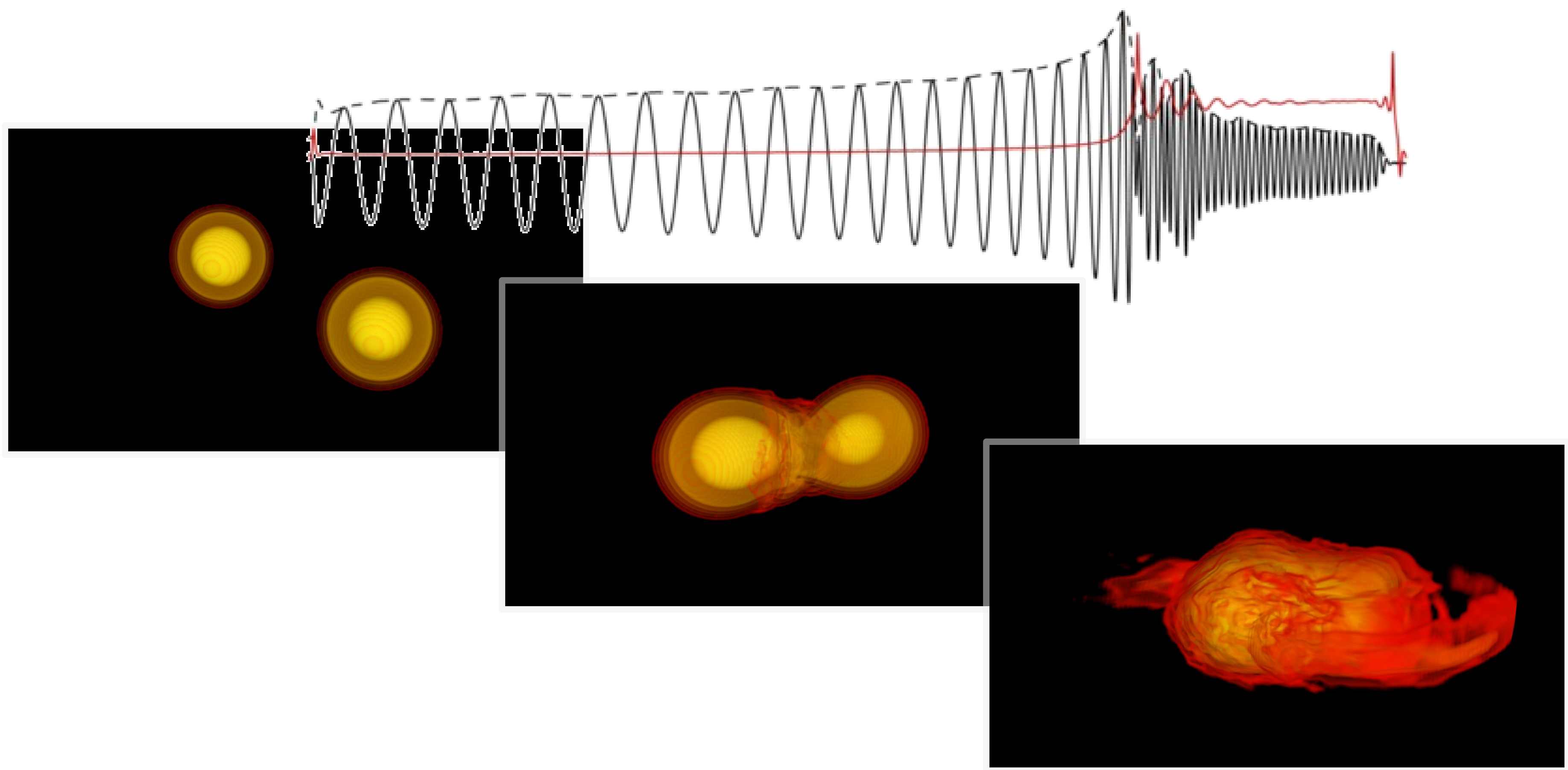
UNIVERSITÀ DI PARMA

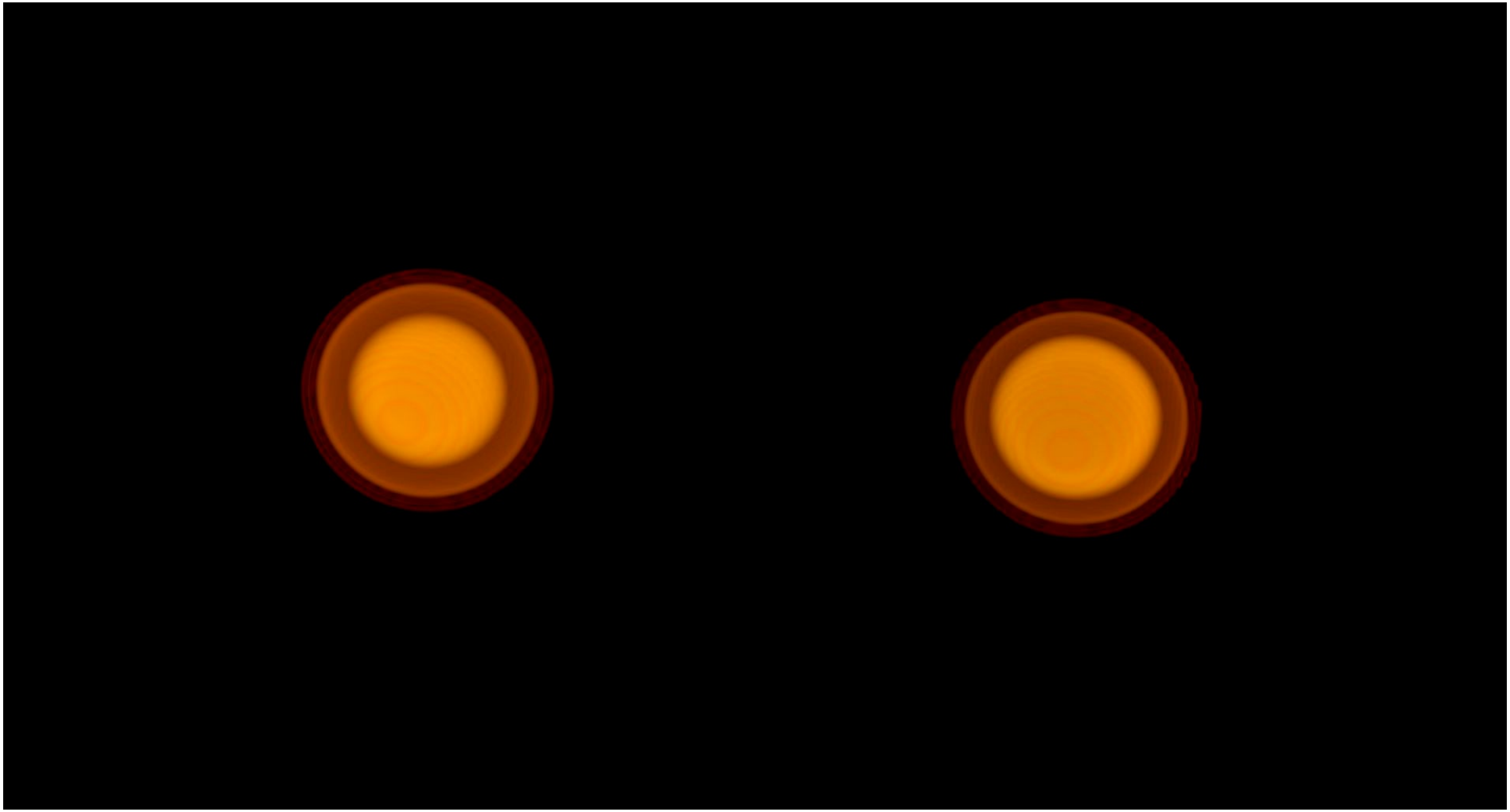
INFN

S.Bernuzzi CINECA Nov 15th 2017

Numerical relativity in the gravitational-wave astronomy era



First numerical relativity simulation of neutron star merger with precessing spins: the double pulsar case

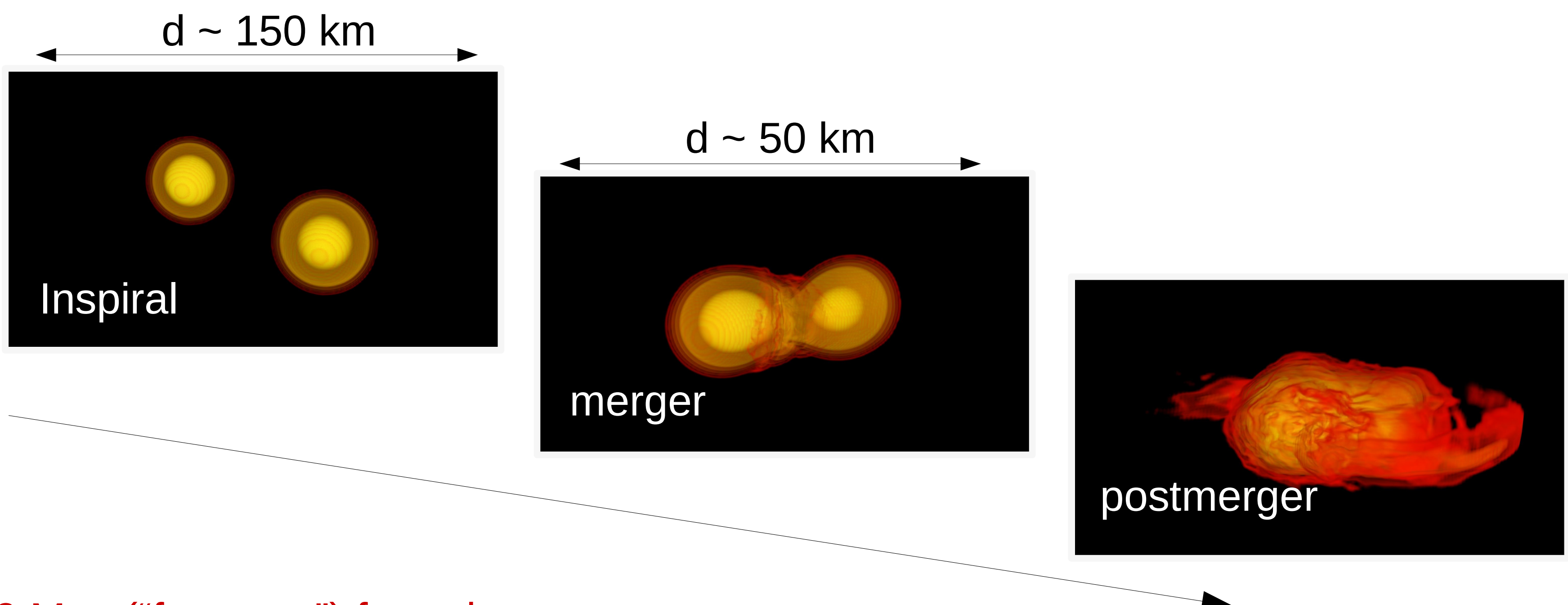


Viz by T.Dietrich

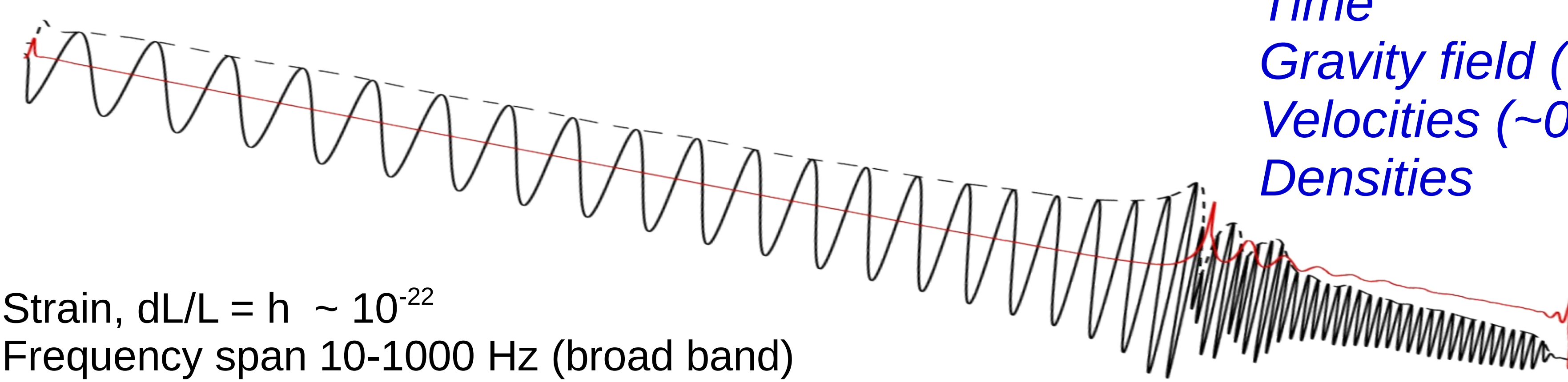
Baryon mass density

GWs: Tiny signatures of extreme events

Collision of neutron stars [Mass~1.4 Msun, Radius~10 km]:



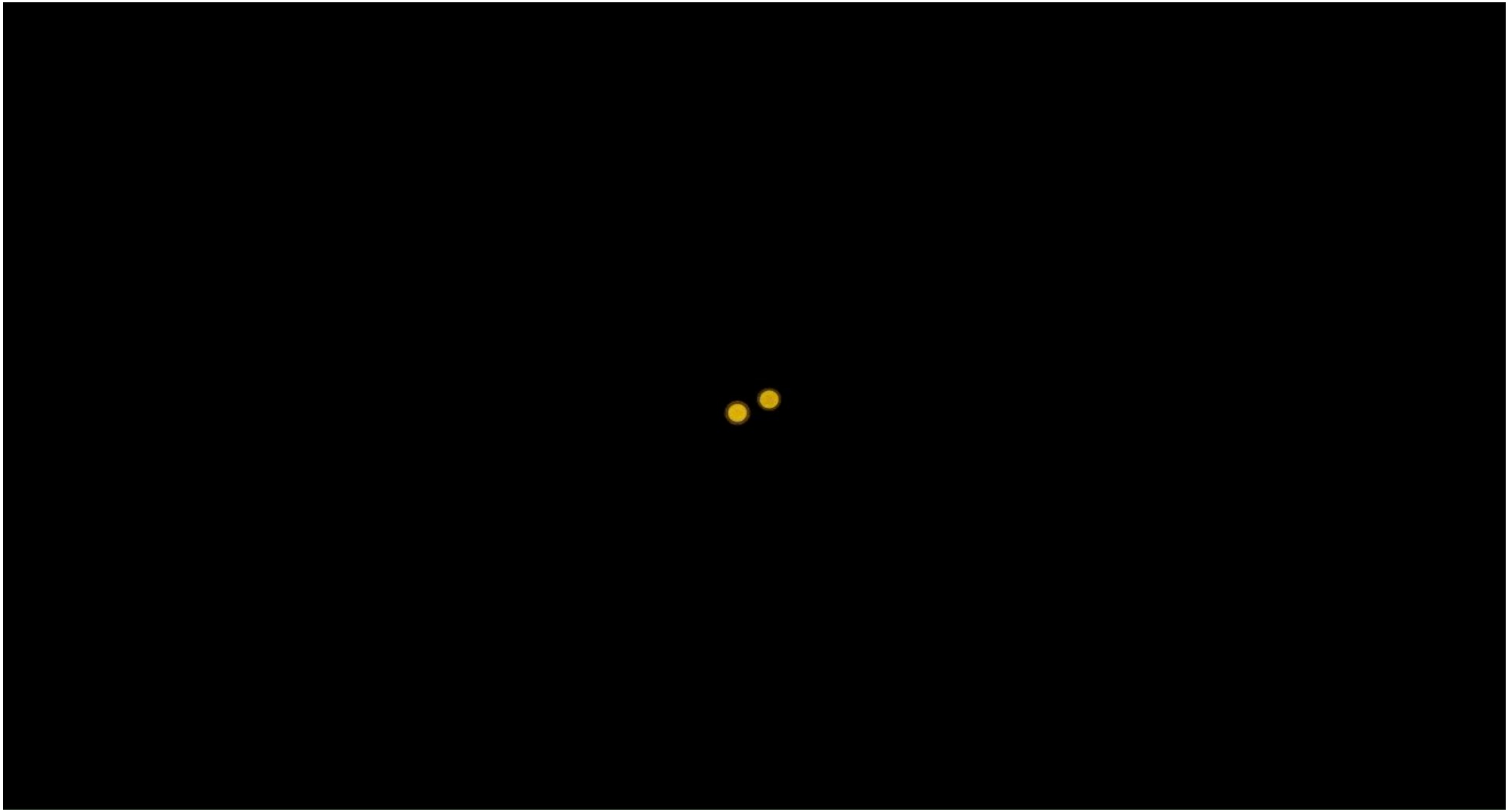
D~200 Mpc ("far away") from the source:



Time
Gravity field (~M/d)
Velocities (~0.1 c)
Densities

Strain, dL/L = h ~ 10^{-22}
Frequency span 10-1000 Hz (broad band)

First numerical relativity simulation of neutron star merger with precessing spins: the double pulsar case

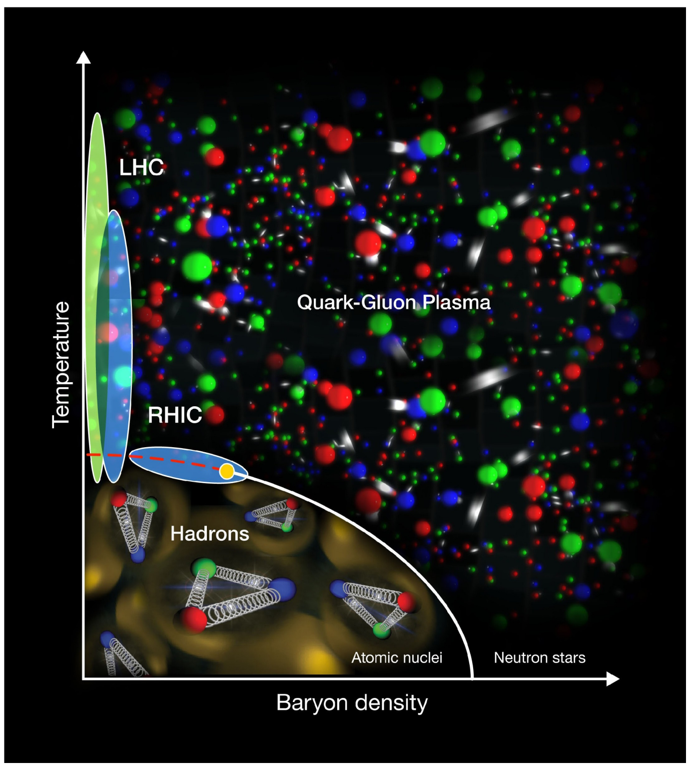


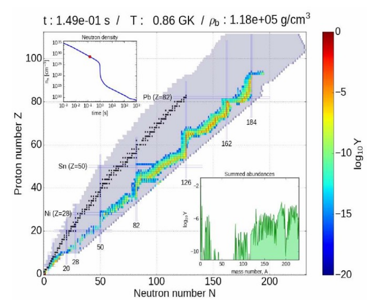
Viz by T.Dietrich

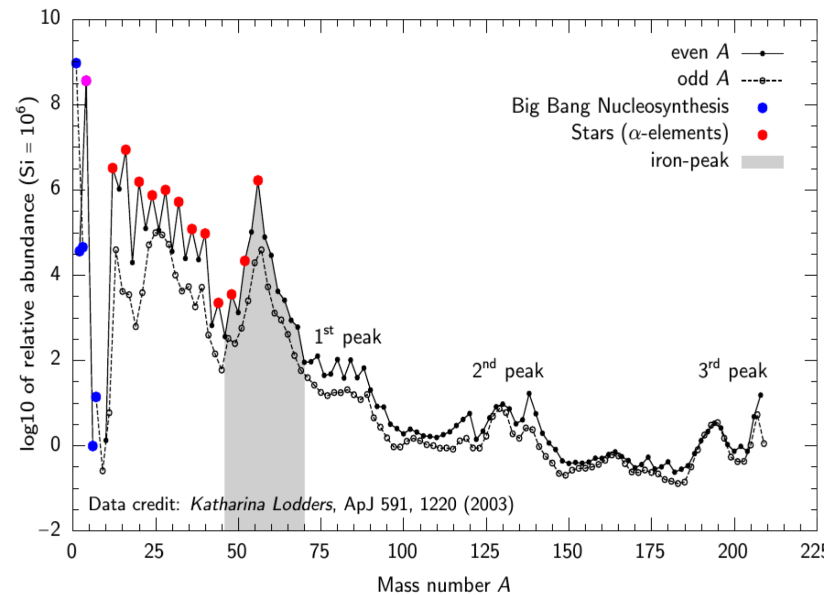
Weyl curvature scalar

What can we learn from neutron star mergers?

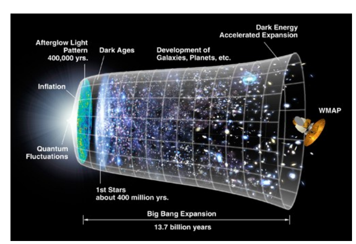
FUNDAMENTAL PHYSICS
Strong-field tests GR (dynamics)
Structure of bulk matter at supranuclear densities
Heavy elements nucleosynthesis

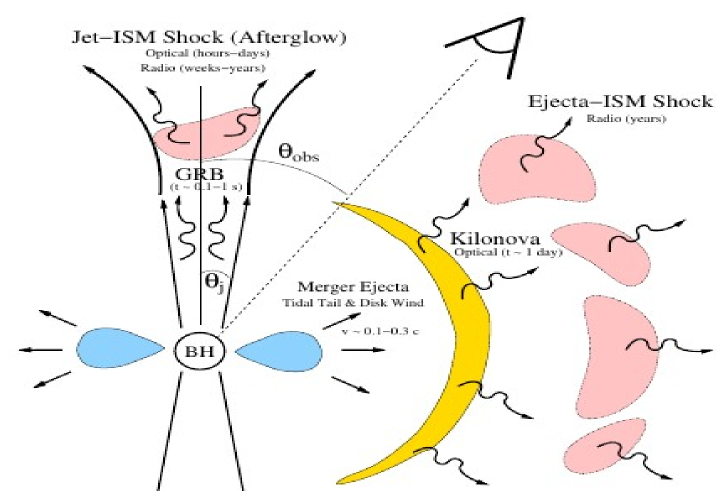






ASTROPHYSICS (Multi-messenger)
Origin of gamma-ray burst
Origin of kilonovae, site for r-processes

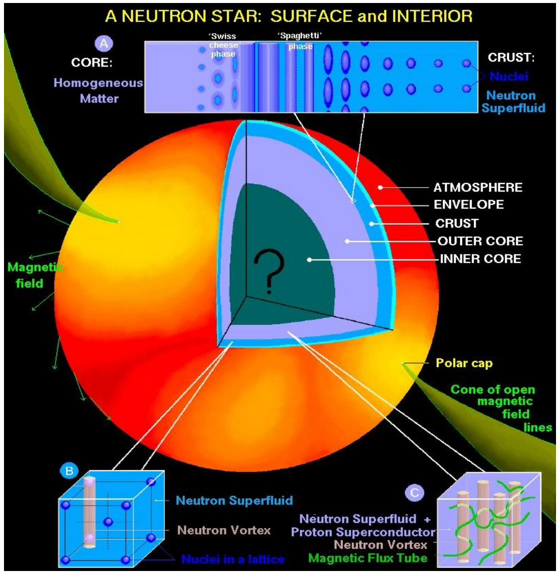




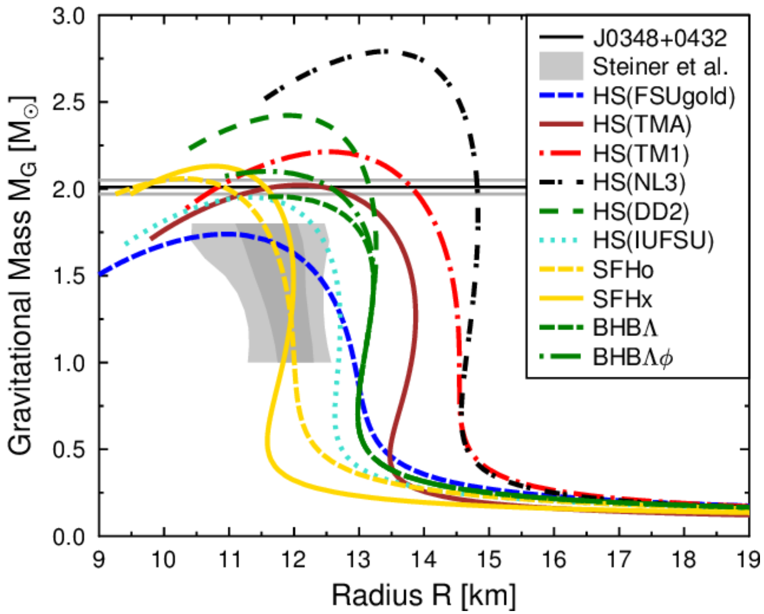
COSMOGRAPHY
Measure Hubble constant
Standard sirens, Calibrate cosmic distance ladder

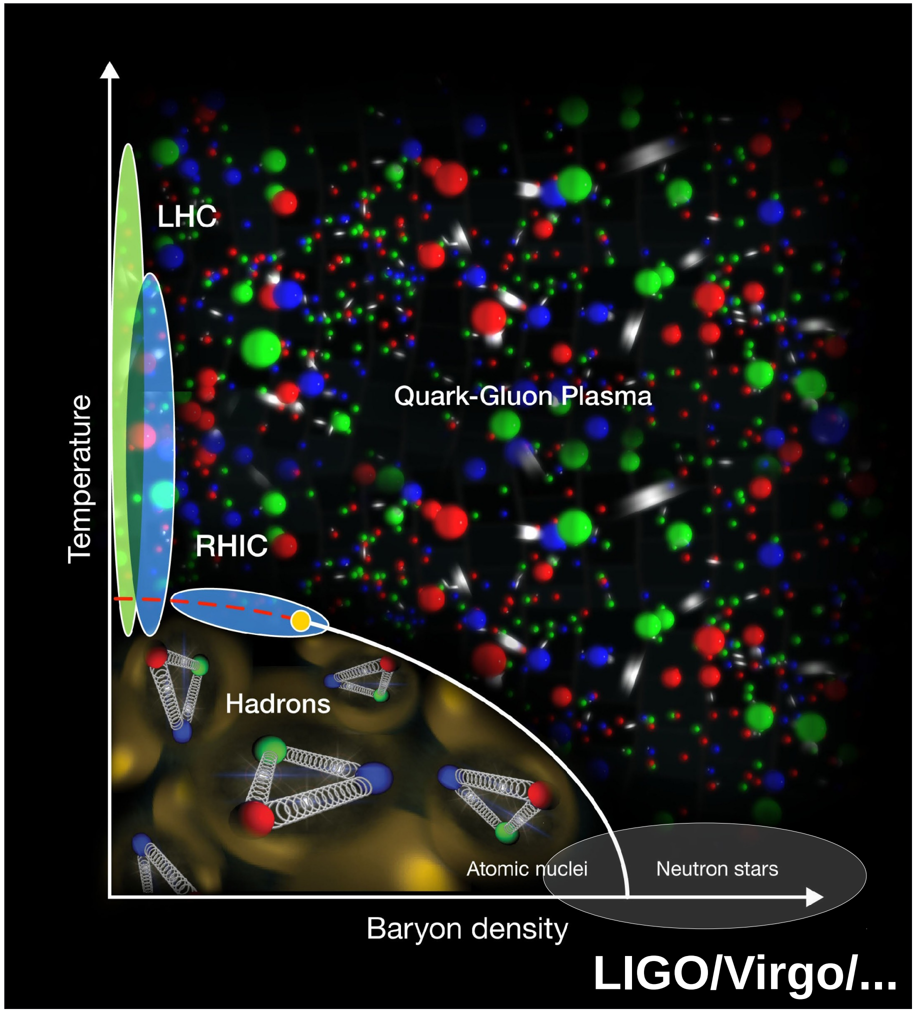
Fundamental physics

Constraining the Equation of State of matter at supranuclear densities



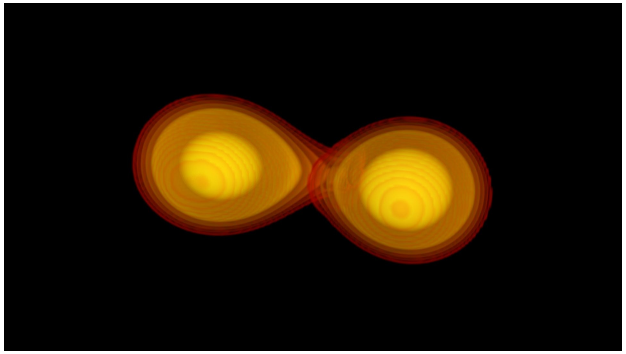
Different EOS → different star's structure

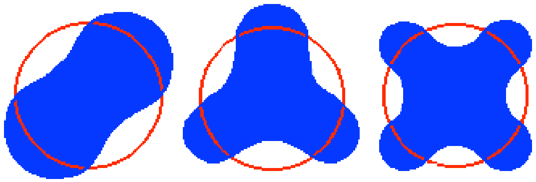




Binary neutron star mergers

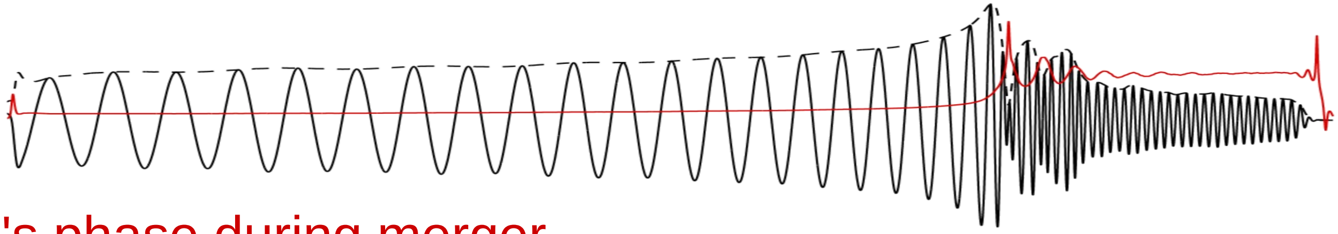
Example: observing tidal effects in GWs tells us about the neutron star matter





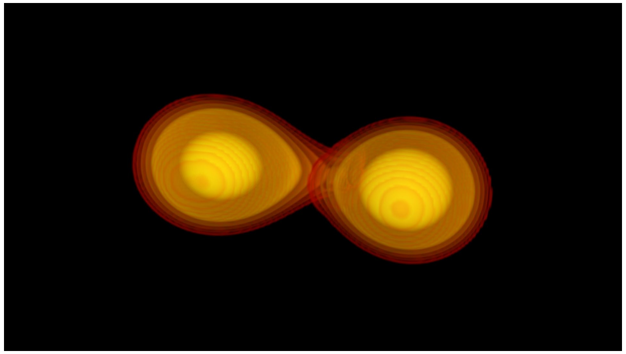
Tides depend critically on EOS

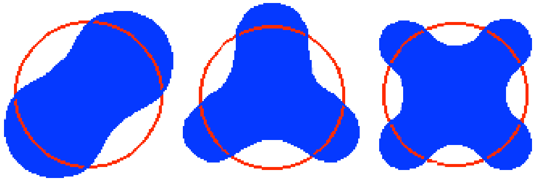
$$Q_{ij} = \lambda_2 G_{ij} \sim \lambda_2 \partial_i \partial_j \phi$$



Tides determine the wave's phase during merger

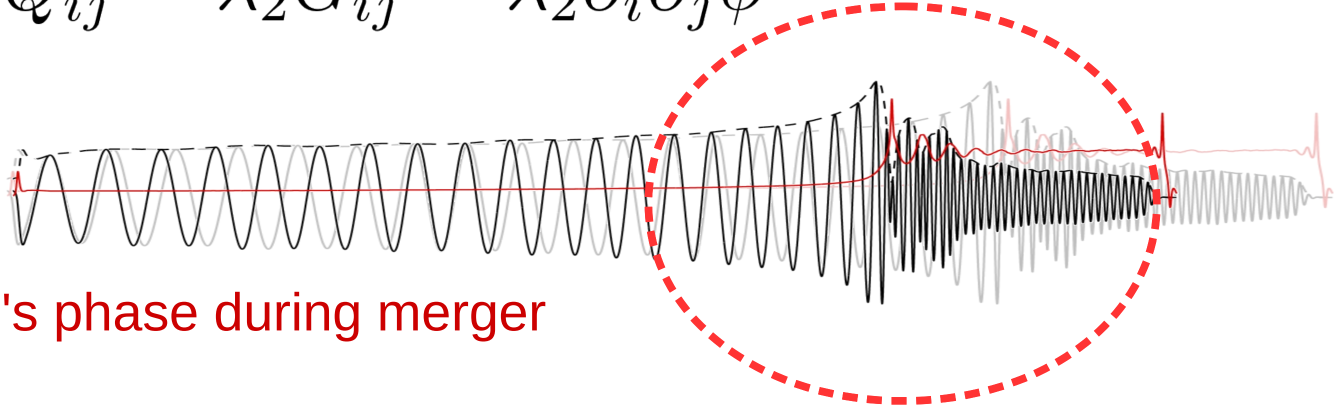
Example: observing tidal effects in GWs tells us about the neutron star matter





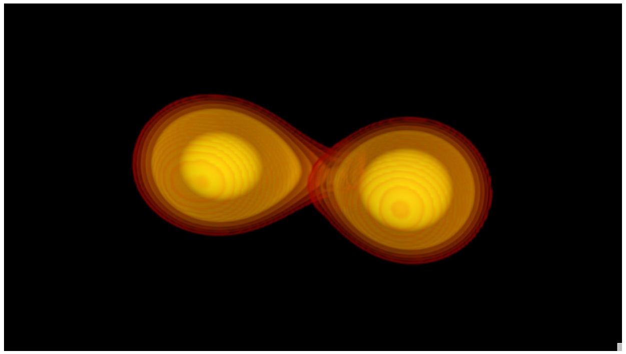
Tides depend crucially on EOS

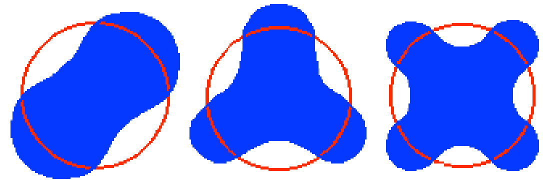
$$Q_{ij} = \lambda_2 G_{ij} \sim \lambda_2 \partial_i \partial_j \phi$$



Tides determine the wave's phase during merger

Example: observing tidal effects in GWs tells us about the neutron star matter





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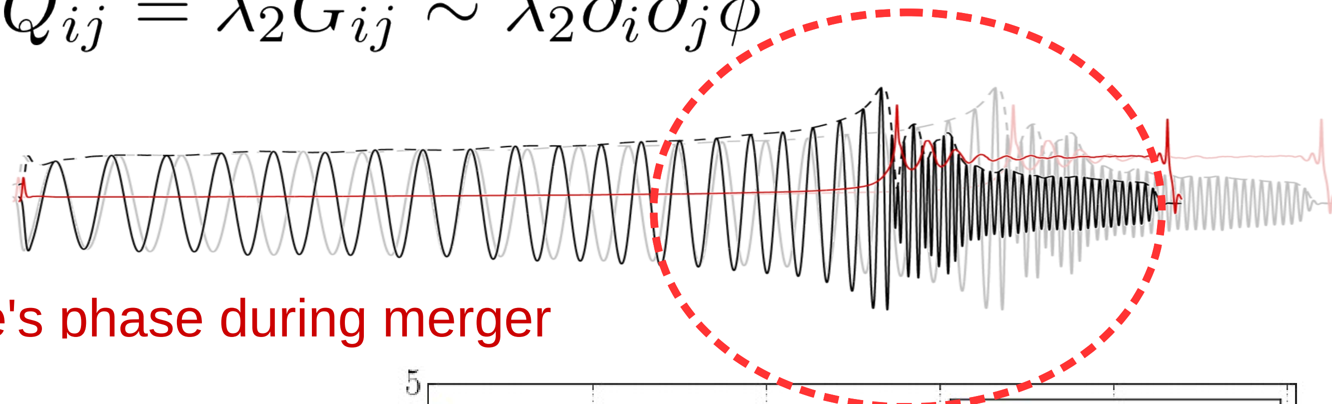
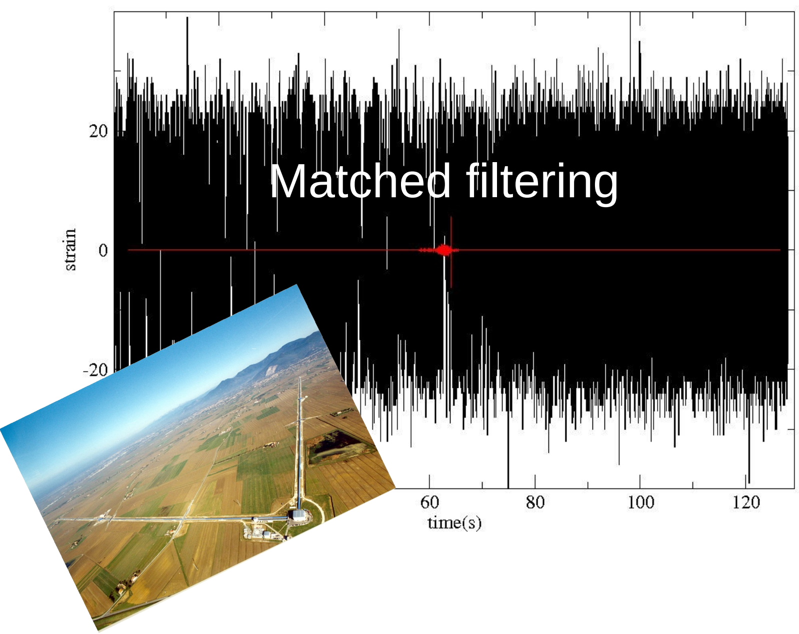
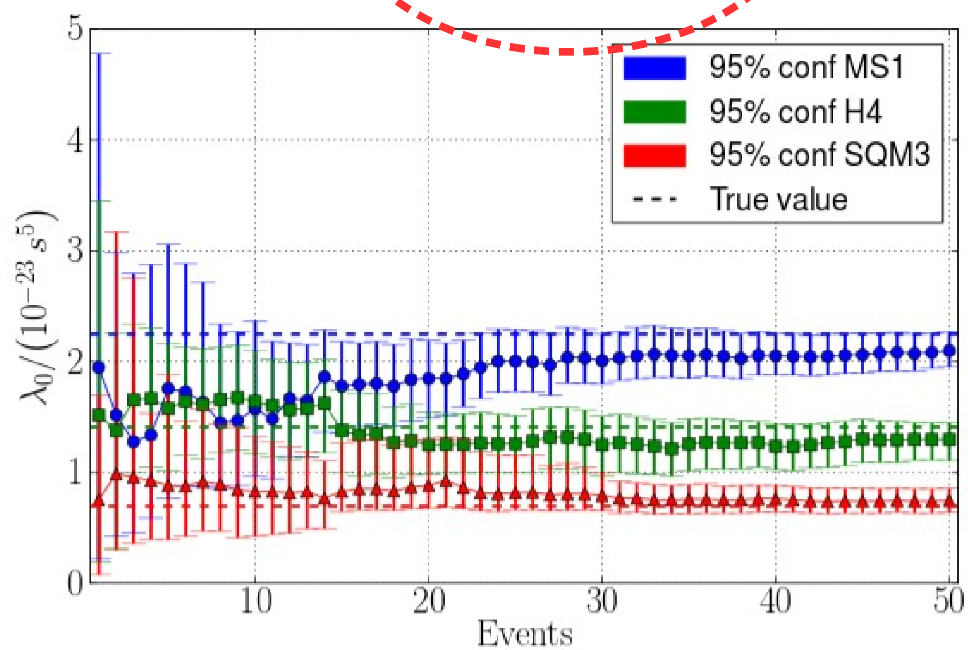


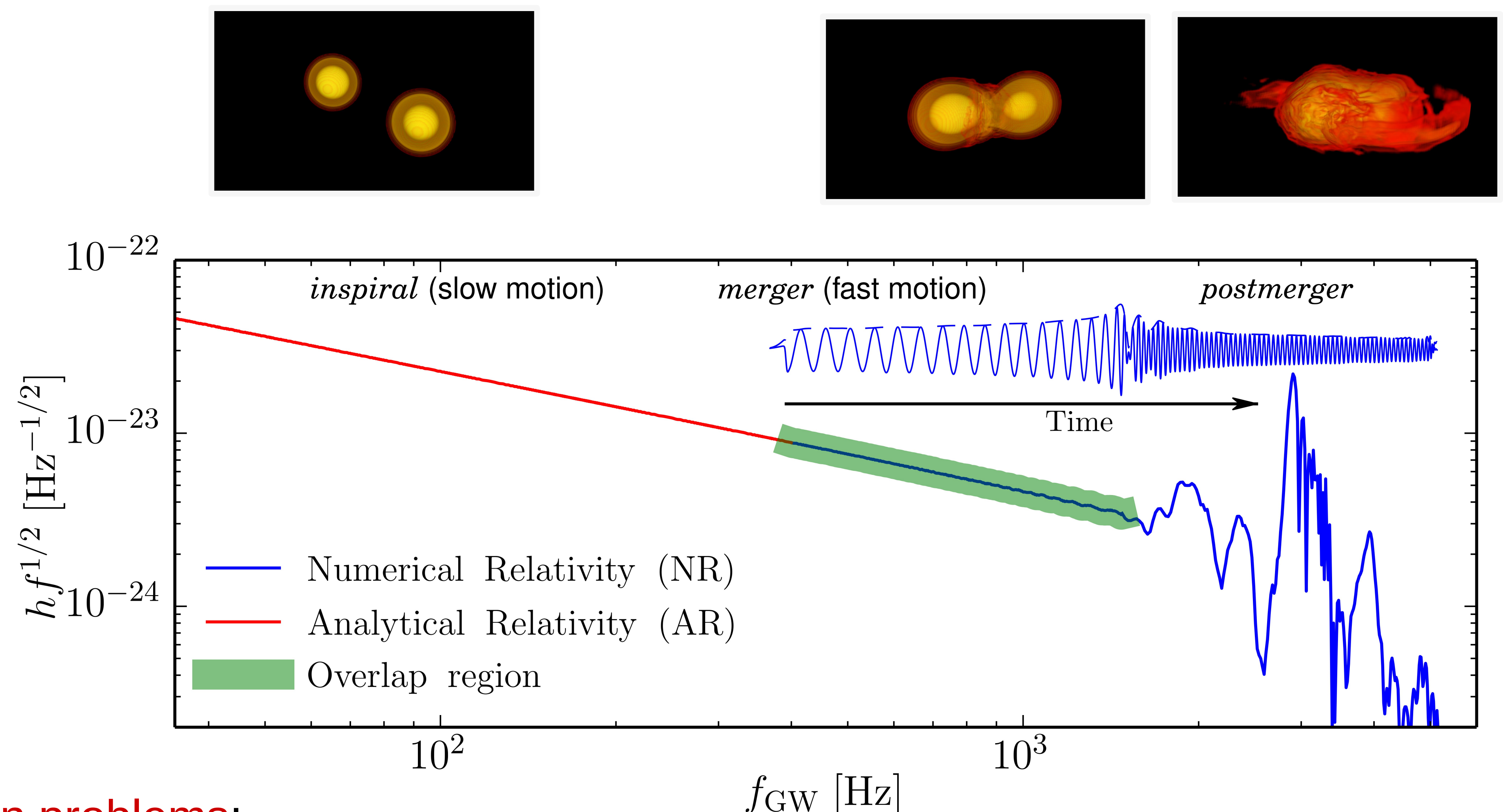
Tides determine the wave's phase during merger





[Del Pozzo+ PRL 111 (2013)]

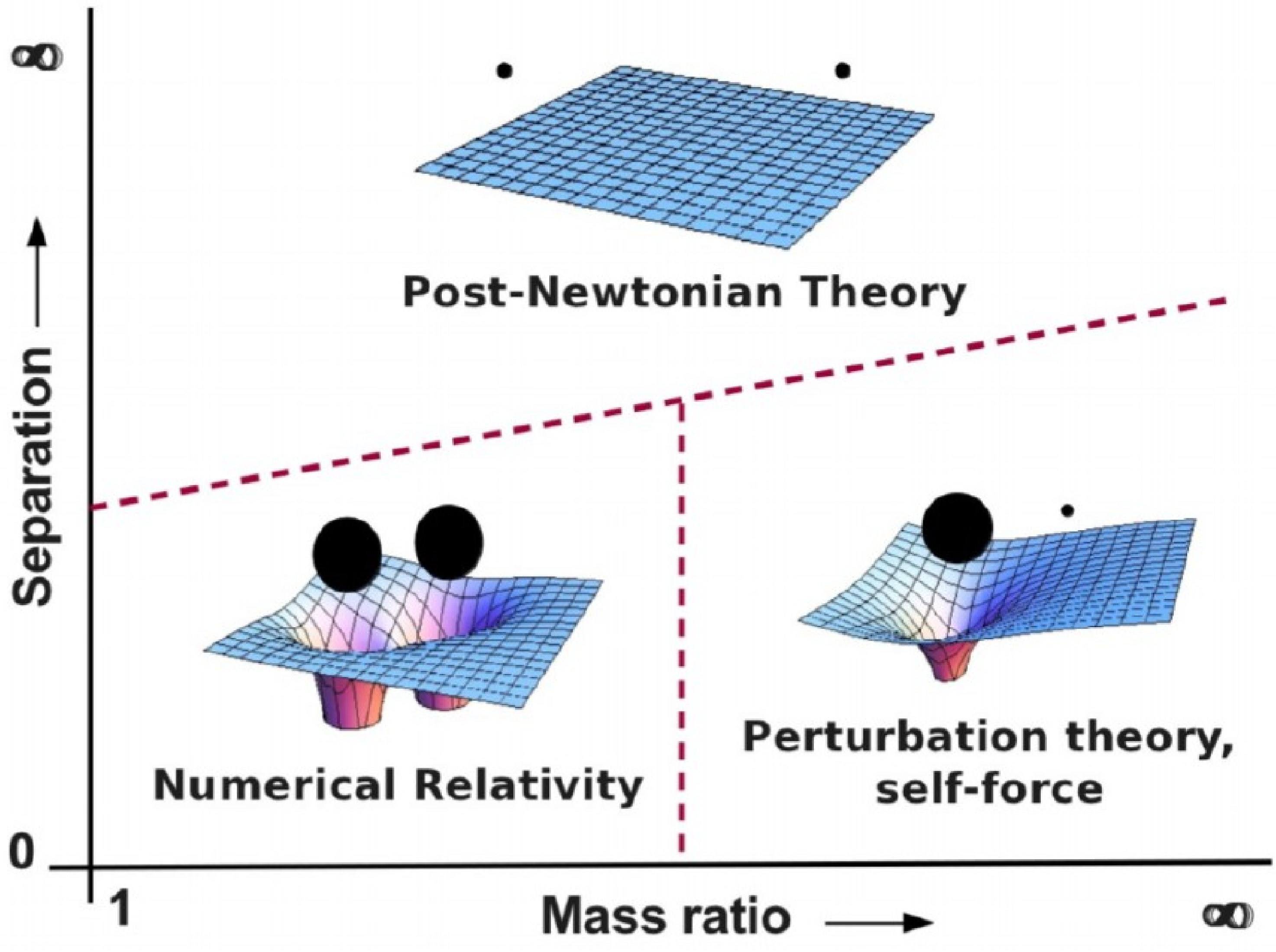
The GW spectrum of binary neutron stars



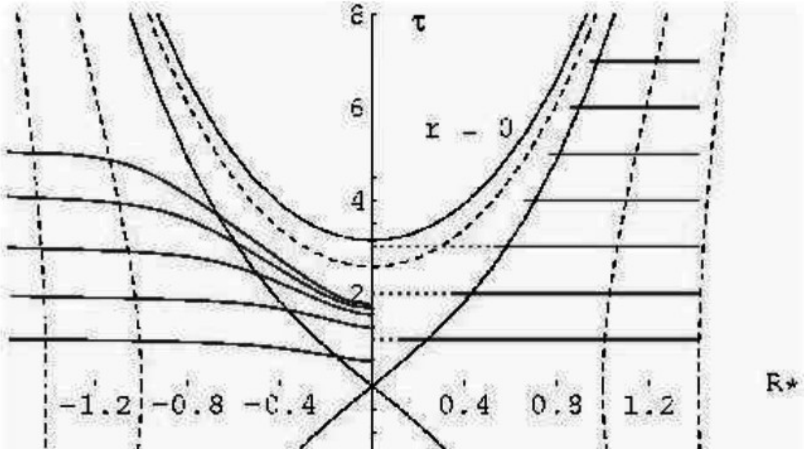
Open problems:

- Faithful and **complete waveform model** (*inspiral+merger+postmerger*)
- Coverage of the **parameter space** (mass, spins, EOS, …)
- Precise prediction of the merger remnant (e.g. collapse, black hole)

Methods for the GR 2-body problem



$$
\begin{aligned}
\partial_t \tilde{\Gamma}^i &= -2\,\tilde{A}^{ij}\,\partial_j \alpha + 2\,\alpha \left[\tilde{\Gamma}^i{}_{jk}\,\tilde{A}^{jk} - \frac{3}{2}\,\tilde{A}^{ij}\,\partial_j \ln(\chi) \right.\\
&\quad \left. - \frac{1}{3}\,\tilde{\gamma}^{ij}\,\partial_j (2\,\hat{K} + \Theta) - 8\,\pi\,\tilde{\gamma}^{ij}\,S_j \right] + \tilde{\gamma}^{jk}\,\partial_j \partial_k \beta^i \\
&\quad + \frac{1}{3}\,\tilde{\gamma}^{ij}\partial_j \partial_k \beta^k + \beta^j\,\partial_j \tilde{\Gamma}^i - (\tilde{\Gamma}_{\rm d})^j\,\partial_j \beta^i \\
&\quad + \frac{2}{3}\,(\tilde{\Gamma}_{\rm d})^i\,\partial_j \beta^j - 2\,\alpha\,\kappa_1\,[\tilde{\Gamma}^i - (\tilde{\Gamma}_{\rm d})^i]\,, \\
\partial_t \Theta &= \frac{1}{2}\,\alpha\,[R - \tilde{A}_{ij}\,\tilde{A}^{ij} + \frac{2}{3}\,(\hat{K} + 2\,\Theta)^2] \\
&\quad - \alpha\,[8\,\pi\,\rho + \kappa_1\,(2 + \kappa_2)\,\Theta] + \beta^i \partial_i \Theta\,,
\end{aligned}
$$



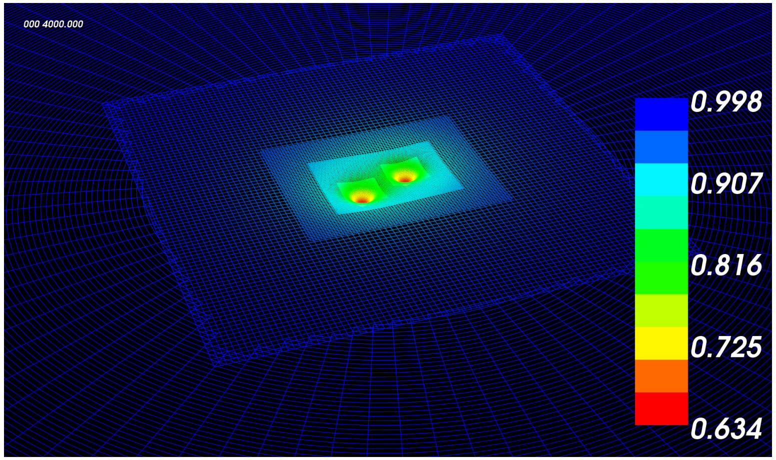
GR Formulation and Cauchy problem
+ GR hydrodynamics

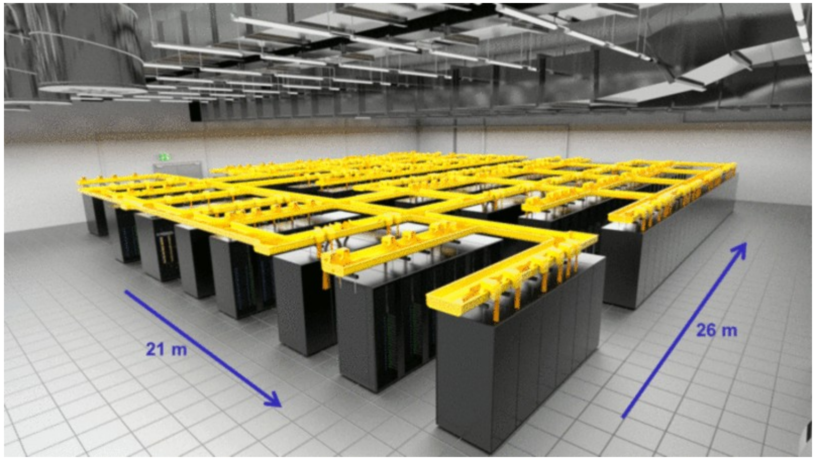
Coordinates and Singularities

Numerical relativity in a nutshell

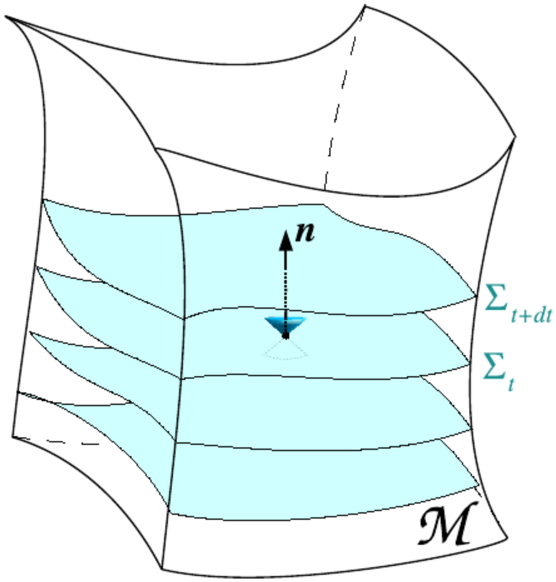
Numerical methods for PDEs on adaptive grids

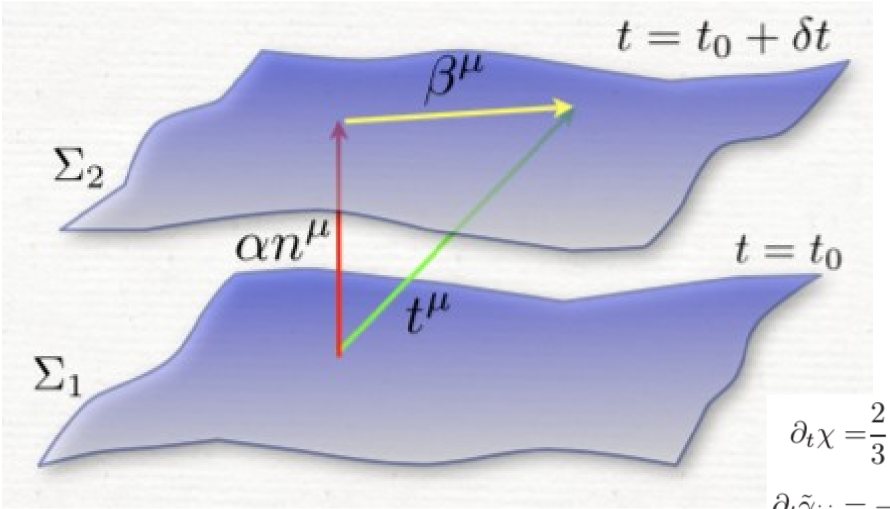
High-performance-computing (HPC)





Numerical relativity: Cauchy problem in GR



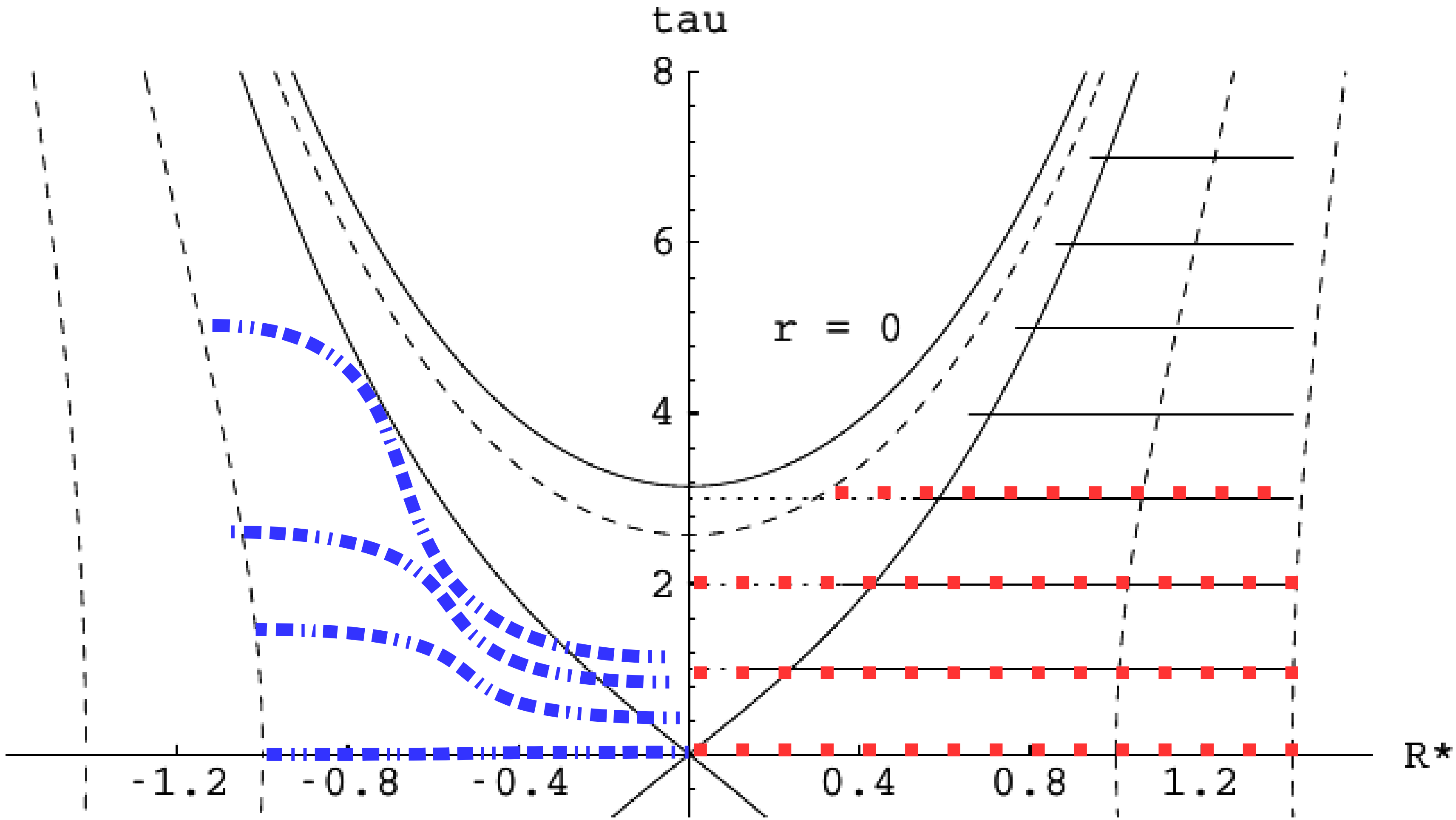


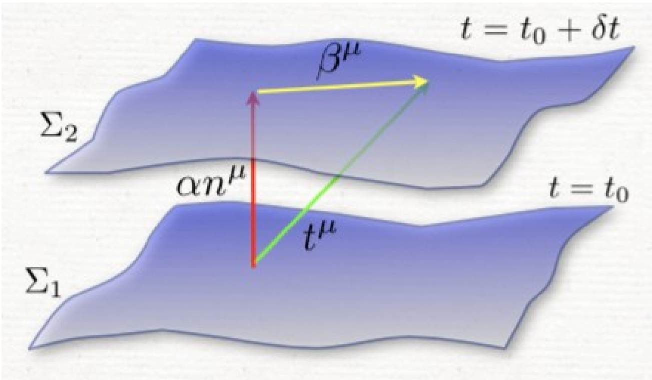
$$
\begin{aligned}
\partial_t\chi =& \frac{2}{3}\chi[\alpha(\hat{K}+2\Theta)-D_i\beta^i],\\
\partial_t\tilde{\gamma}_{ij} =& -2\alpha\tilde{A}_{ij}+\beta^k\tilde{\gamma}_{ij,k}+2\tilde{\gamma}_{k(i}\beta^k_{,j)}-\frac{2}{3}\tilde{\gamma}_{ij}\beta^k_{,k}\ ,\\
\partial_t\hat{K} =& -D^iD_i\alpha+\alpha[\tilde{A}_{ij}\tilde{A}^{ij}+\frac{1}{3}(\hat{K}+2\Theta)^2]\\
&+4\pi\alpha[S+\rho_{\mathrm{ADM}}]+\alpha\kappa_1(1-\kappa_2)\Theta+\beta^i\hat{K}_{,i}\\
\partial_t\tilde{A}_{ij} =& \chi[-D_iD_j\alpha+\alpha(R_{ij}-8\pi S_{ij})]^{\mathrm{tf}}\\
&+\alpha[(\hat{K}+2\Theta)\tilde{A}_{ij}-2\tilde{A}^k{}_i\tilde{A}_{kj}]\\
&+\beta^k\tilde{A}_{ij,k}+2\tilde{A}_{k(i}\beta^k_{,j)}-\frac{2}{3}\tilde{A}_{ij}\beta^k{}_{,k}\\
\partial_t\tilde{\Gamma}^i =& -2\tilde{A}^{ij}\alpha_{,j}+2\alpha[\tilde{\Gamma}^i_{jk}\tilde{A}^{jk}-\frac{3}{2}\tilde{A}^{ij}\ln(\chi)_{,j}\\
&-\frac{1}{3}\tilde{\gamma}^{ij}(2\hat{K}+\Theta)_{,j}-8\pi\tilde{\gamma}^{ij}S_j]+\tilde{\gamma}^{jk}\beta^i_{,jk}\\
&+\frac{1}{3}\tilde{\gamma}^{ij}\beta^k_{,kj}+\beta^j\tilde{\Gamma}^i_{,j}-\tilde{\Gamma}_{\mathrm{d}}{}^j\beta^i_{,j}+\frac{2}{3}\tilde{\Gamma}_{\mathrm{d}}{}^i\beta^j_{,j}\\
&-2\alpha\kappa_1(\tilde{\Gamma}^i-\tilde{\Gamma}_{\mathrm{d}}{}^i),\\
\partial_t\Theta =& \alpha[\frac{1}{2}R-\frac{1}{2}\tilde{A}_{ij}\tilde{A}^{ij}+\frac{1}{3}(\hat{K}+2\Theta)^2\\
&-8\pi\rho_{\mathrm{ADM}}-\kappa_1(2+\kappa_2)\Theta]+\mathcal{L}_\beta\Theta.
\end{aligned}
$$

- 3+1 formulation (hyperboloidal slices?)
- Initial data (Lichnerowic, York, ...)
- Evolution schemes (GHG; ADM →BSSN, Z4c)
- Well posedness (Choquet-Bruhat; Friedrich; …. Gundlach&Martin-Garcia) [*need gauge fix*]

[SB, Hilditch arXiv:0912.2920]

Numerical relativity: singularities & crash tests

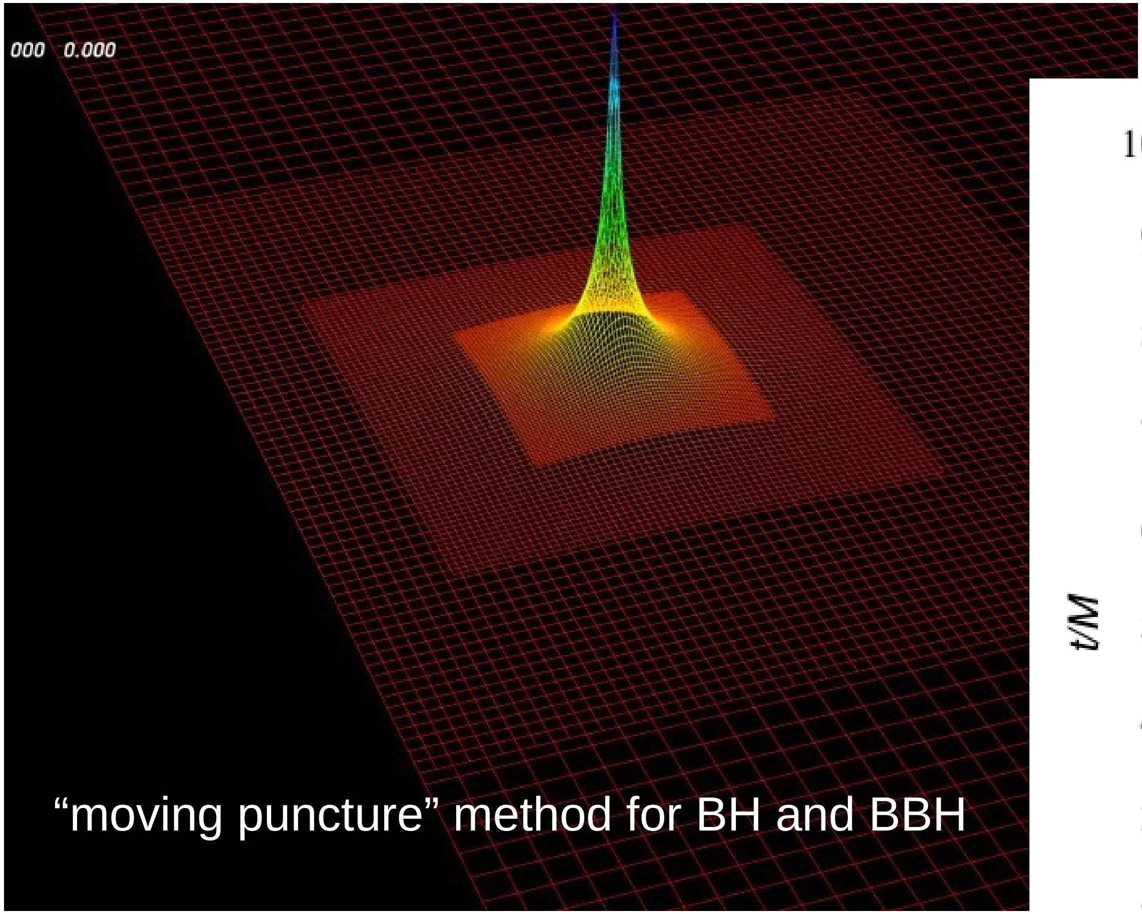




- Crash at tau=pi (geodesic slicing)
- Lapse collapse, slice stretching (1+log, shift=0)

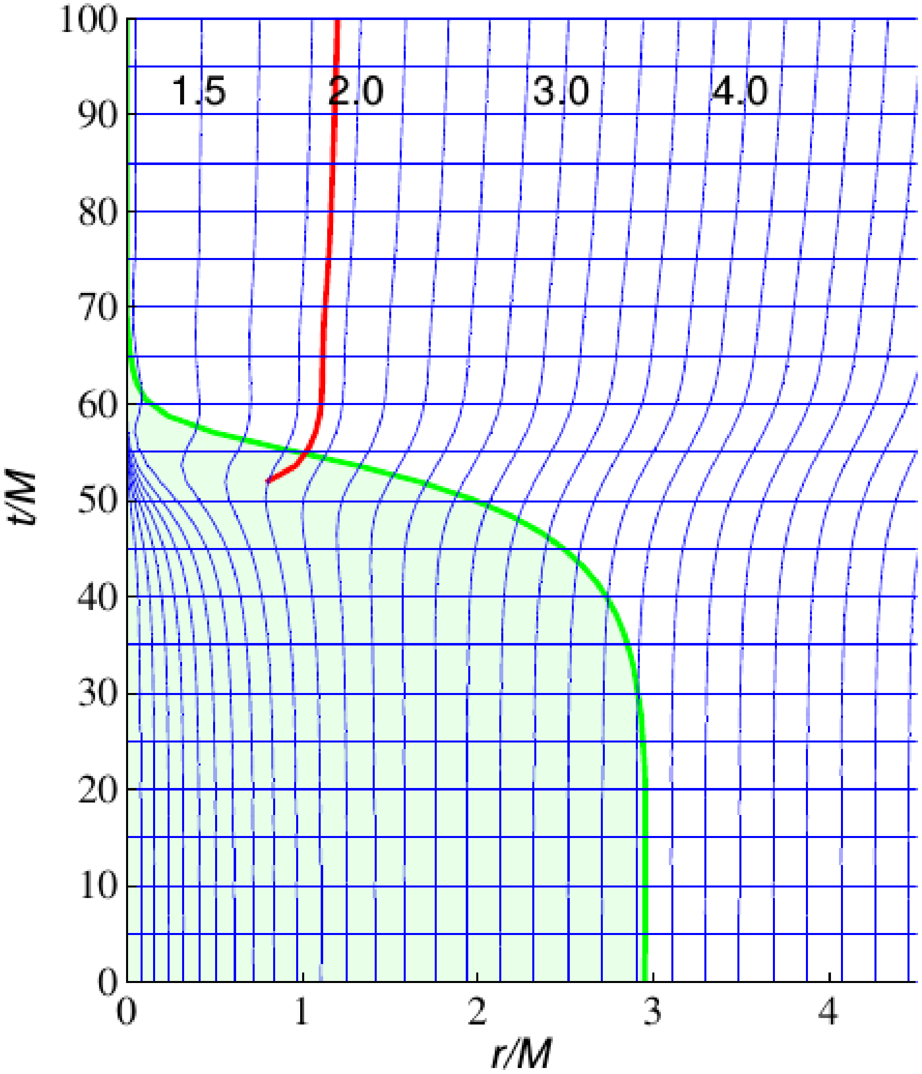
e.g. [Bruegmann arXiv:9912009]

Numerical relativity: singularities & coordinates



[Brandt&Bruegmann, arXiv:gr-qc/9703066,
Baker+ arXiv:gr-qc/0511103,
Campanelli+ arXiv:gr-qc/0511048]

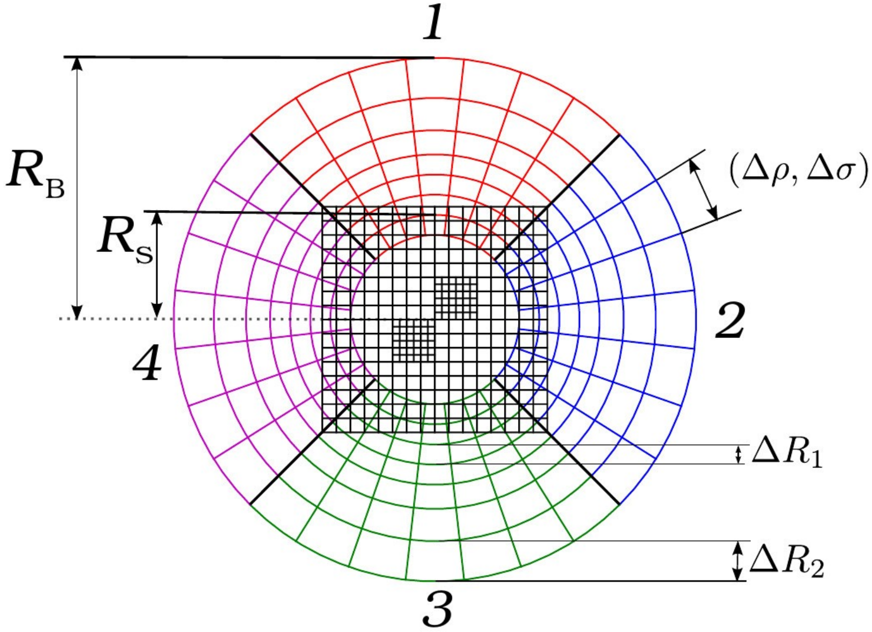
Gravitational collapse to BH

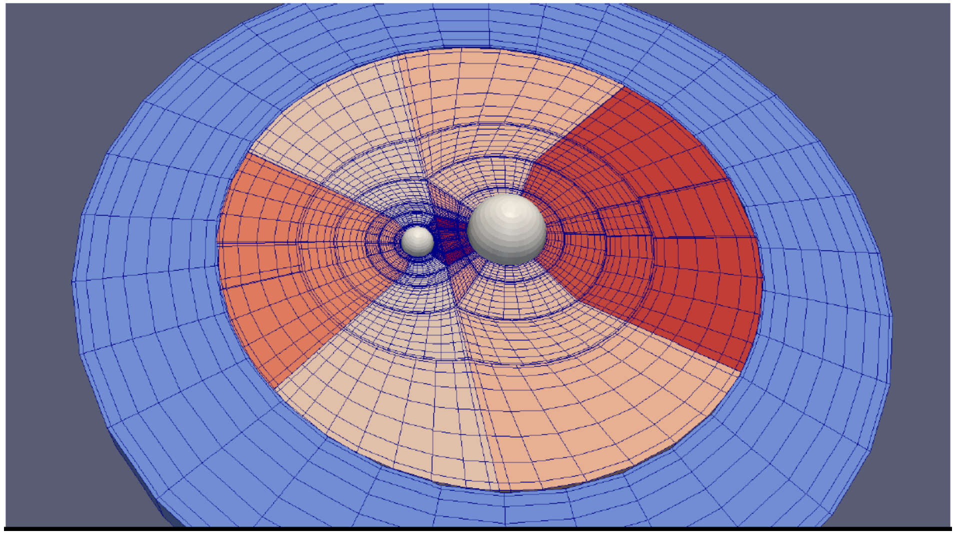


[Thierfelder, SB, Hilditch, Bruegmann, Rezzolla
arXiv:1012.3703]

Numerical relativity: numerical methods (some)

Adaptive mesh refinement (AMR) → resolve multiple scales



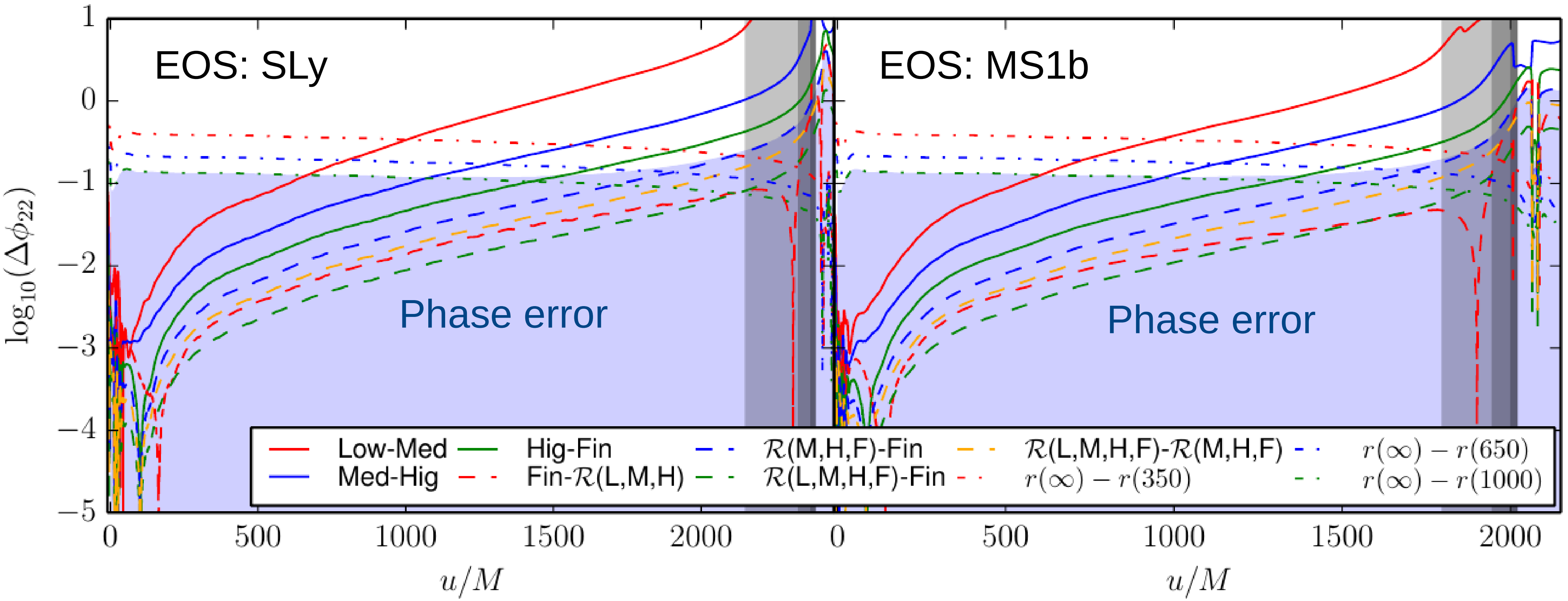


Grid based, AMR Berger-Oliger
Method of line w\ Runge-Kutta (Subcycling)
Finite differencing and finite volumes
Numerical relativity specs

- R.H.S. complexity (derivatives and contractions)
→ stencil (“horizontal”) + pointwise (“vertical”) ops
- High-order operators (large 3D stencils > 5 pts/direction)
→ communication overhead for distributed computations
- Memory: >~ O(100) 3D grid function per time level

Improved NR GW with high-order WENO schemes

[SB,Dietrich PRD94 064062 (2016)]

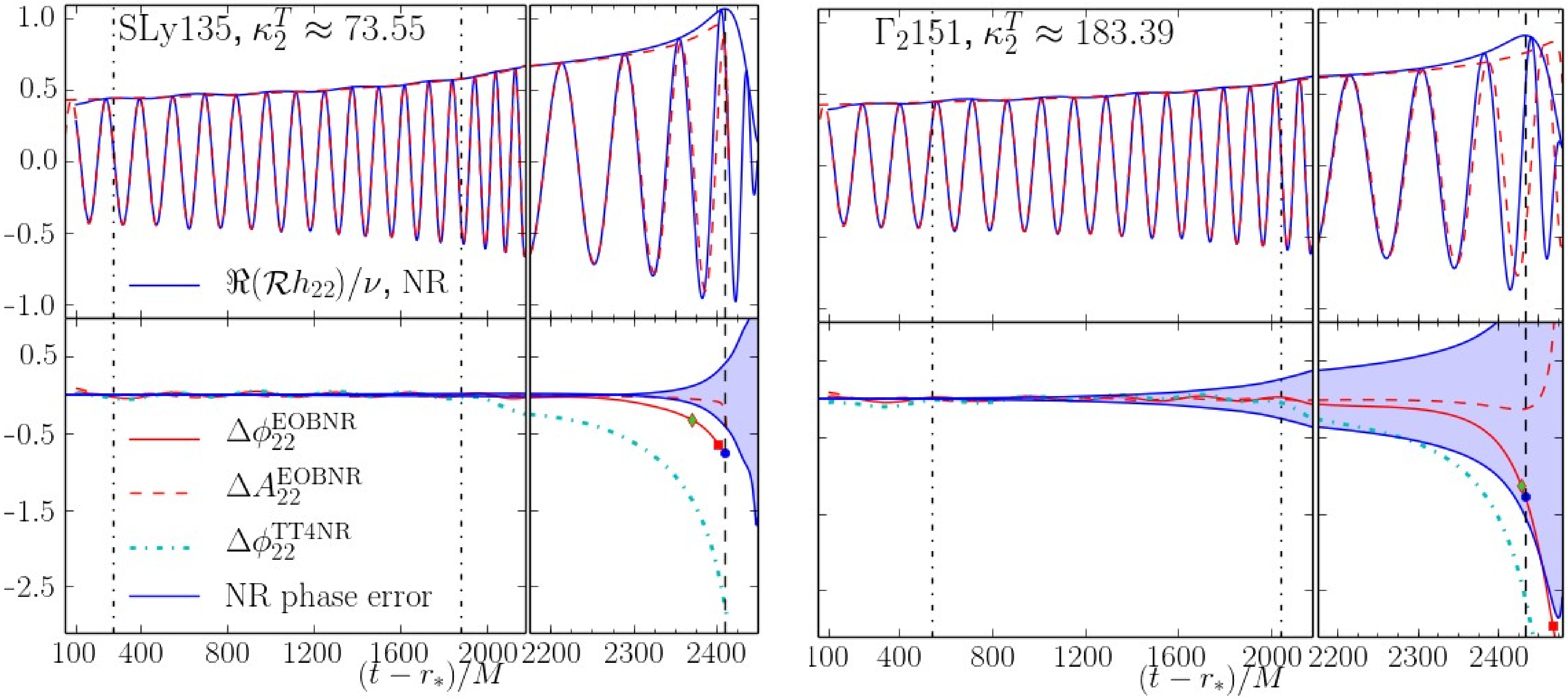


- Robust convergence assessment (although not 5th order)
- Large resolution span (64^3 -192^3), no alignment
- Error budget: significant improvement wrt FV schemes

See also [SB+ arXiv:1205.3403] [Radice+ arxiv:1306.6052]

First waveform model for inspiral → merger

[SB,Nagar,Dietrich,Damour PRL 114 (2015)]

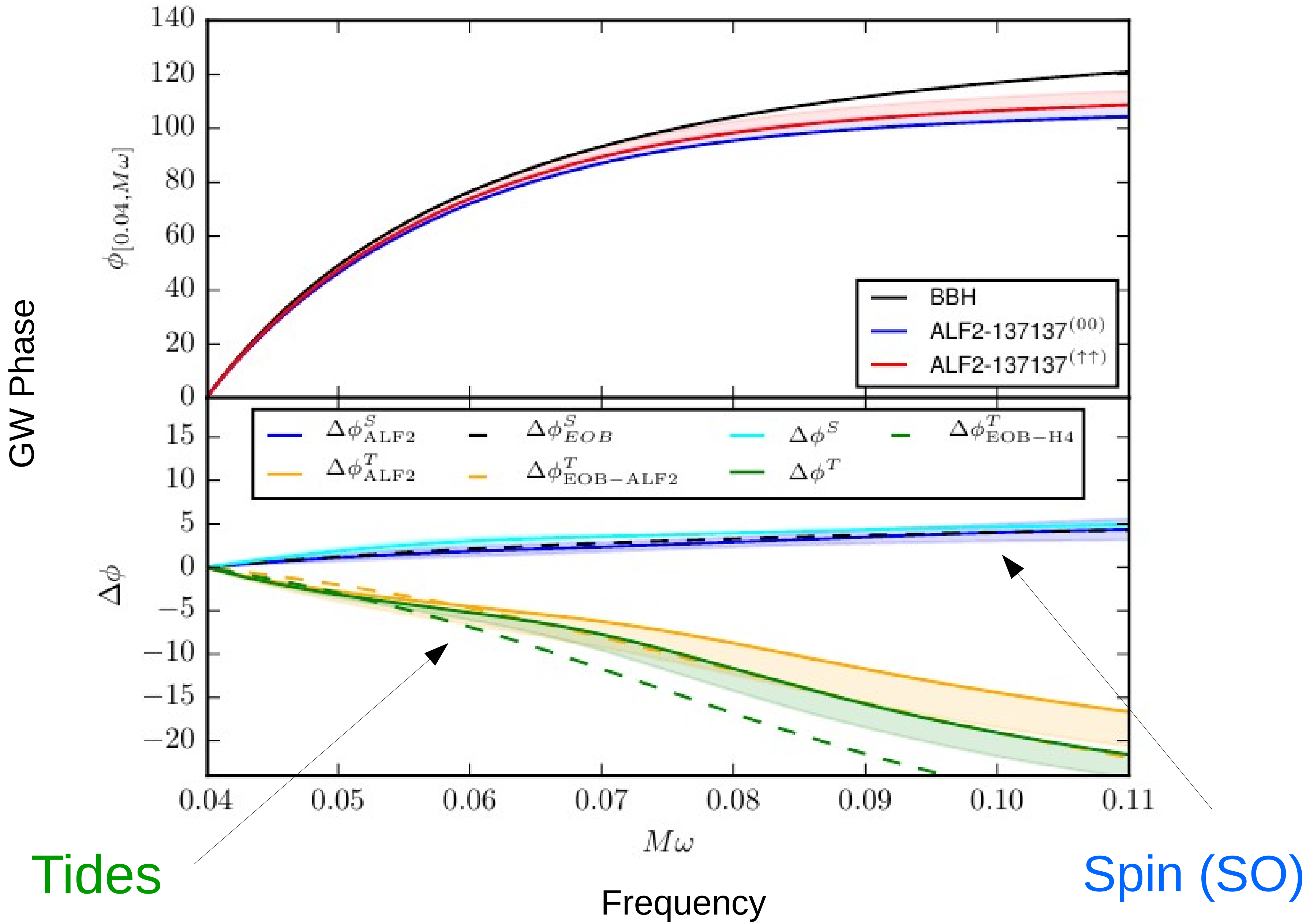


- Effective-one-body model with tides, GSF Resummed approach [Bini+ 2014]
- Valid from low frequencies to merger, PREDICT the merger waveform
- Accuracy: uncertainties of the numerical data (improve simulations!)

See [Hinderer+ PRL 116 (2016)] for an alternative approach

Spins & tides during merger: phasing

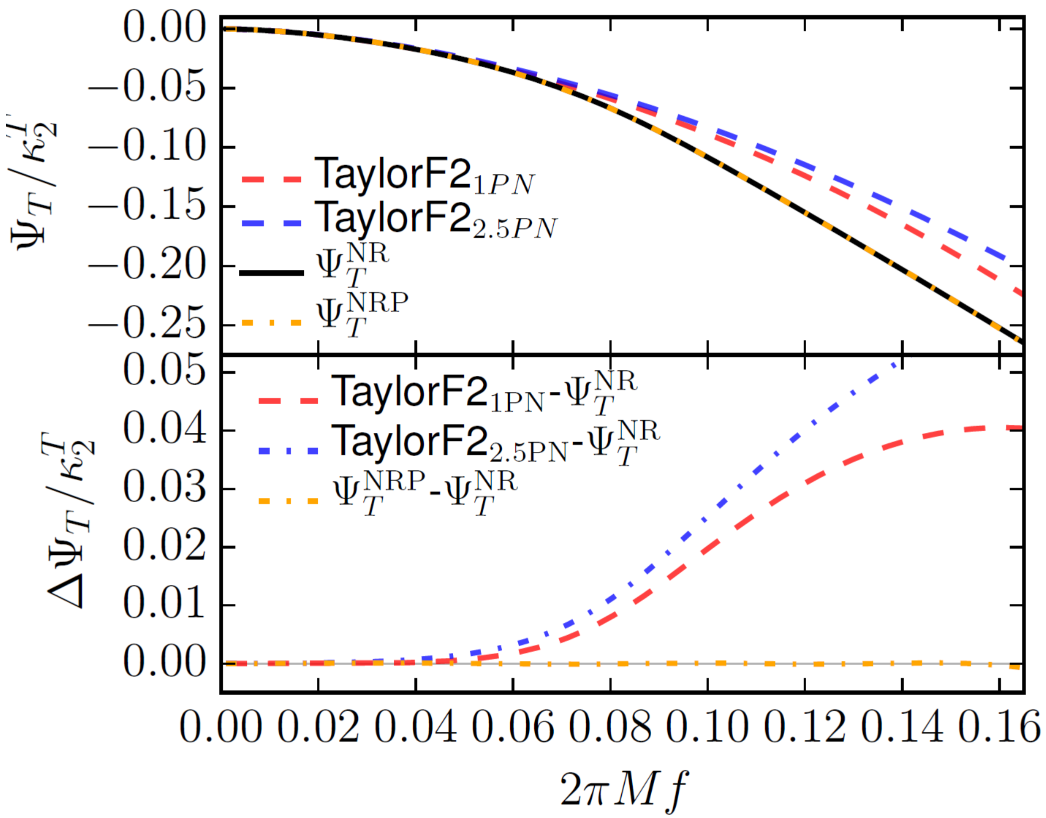
[Dietrich, SB, Ujevic, Tichy PRD 95 (2017)]

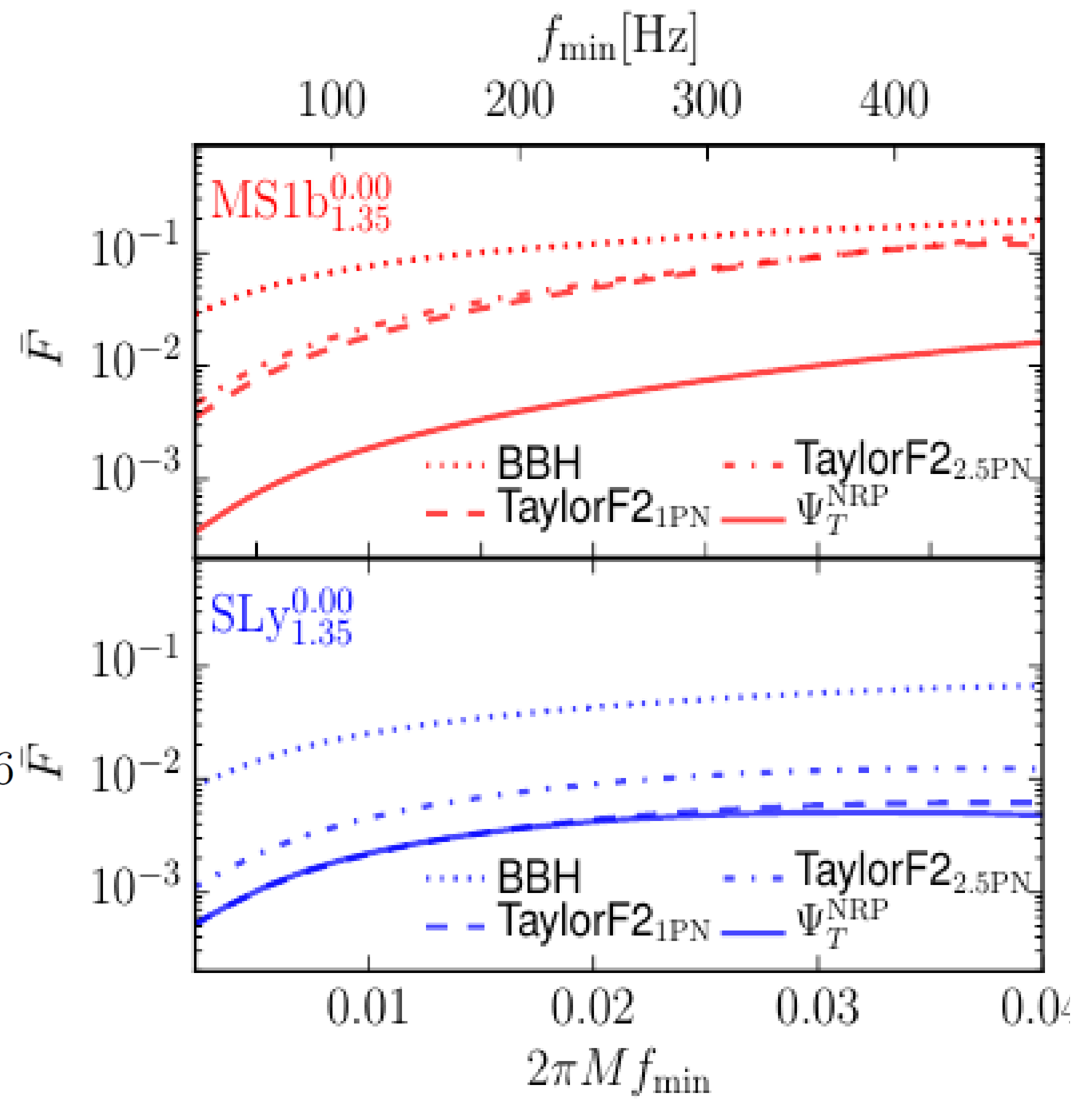


Frequency-domain tidal wf approximant

[Dietrich, SB, Tichy arXiv:1706.02969]

First NR-based tidal approximant
Fast, flexible, accurate

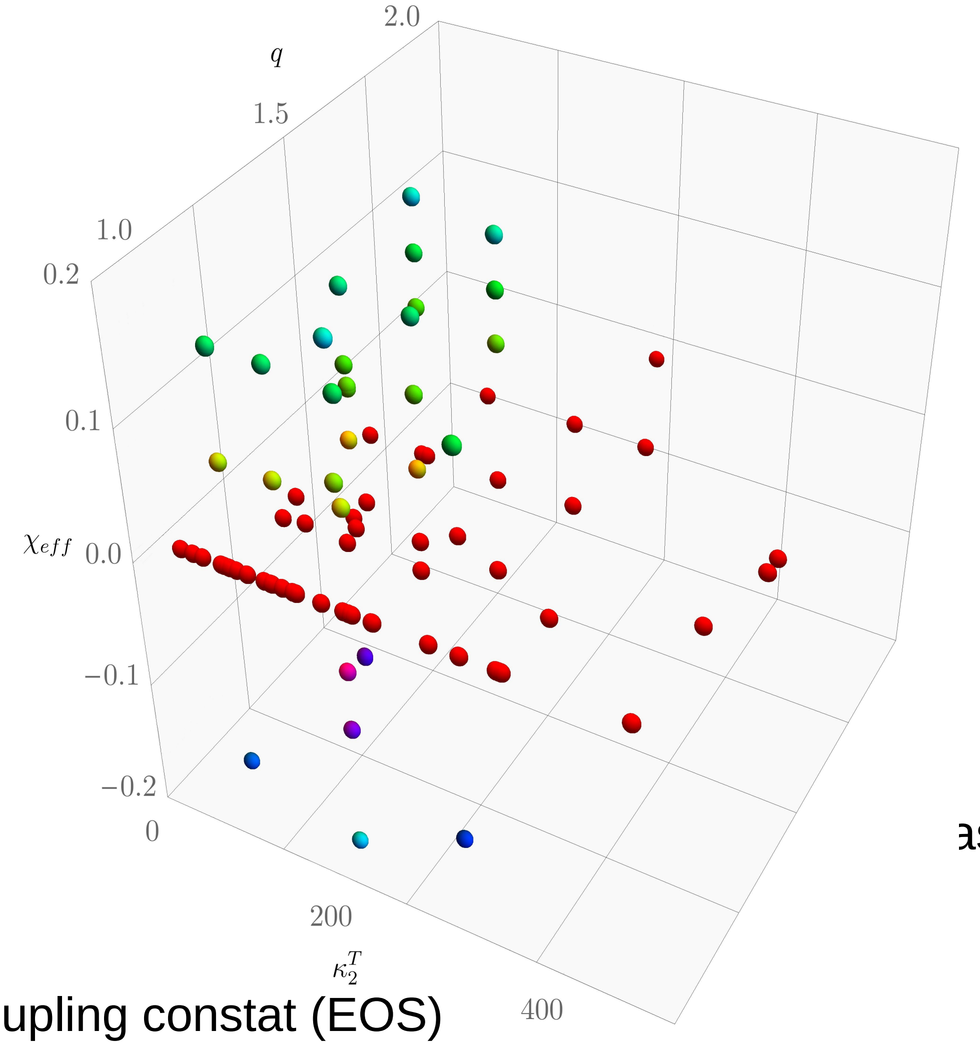




Used for GW170817 analysis!

Exploring the BNS parameter space

Largest exploration of parameter space in strong-field regime available to date



Effective Spin

ass ratio

Tidal coupling constat (EOS)

- 130 BNS
- 330 dataset (multiple resolutions)
- >= 10 orbits + post merger
- Variation of M, q, EOS, spins
- Variation of input physics

More data:
[Bernuzzi+ PRL (2015), Dietrich+ PRD91 (2015), SB+ PRD94 (2016), Radice+ PRD94 (2016), SB&Dietrich PRD94 (2016), Dietrich+ PRD95 024029 (2017), Radice+ ApJL 842 (2017), ….]

HPC time usage

Usage in 2015/2016

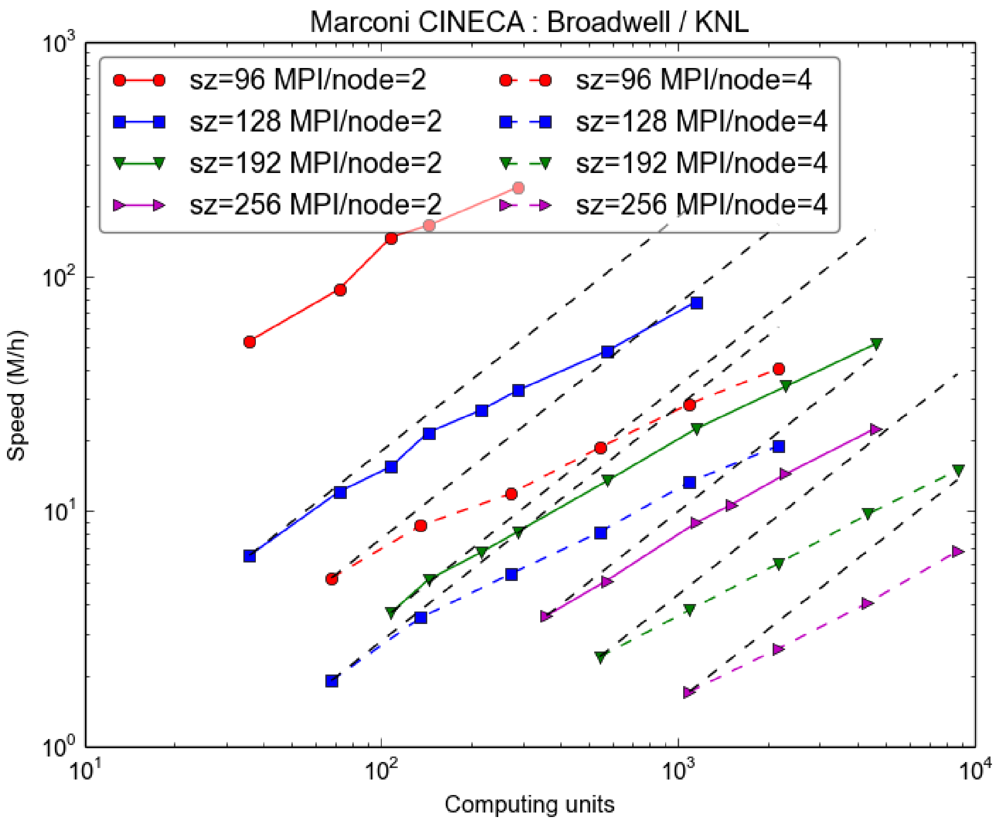
Production codes & parallelization

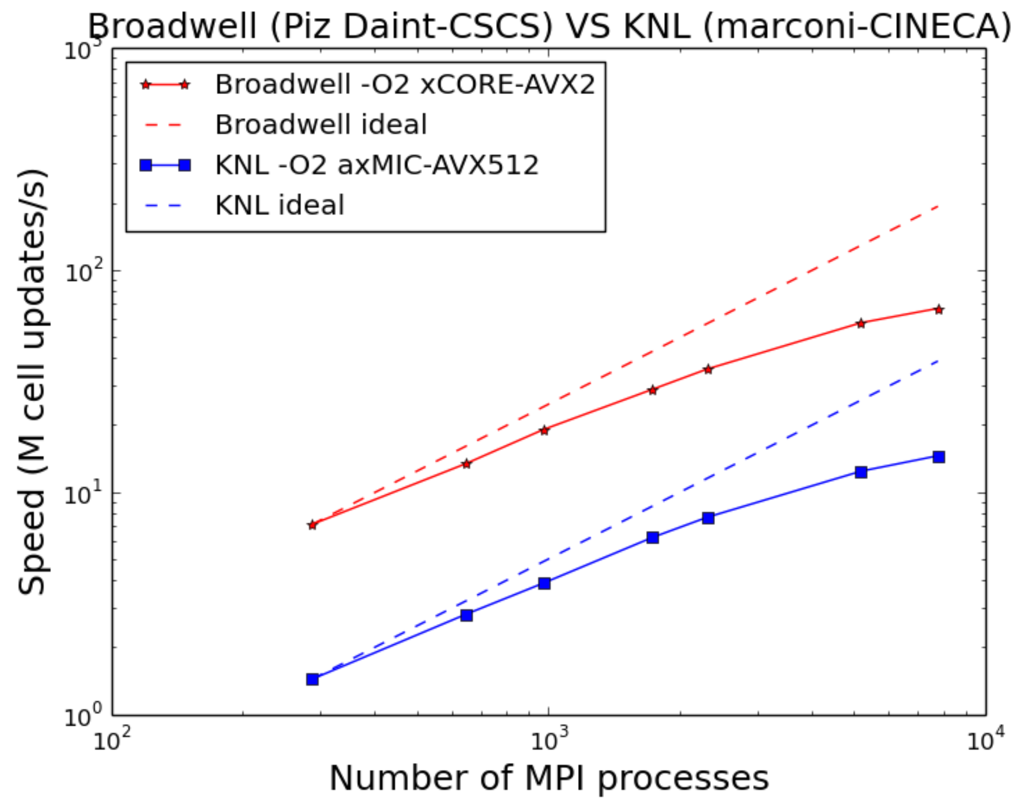
- BAM [Bruegmann (Jena) + Tichy (Florida Atlantic) + Bernuzzi (Parma) and others]
- THC [Radice (Princeton), based on Cactus & ET/CTGamma]

- MPI: domain decomposition for each level
- OMP threads on MPI job
- None or poor vectorization
- Numerical relativity specs:
  - R.H.S. complexity (derivatives and contractions) → stencil ("horizontal") + pointwise ("vertical") ops
  - High-order operators (large 3D stencils > 5 pts/direction) → communication overhead for distributed computations
  - Memory: >~ O(100) 3D grid function per time level

No NR production code scales to >~ 10k cores

Porting to KNL: first strong scaling results





BAM (MPI+OMP)
Numerical relativity,
Compact binaries simulations

FISH-ASL (MPI)
Newtonian gravity + hydro
Supernova core-collapse
and disk winds

NOTE: no attempt to optimize code, just re-compile and run

Linearized Einstein

The spacetime metric reads

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

where $g_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ is a small perturbation.

We adopt the following gauge

$$g_{00} = 1, \quad g_{0i} = 0,$$

so that the linearized Einstein equations read

$$\partial_t^2 h_{ij} = \eta^{kl} \partial_k \partial_l h_{ij}.$$

The equations are solved in first order in time form

$$\begin{aligned} \partial_t h_{ij} &= -2K_{ij} \\ R_{ij} &= -\frac{1}{2}\eta^{kl}\partial_k \partial_l h_{ij}, \\ \partial_t K_{ij} &= R_{ij}. \end{aligned}$$

Tensor wave equations

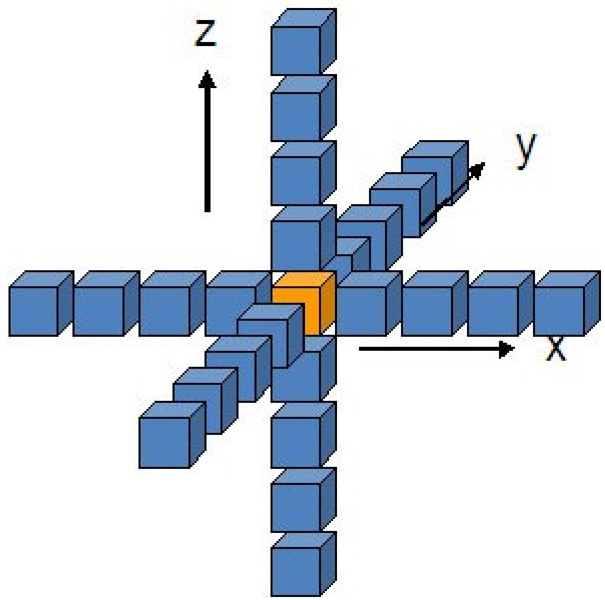
Operations

$$R_{ij} = -\frac{1}{2}\eta^{kl}\partial_k\partial_l h_{ij}$$

Tensor contractions

```
R23
=
(-0.5*deldelg1123 + gamma113*gammado211 + gamma213*gammado212 +
     (gamma212 + gamma313)*gammado213 +
     gamma112*(gammado113 + gammado311) + gamma212*gammado312 +
     2.*gamma312*gammado313)*ginv11 +
  (-deldelg1223 + gamma123*gammado211 + (gamma113 + gamma223)*gammado212 +
     (gamma222 + gamma323)*gammado213 + gamma213*gammado222 +
     (gamma212 + gamma313)*gammado223 +
     gamma122*(gammado113 + gammado311) + gamma222*gammado312 +
     gamma112*(gammado123 + gammado312) + gamma212*gammado322 +
     2.*(gamma322*gammado313 + gamma312*gammado323))*ginv12 +
  (-deldelg1323 + gamma133*gammado211 + gamma233*gammado212 +
     (gamma113 + gamma223 + gamma333)*gammado213 + gamma213*gammado223 +
     (gamma212 + gamma313)*gammado233 +
     gamma123*(gammado113 + gammado311) + gamma223*gammado312 +
     gamma112*(gammado133 + gammado313) + gamma212*gammado323 +
     2.*(gamma323*gammado313 + gamma312*gammado333))*ginv13 +
  (-0.5*deldelg2223 + gamma123*gammado212 + gamma223*gammado222 +
     (gamma222 + gamma323)*gammado223 +
     gamma122*(gammado123 + gammado312) + gamma222*gammado322 +
     2.*gamma322*gammado323)*ginv22 +
  (-deldelg2323 + gamma133*gammado212 + gamma233*gammado222 +
     (2.*gamma223 + gamma333)*gammado223 +
     (gamma222 + gamma323)*gammado233 +
     gamma123*(gammado123 + gammado213 + gammado312) +
     gamma122*(gammado133 + gammado313) + gamma223*gammado322 +
     (gamma222 + 2.*gamma323)*gammado323 + 2.*gamma322*gammado333)*ginv23 +
  (-0.5*deldelg3323 + gamma133*gammado213 + gamma233*gammado223 +
     (gamma223 + gamma333)*gammado233 +
     gamma123*(gammado133 + gammado313) + gamma223*gammado323 +
     2.*gamma323*gammado333)*ginv33 +
  0.5*((gammado213 + gammado312)*Gfromg1 +
     (gammado223 + gammado322)*Gfromg2 + (gammado233 + gammado323)*Gfromg3 +
     delG31*g12[ijk] + delG21*g13[ijk] + delG32*g22[ijk] +
     (delG22 + delG33)*g23[ijk] + delG23*g33[ijk])
;
```

Partial derivatives (3D) *



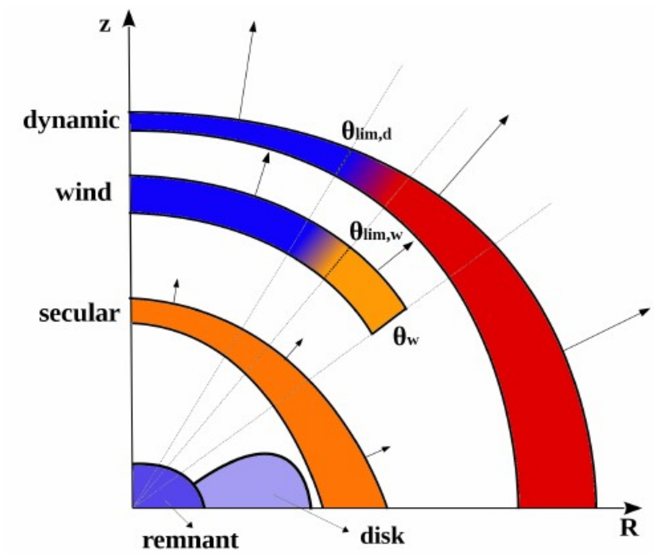

```
U[i,j,k] = C[0]*V[i,j,k];
for (r=1; r<=4; r++)
    U[i,j,k]+=C[r]*(V[i-r,j,k]+V[i,j-r,k]+V[i,j,k-r]+
                    V[i+r,j,k]+V[i,j+r,k]+V[i,j,k+r]);
```

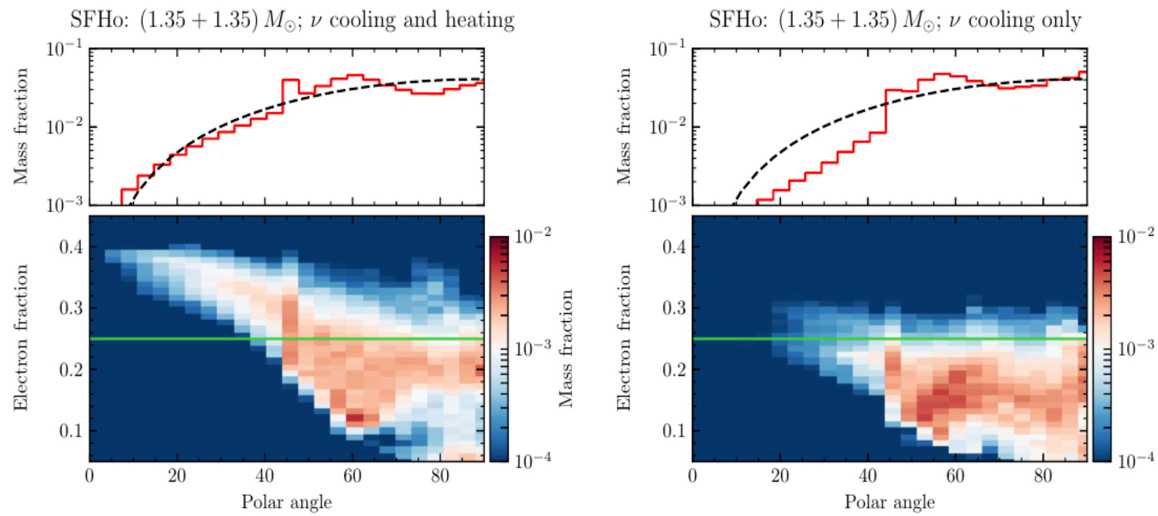
```
bernuzzi@ThinkPad-T440s:~/Codes/BAM/src/projects/z4$ wc -l z4_rhs_movpunc_N.c
10854 z4_rhs_movpunc_N.c
```

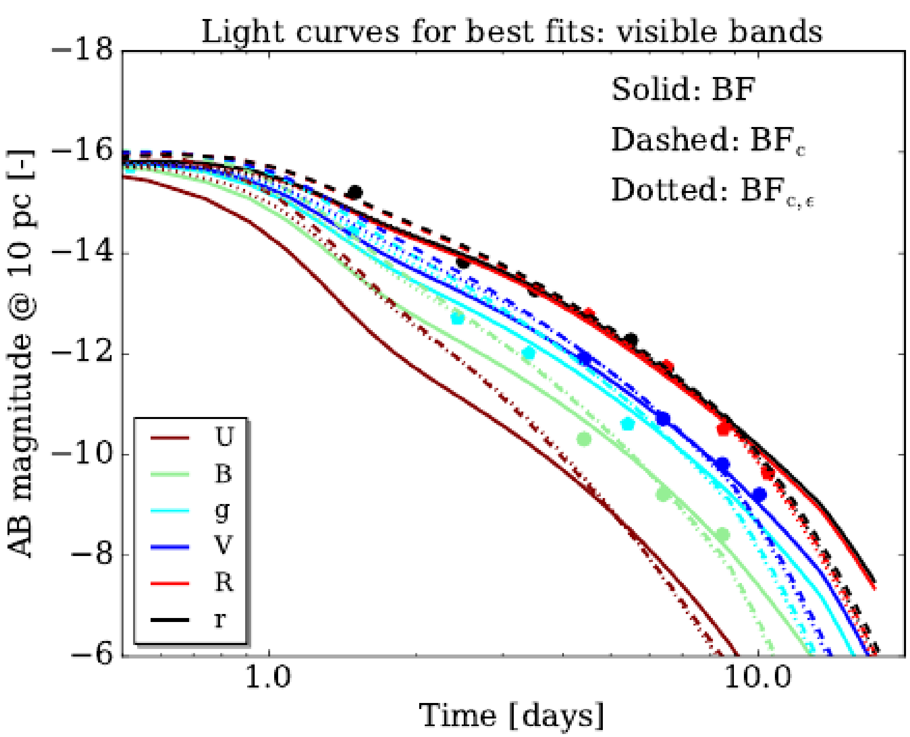
*[https://software.intel.com/en-us/articles/3d-finite-differences-on-multi-core-processors/]

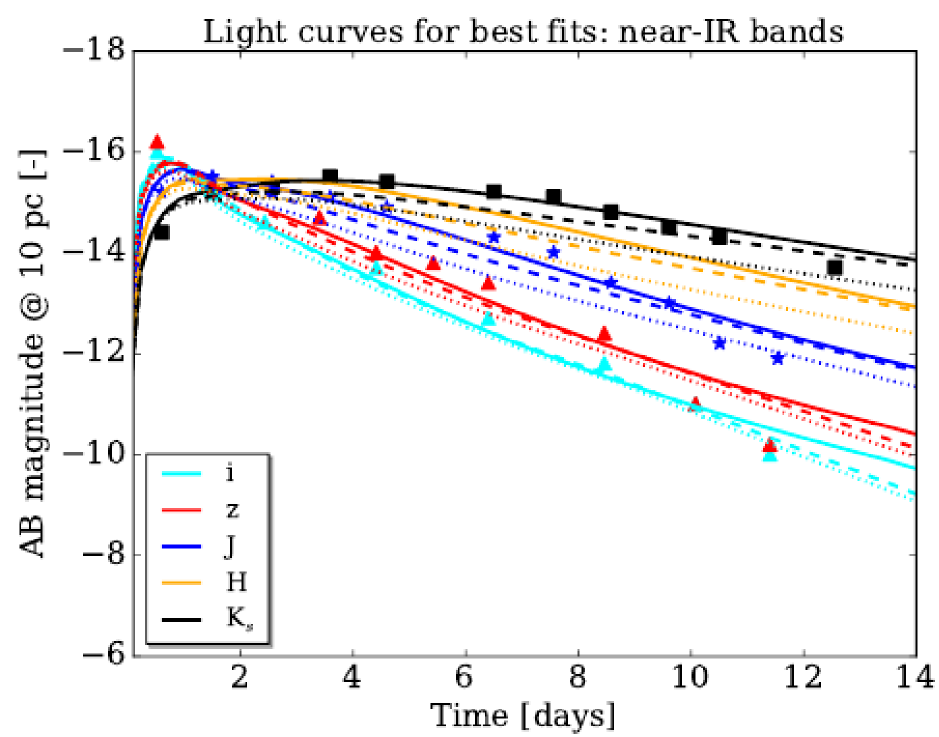
An anisotropic and three-components kilonova counterpart of GW170817

[Perego, Radice, Bernuzzi ApJL (2017)]

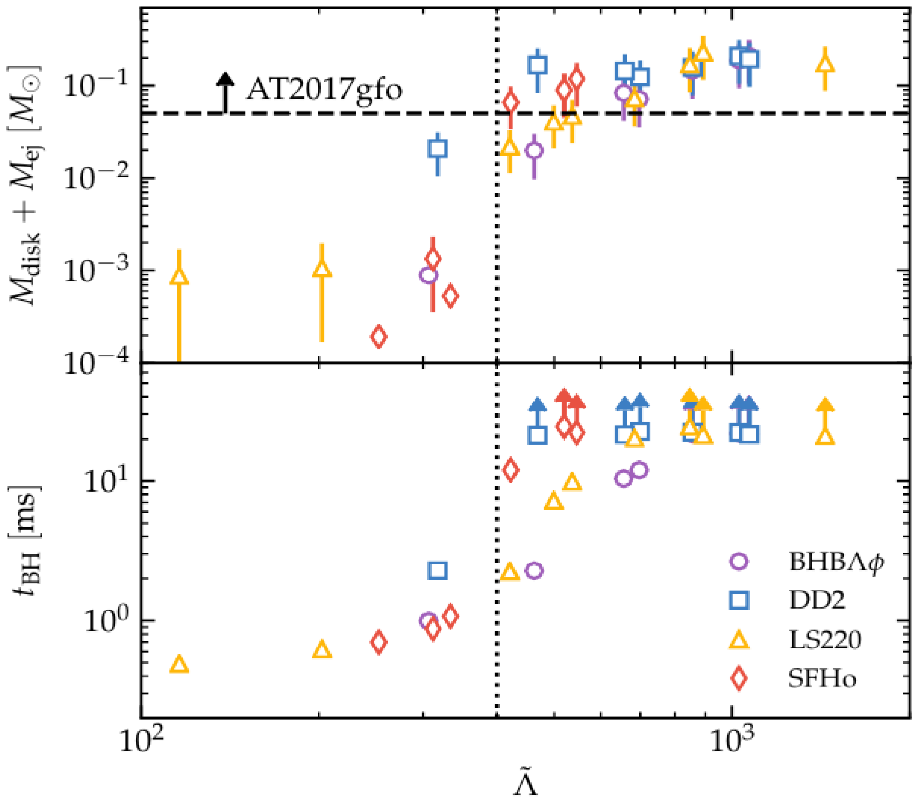




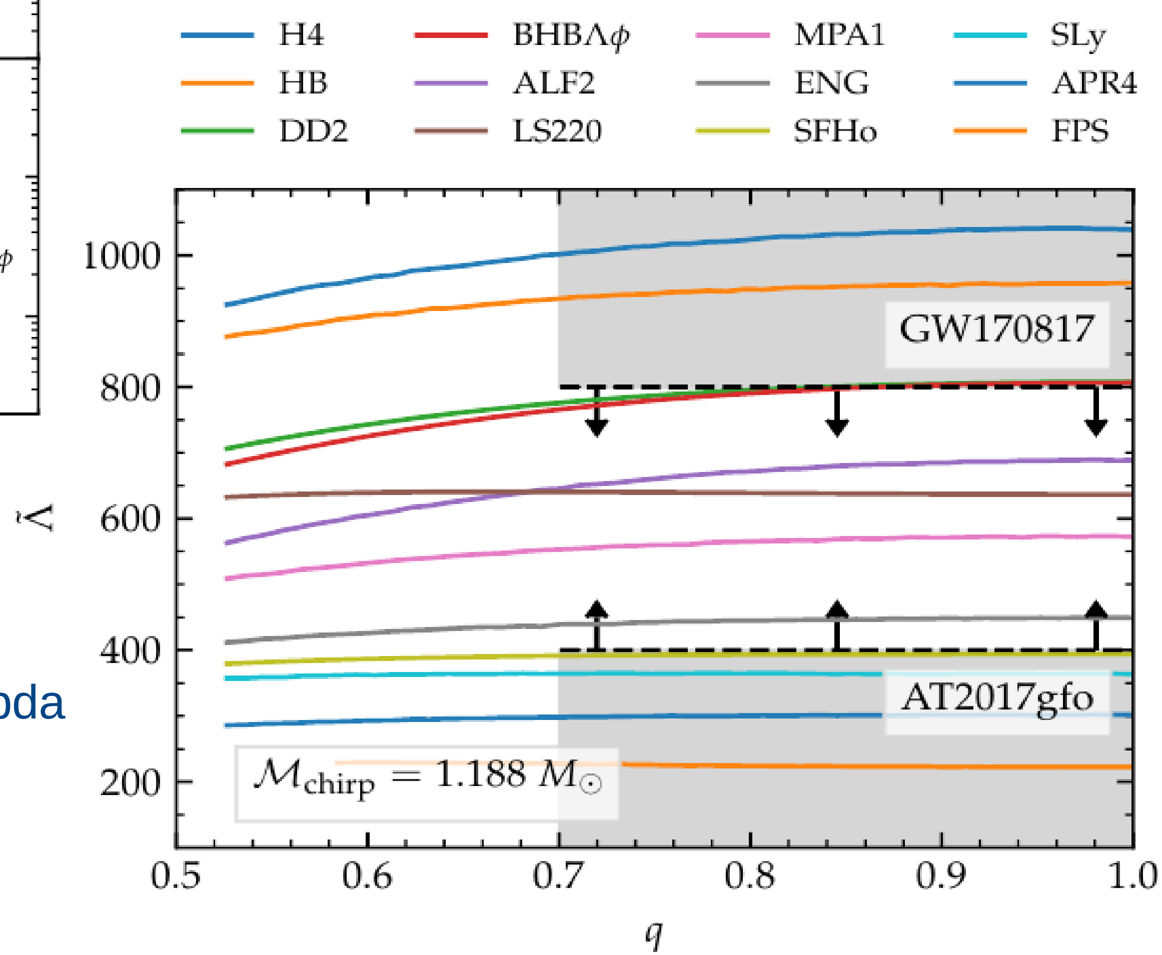




Joint constraint on the neutron star equation of state from multimessenger observations



[Radice, Perego, Zappa arxiv:1711.03647]



- kN model → lower bound on Mdisk
- Numerical relativity → Mdisk(Lambda)
- EM+NR analysis → lower bound on Lambda
- GW analysis → upper bound on Lambda

Summary

- Numerical relativity is key for
  - Waveform modeling
  - Exploting multi-messenger astronomy info
- No standard but dedicated codes
- New challenges need
  - Improve performances
  - New solutions