- N_p can be modified safely
- *adjust Φ by changing N_p*

Remarks about the mass load Φ

In experiments or numerics fix $\left\{ Re_0;\ St_\eta;\ \frac{\rho_p}{\rho_f};\ \frac{d_p}{\eta};\ \Phi;\ N_p \right\}$

$$St_\eta = \frac{1}{18}\frac{\rho_p}{\rho_f}\left(\frac{d_p}{\eta}\right)^2 \qquad \Phi = 9\sqrt{2}\pi\,\frac{N_p}{N_c}\,\frac{St_\eta^{3/2}}{(\rho_p/\rho_f)^{1/2}}$$

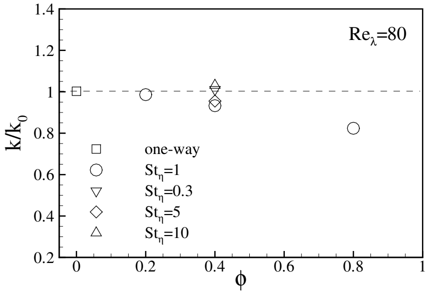


In the PIC

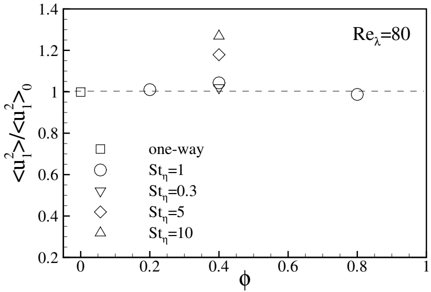
- $N_p/N_c \geq 1$ to achieve a smooth forcing
- $N_c \propto Re_0^{9/4}$ is fixed
- *Φ can not be changed anymore!*

Turbulence modulation: velocity variances

Turbulent kinetic energy



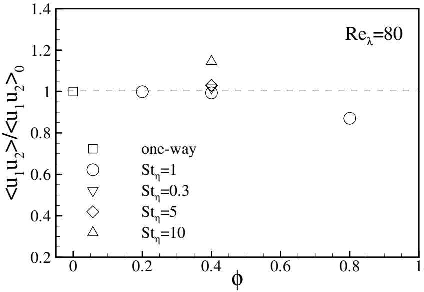
Stream-wise velocity variance



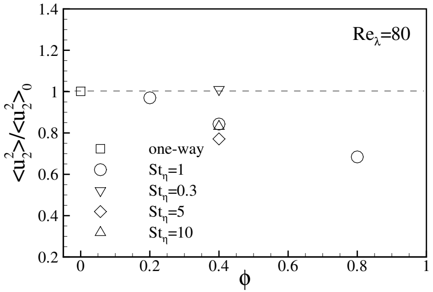
- Remarks
  - Fluctuations attenuated at increasing mass loadings
  - Selective turbulence modification
  - Effect of St_η on anisotropic turbulence modulation

Turbulence modulation: velocity variances

Reynolds shear stresses



Cross-flow velocity variance

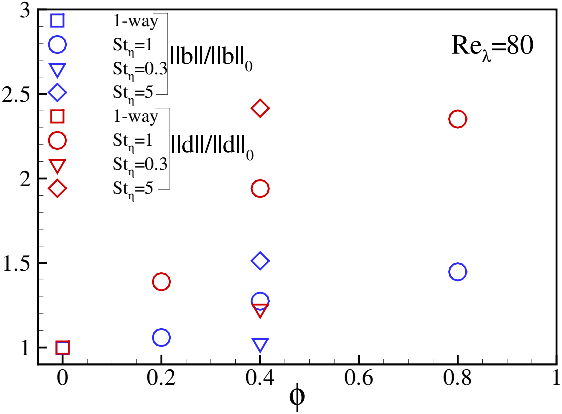


- Remarks
  - Fluctuations attenuated at increasing mass loadings
  - Selective turbulence modification
  - Effect of St_η on anisotropic turbulence modulation

Small scale isotropy recovery?

- Large & Small scale anisotropy indicator

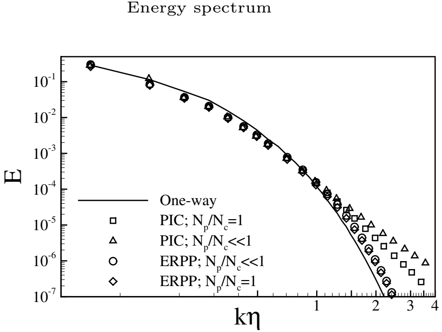
$$b_{\alpha\beta} = \frac{\langle u_\alpha u_\beta \rangle}{\langle u_\gamma u_\gamma \rangle} - \frac{1}{3}\delta_{\alpha\beta} \qquad d_{\alpha\beta} = \frac{\epsilon_{\alpha\beta}}{\epsilon_{\gamma\gamma}} - \frac{1}{3}\delta_{\alpha\beta}$$

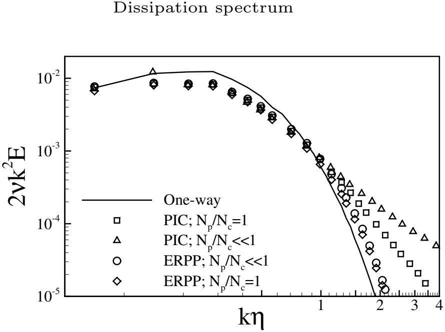


Anisotropy enhanced in the two-way coupling regime

Energy spectrum

- ERPP *vs.* PIC @ $\Phi = 0.4,\ St_\eta = 1$



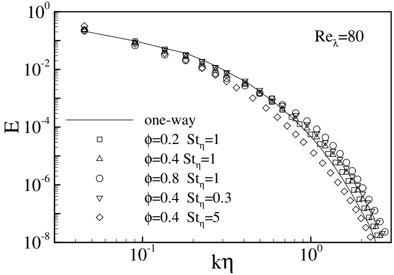


- Remarks
  - $E(k)$ and $2\nu\, k^2\, E(k)$ nicely smooth at small scales
  - ERPP prediction independent on N_p/N_c
  - Small scale fluctuations energized by the backreaction
  - Turbulence modulation controlled by Φ and St_η

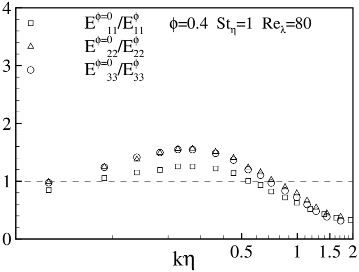
Turbulent spectra

- Effect of Mass loading Φ and Stokes number St_η

Energy spectrum



Velocity components spectrum

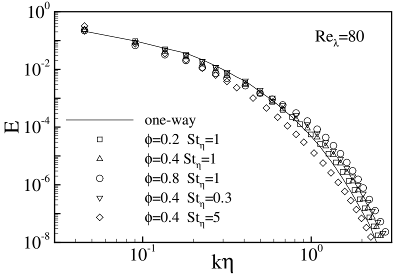


- Remarks
  - Small scale fluctuations energized at $St_\eta = 1$
  - Energy attenuation at all scales for $St_\eta = 5$
  - Anisotropic turbulence modulation at different scales

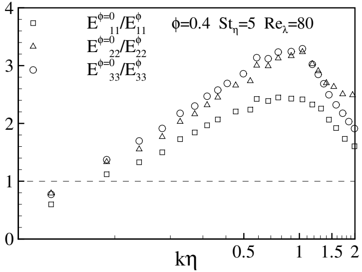
Turbulent spectra

- Effect of Mass loading Φ and Stokes number St_η

Energy spectrum



Velocity components spectrum



- Remarks
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