Marco Baldi
Bologna University

Numerical methods for standard and non-standard cosmological simulations

HPC methods for Computational Fluid Dynamics and Astrophysics

CINECA, 13-15 XI 2017

BRIEFLY INTRODUCING MYSELF...

Marco Baldi, Ph.D.

Present position: Researcher @ DIFA, Bologna
P.I. of the SIR project SIMCODE (www.marcobaldi.it/SIMCODE)

In the past: PhD @ Max Planck Institute for Astrophysics, Garching (DE)
Post-Doc @ Excellence Cluster Universe, Munich (DE)

Main research interests: theoretical cosmology, dark energy, structure formation, cosmological N-body simulations

Collaborations: Member of the Euclid consortium since January 2010
Coordinator of the WP "Numerical tools for non-standard cosmological models" of the Euclid Cosmological Simulations Working Group

Numerical Projects: PI of the PRACE Tier-0 projects SIBEL1 (8.4 M CPU hours) and SIMCODE1 (20 M CPU hours)

OUTLINE (1ST PART)

Cosmological Simulations of structure formation: **the standard case**

1. A brief **introduction to cosmology**

2. **Structure formation**:
introduction and main concepts

3. Why do we **need** simulations?

4. **The N-body method**:
gravity solvers and time integration schemes

5. The N-body code **GADGET**

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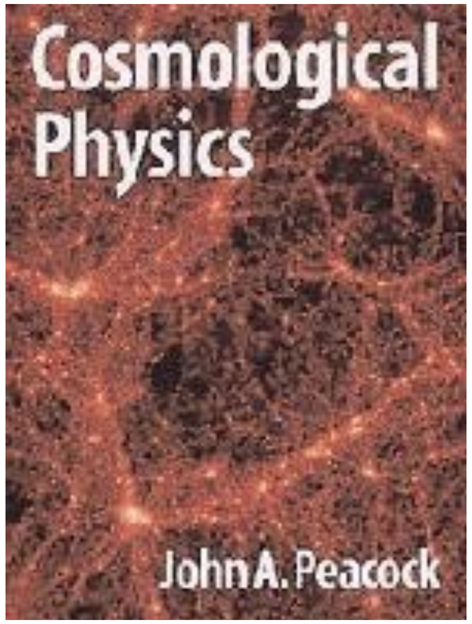

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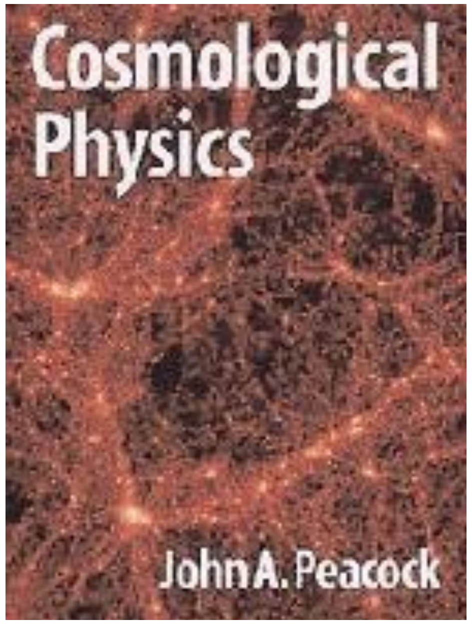

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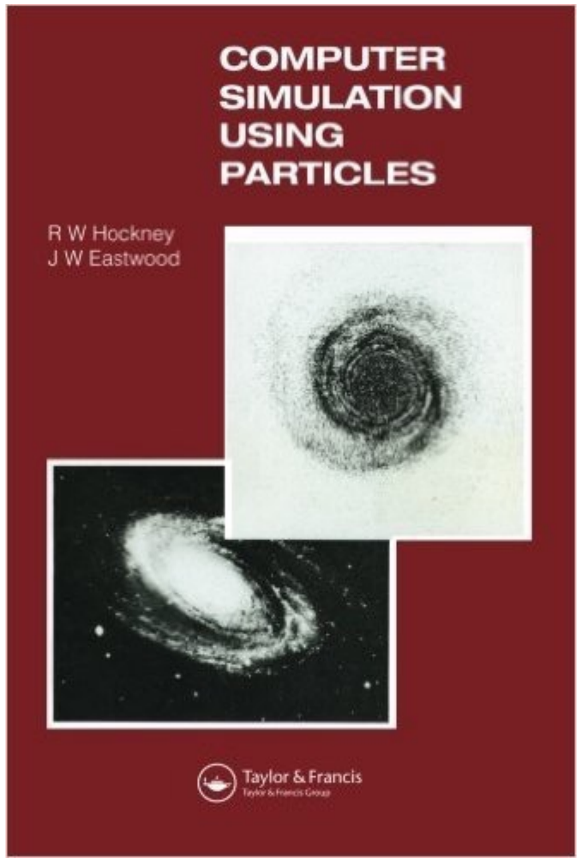

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Cosmological Simulations of structure formation: **the non-standard case**

1. **Non-standard cosmologies**:
Dark Energy, Modified Gravity, Massive neutrinos, Axions, etc…

2. Why do we **need** simulations
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3. **Modified N-body algorithms**:
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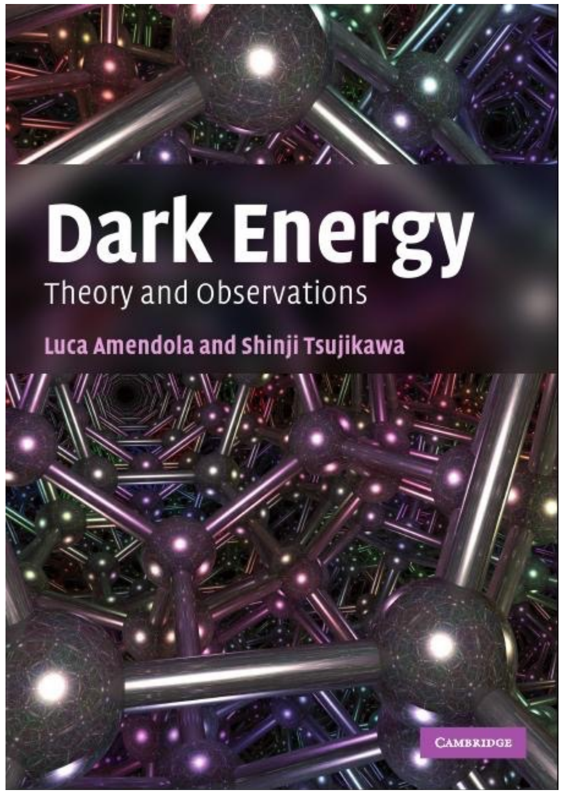

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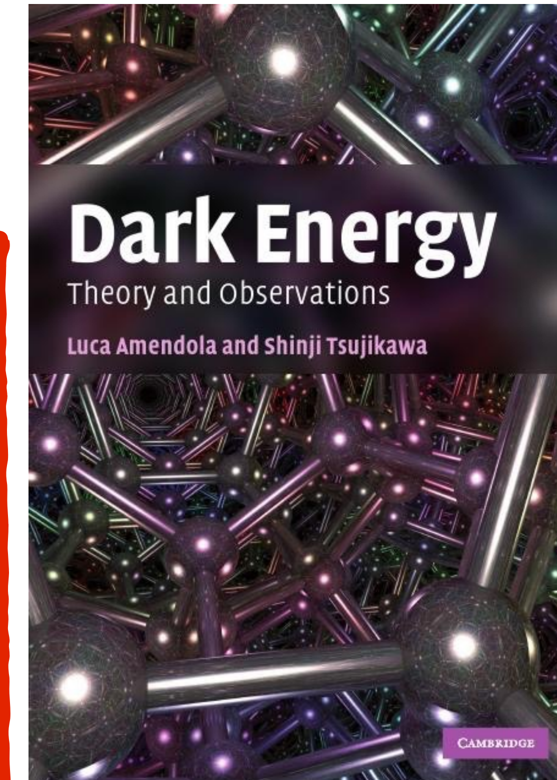

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Contents lists available at SciVerse ScienceDirect

Dark Universe

ELSEVIER

journal homepage: www.elsevier.com/locate/agee

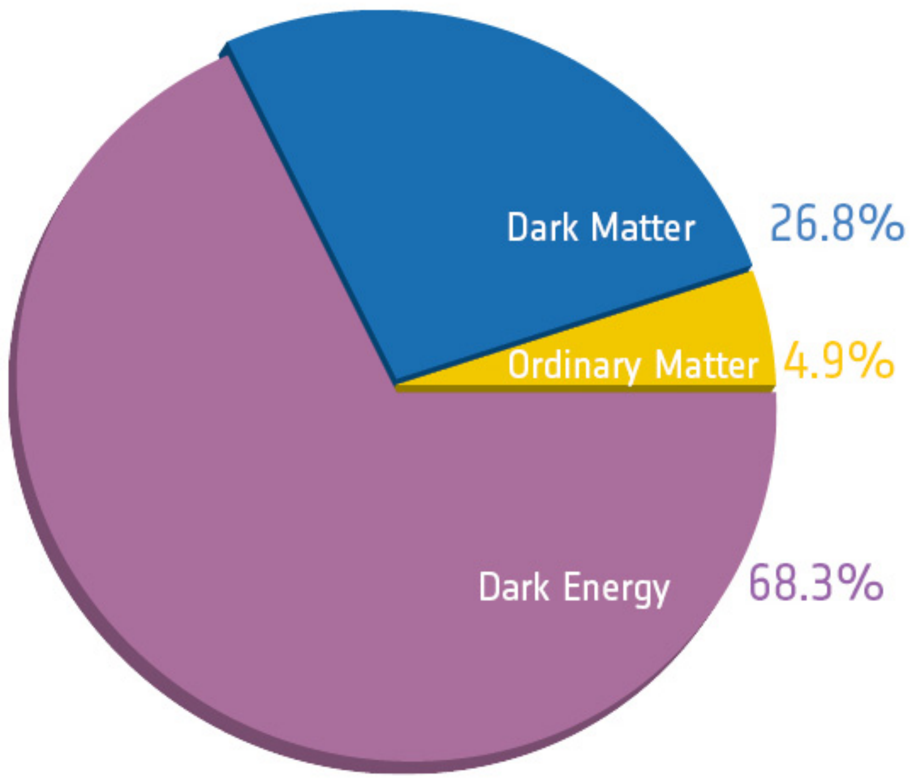
Dark Energy simulations

Marco Baldi

Excellence Cluster Universe, Boltzmannstrasse 2, 85748 Garching bei München, Germany

A (very) brief introduction to cosmology

The standard cosmological model



The Universe after Planck: 6 parameters to fit all data

$$\Omega_{\rm b} = 0.049 \qquad \Omega_{\rm CDM} = 0.265$$

$$\Omega_{\Lambda} = 0.6844 \qquad \sigma_8 = 0.831$$

$$n_s = 0.9645 \qquad \tau = 0.079$$

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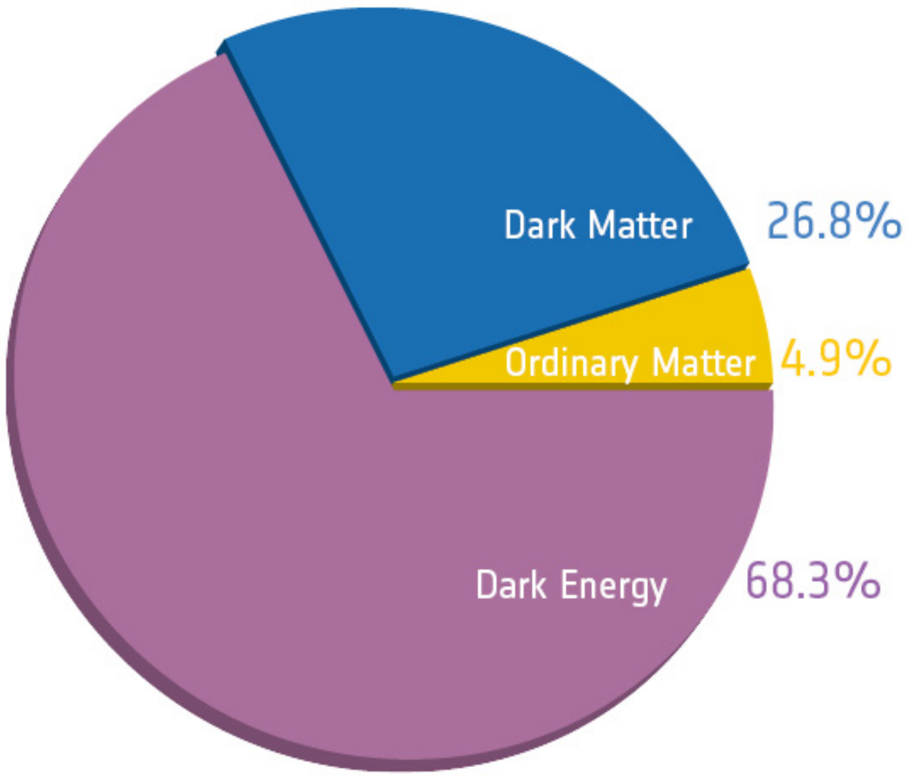

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- Cosmological Principle (homogeneity & isotropy);
- Gaussian and Adiabatic initial conditions;
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- Neutrinos are massless;
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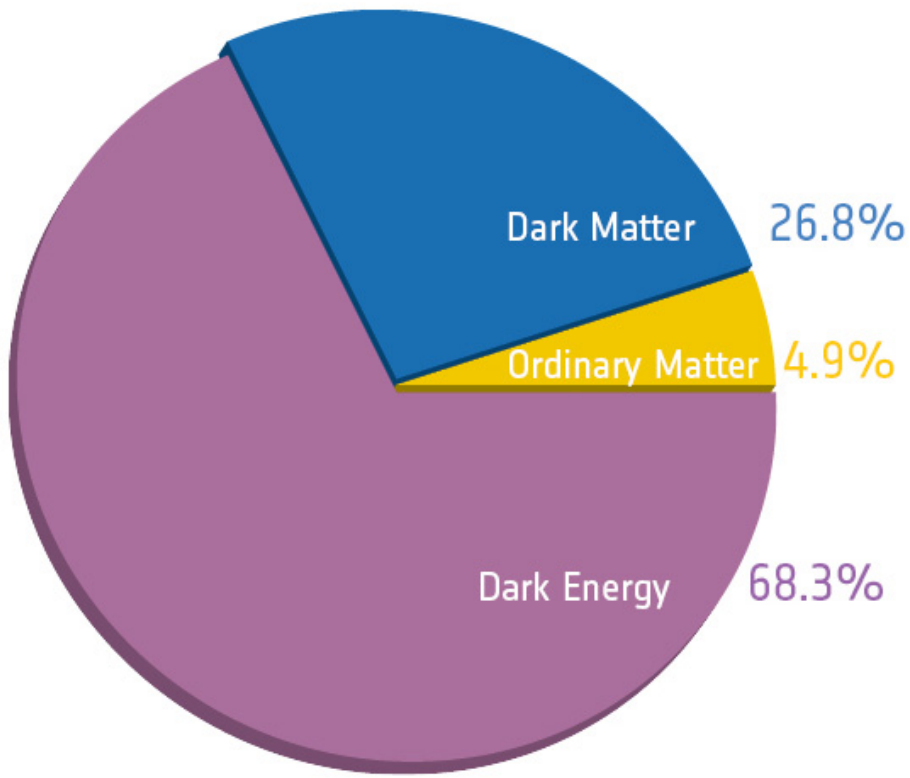

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Precision Cosmology test the model's assumptions with **~1% accuracy**

Structure formation

Linear growth of density perturbations

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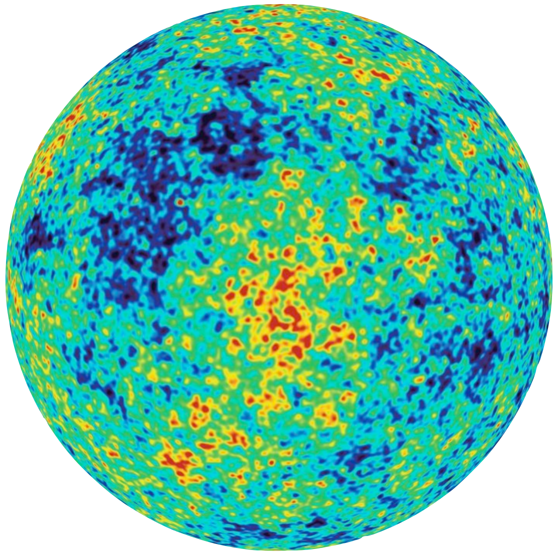

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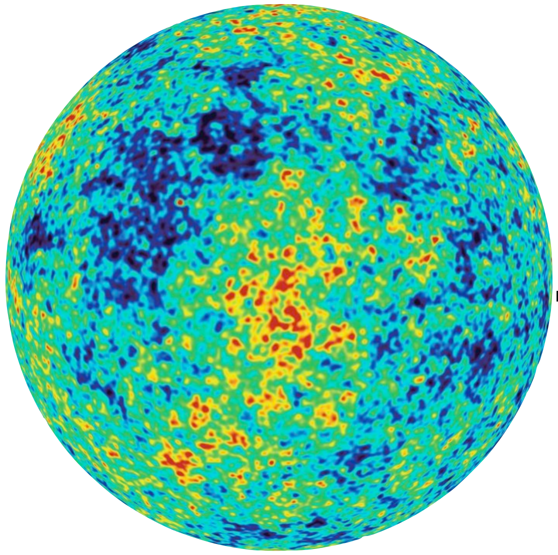

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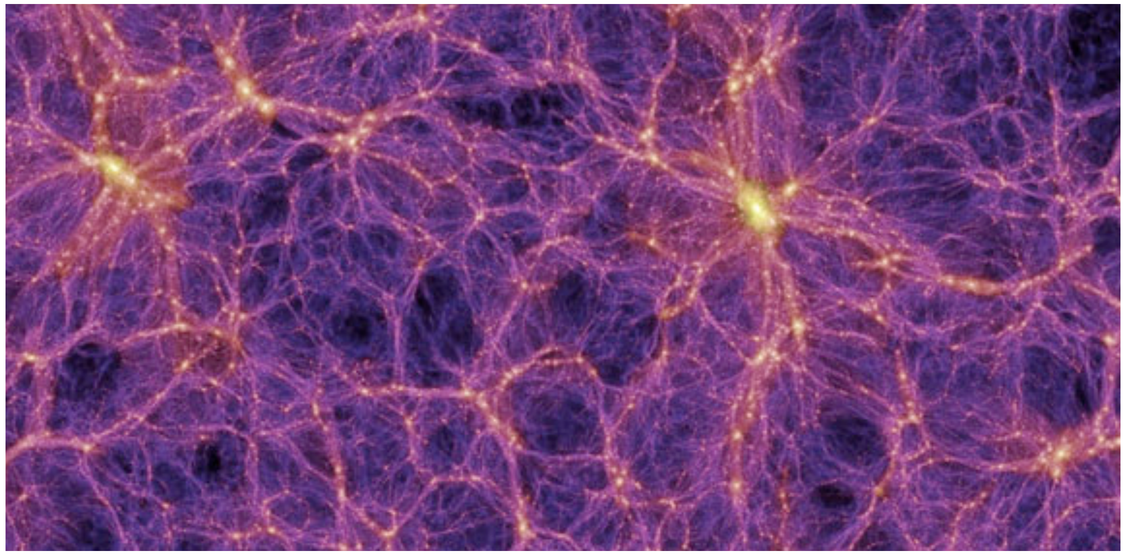
$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}^2\delta = 0$$

$$D_+ \approx 10^3$$

Structures in the present-day Universe

$z_0 = 0, a_0 = 1$



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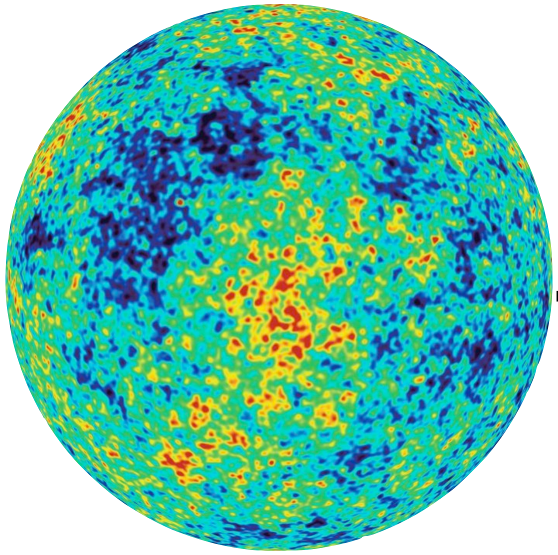

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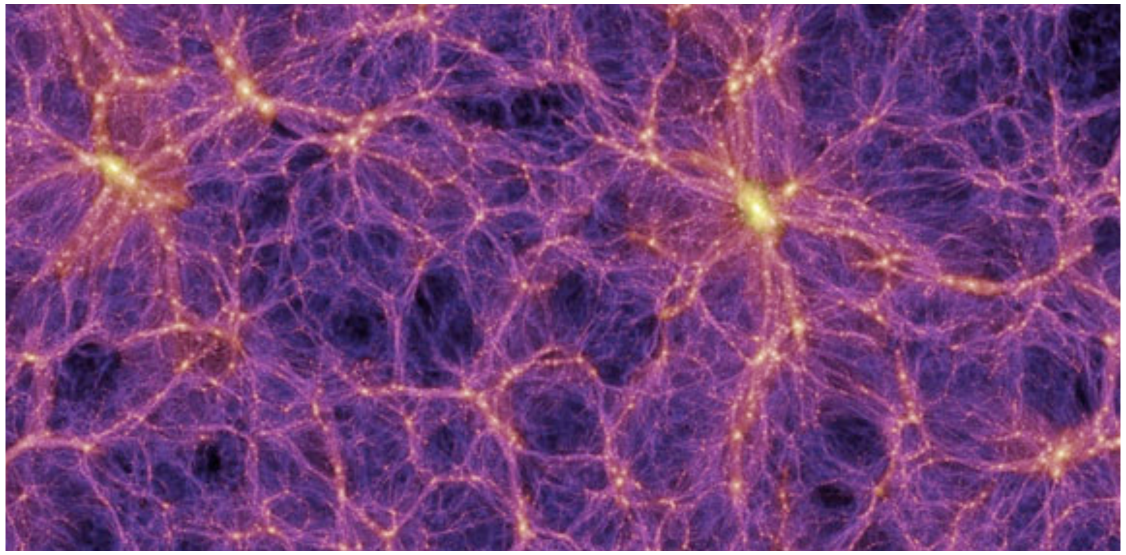

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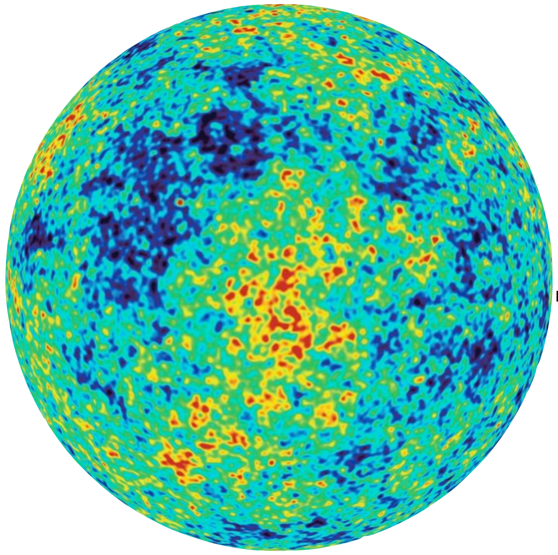

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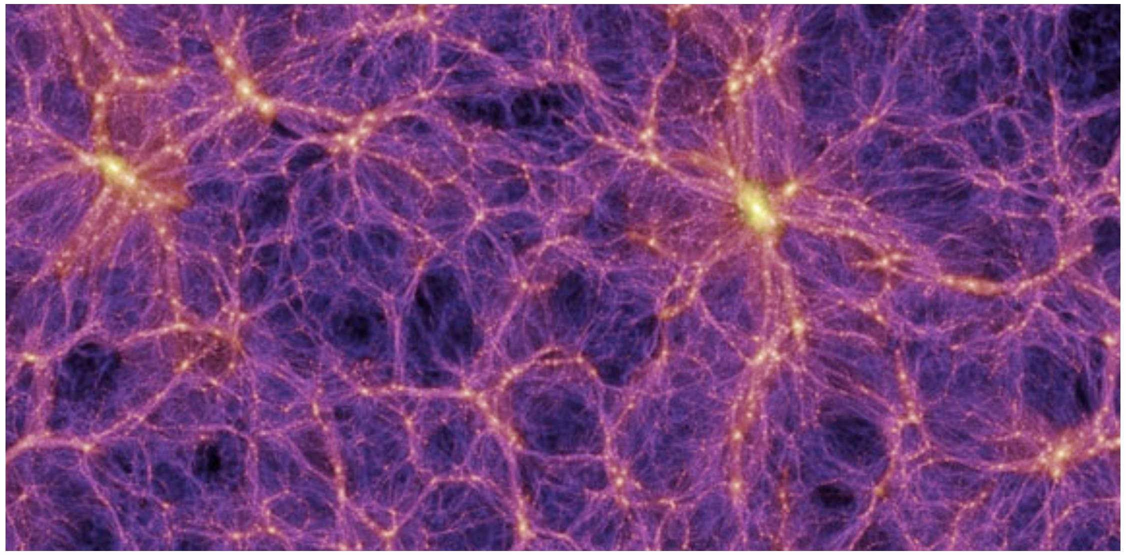

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With a cosmological constant $\Omega_{\rm M} + \Omega_\Lambda = 1$: $\delta_+ \propto a^m$, $m < 1$

With more complicated Dark Energy models also $\mathcal{H}$ changes non-trivially, and additional forces might come to play, so that $m \lesseqgtr 1$

WHY DO WE NEED SIMULATIONS?

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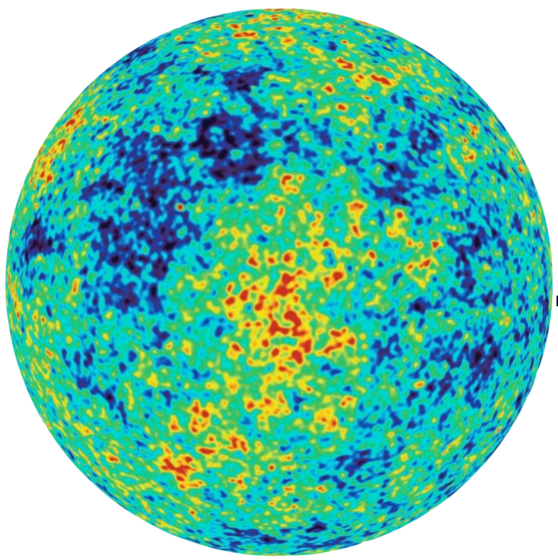

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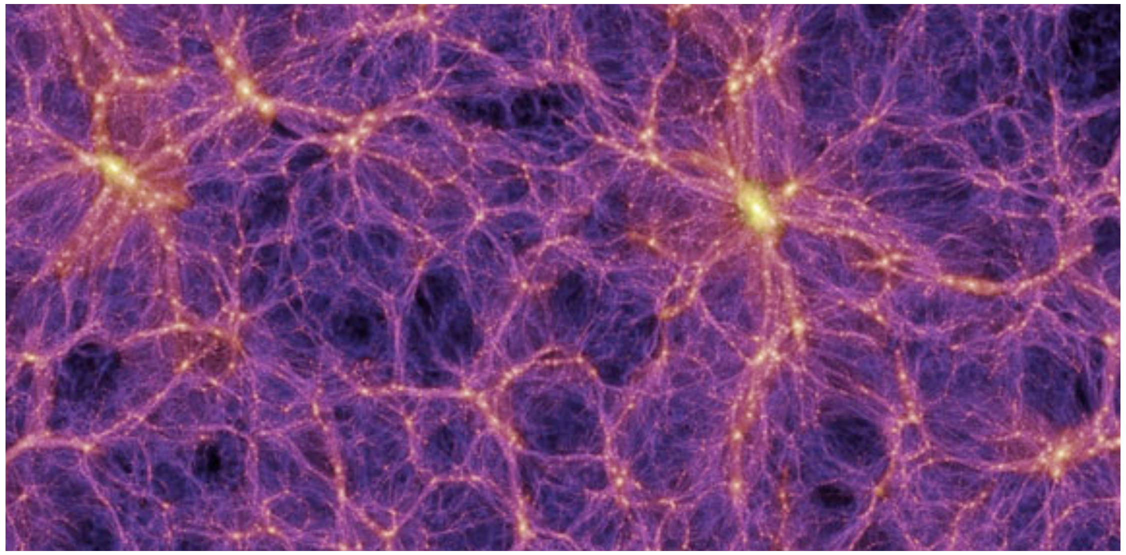

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THE NON-LINEAR REGIME OF STRUCTURE FORMATION

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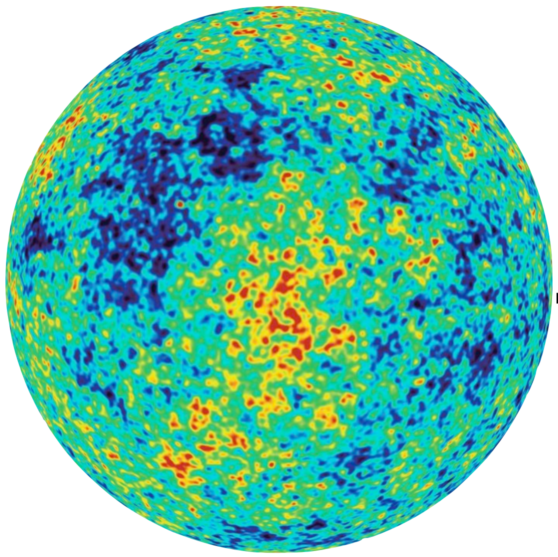

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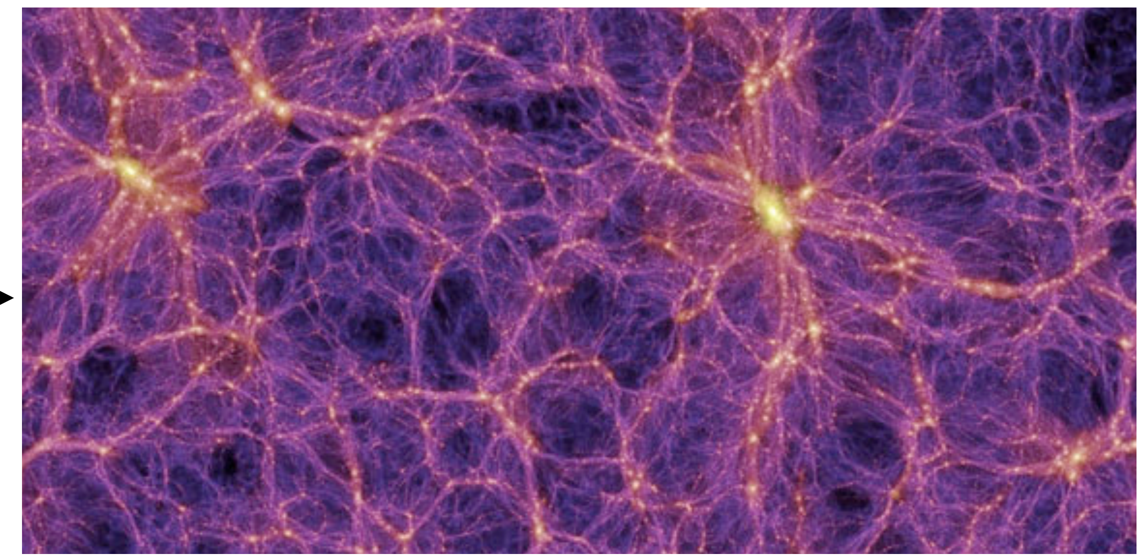

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... we know that in the present Universe density perturbations can reach large values ($\delta \approx 1$ on scales of $\sim 8\mathrm{Mpc}$ and up to $\delta \approx 10^5$ in the center of galaxy clusters).

The assumption of small perturbations does not hold anymore, and linearity no longer applies $\longrightarrow$ need of numerical methods

The N-body method

COSMOLOGICAL SIMULATIONS IN PILLS

Simulate the formation and evolution of structures in the Universe under the effect of gravitational instability.

Use **particles as fluid elements**, assign **initial conditions**, **compute gravitational force** on particles, and **evolve the system** according to some assumptions on the physics (cosmology, astrophysical processes, etc...)

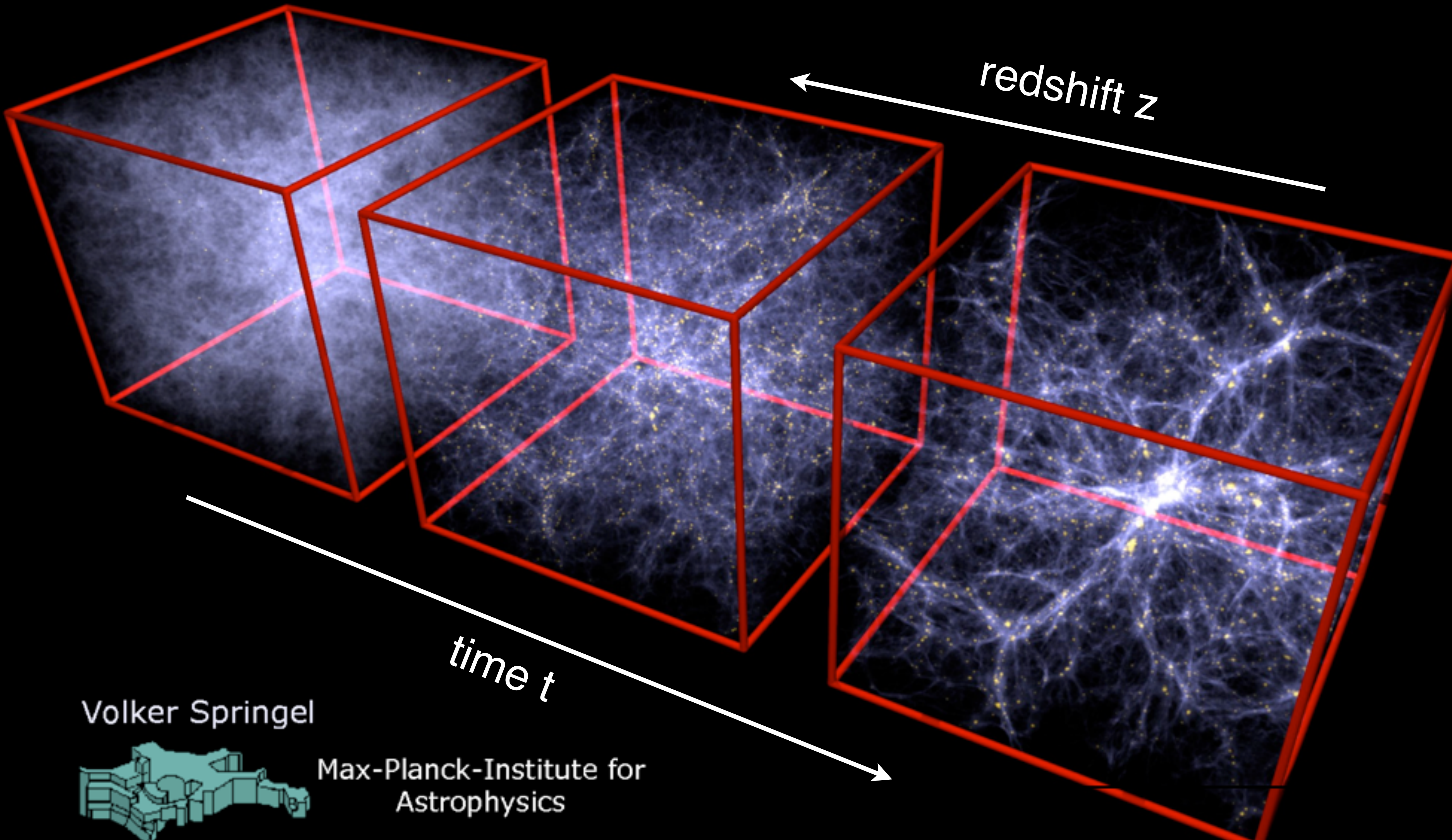
Highly non-linear processes when density perturbations exceed unity (maximum overdensities in the Universe today $\sim 10^5$)

The gravitational evolution of N particles is a $N(N-1) \sim N^2$ problem! Need to **devise approximated solutions** to reduce computational cost (e.g. Particle-Mesh, Tree, Tree-PM, scale as N logN)

Analyze the simulations outputs through post-processing tools (halo finders, ray-tracers, mock galaxy catalogues) and compare with observational data

Cosmological N-body simulations

Integrate the evolution of density perturbations forward in time (starting from a known initial power spectrum) within a periodic, comoving, and cosmologically representative box filled with tracer particles



Millennium Run
Springel et al. 2005


z = 48.4
T = 0.05 Gyr
500 kpc
Millennium Run
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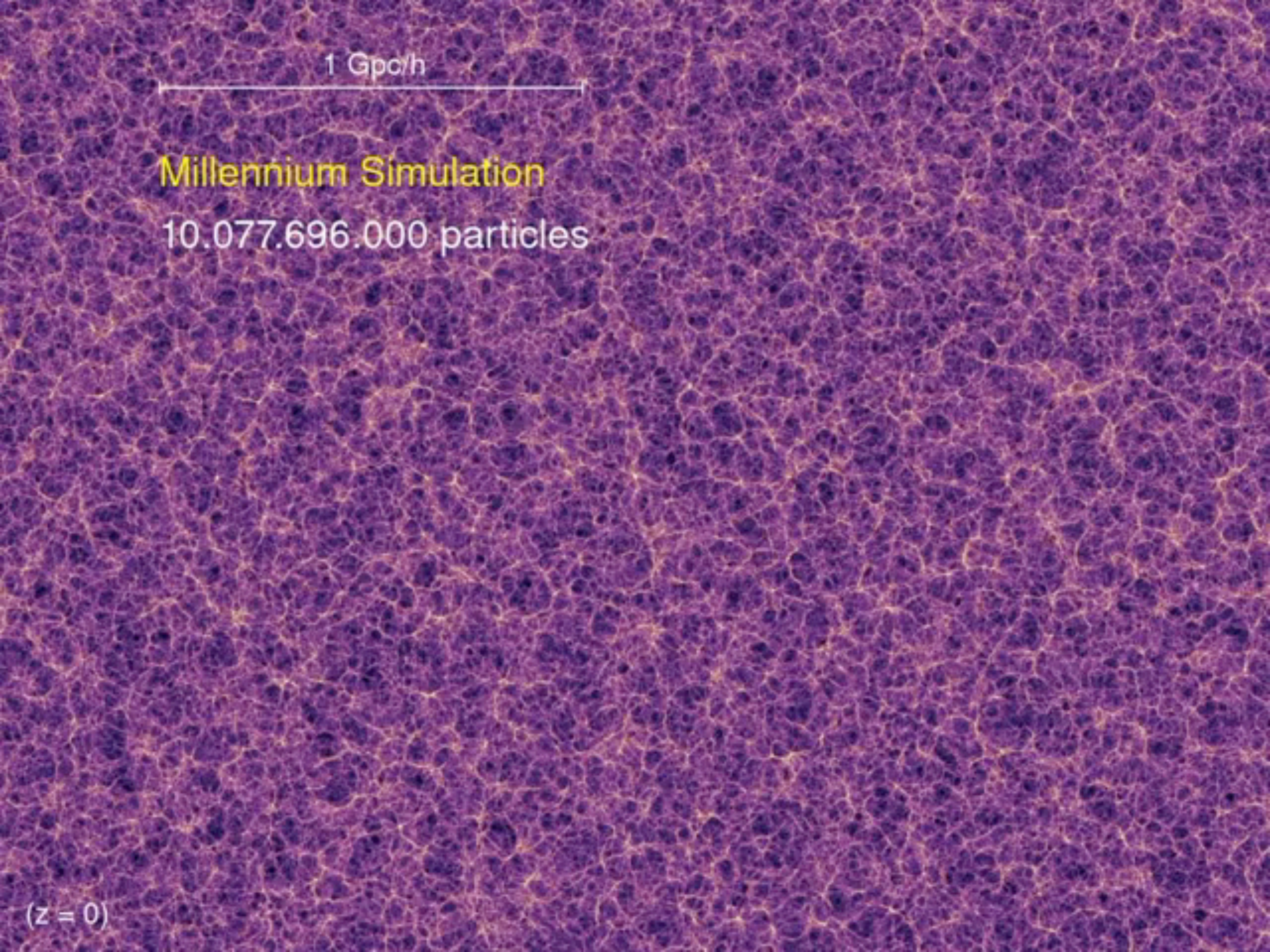
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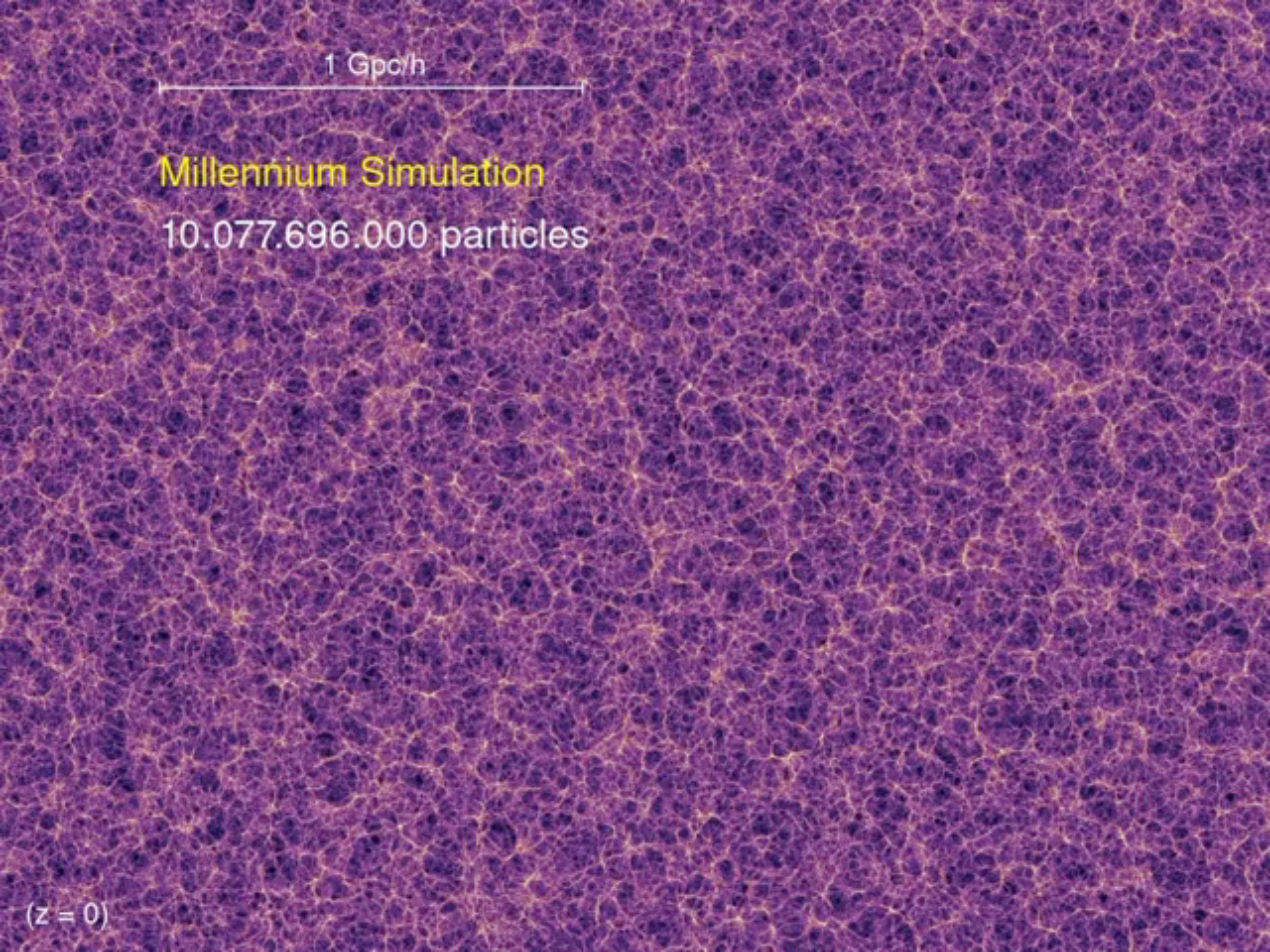
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SIMULATIONS ARE PREDICTIVE!

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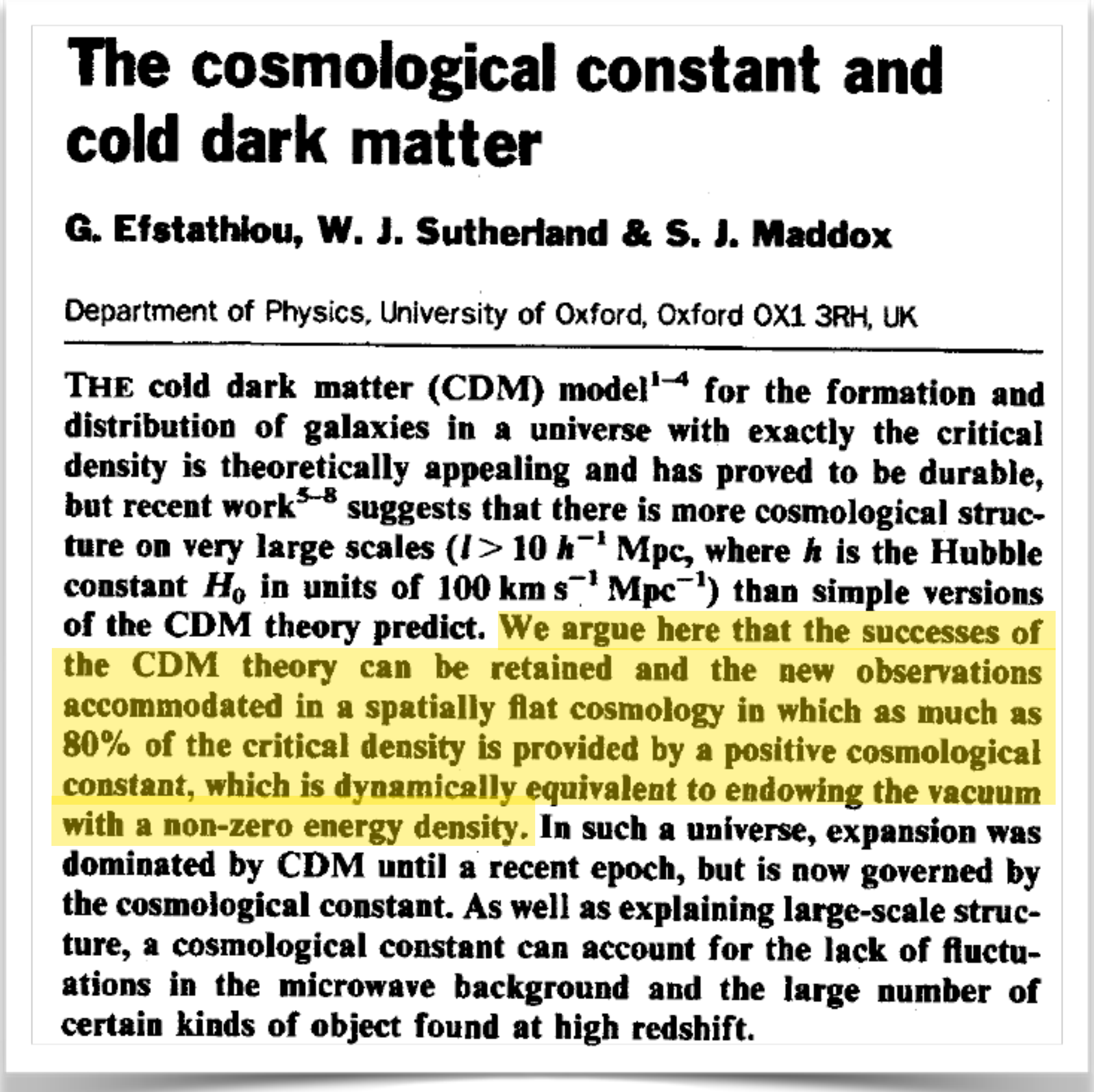

The cosmological constant and cold dark matter

G. Efstathiou, W. J. Sutherland & S. J. Maddox

Department of Physics, University of Oxford, Oxford OX1 3RH, UK

THE cold dark matter (CDM) model[1–4] for the formation and distribution of galaxies in a universe with exactly the critical density is theoretically appealing and has proved to be durable, but recent work[5–8] suggests that there is more cosmological structure on very large scales ($l > 10\ h^{-1}$ Mpc, where h is the Hubble constant H_0 in units of $100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$) than simple versions of the CDM theory predict. We argue here that the successes of the CDM theory can be retained and the new observations accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant, which is dynamically equivalent to endowing the vacuum with a non-zero energy density. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. As well as explaining large-scale structure, a cosmological constant can account for the lack of fluctuations in the microwave background and the large number of certain kinds of object found at high redshift.

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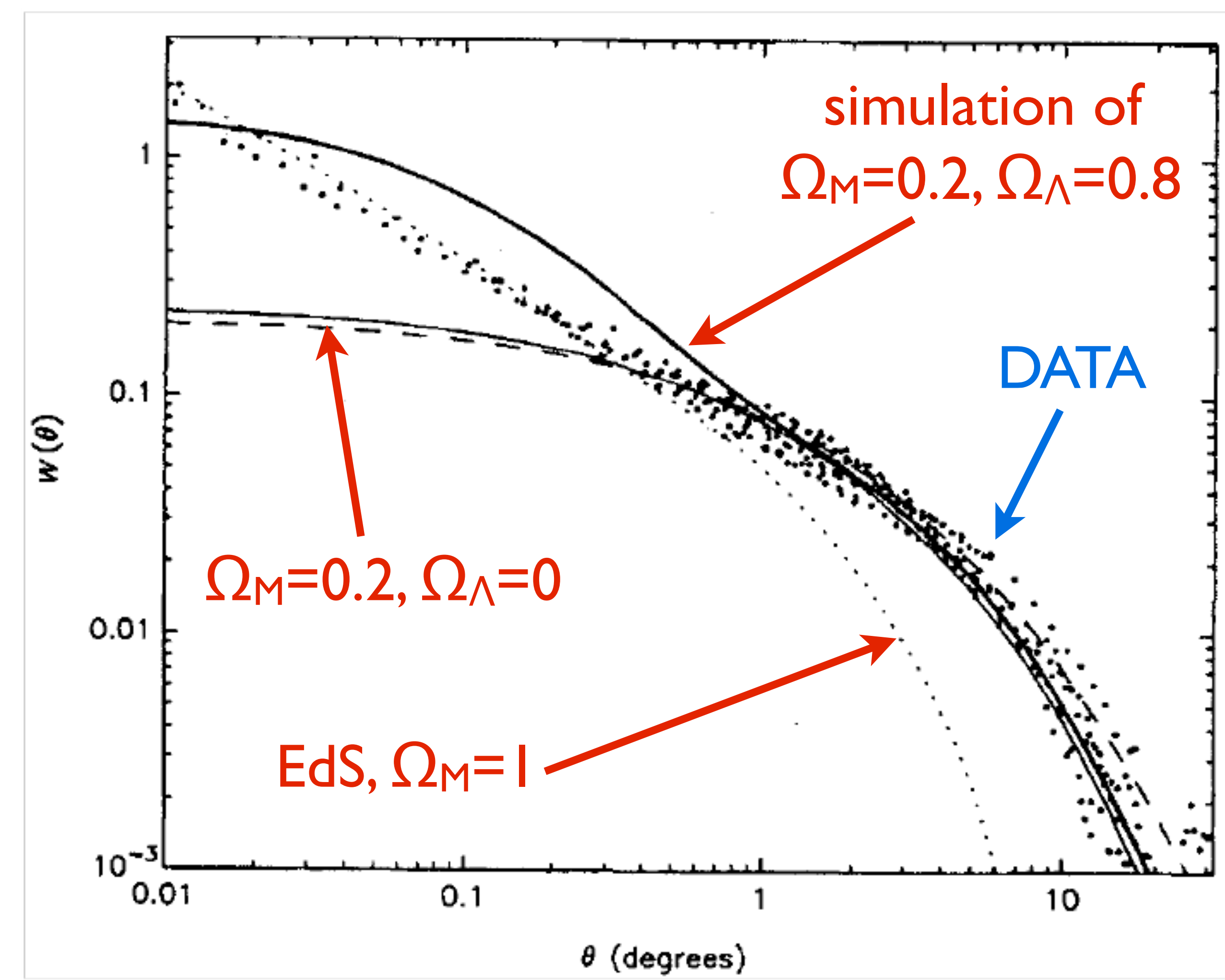

N-body algorithms: PP, PM, Tree

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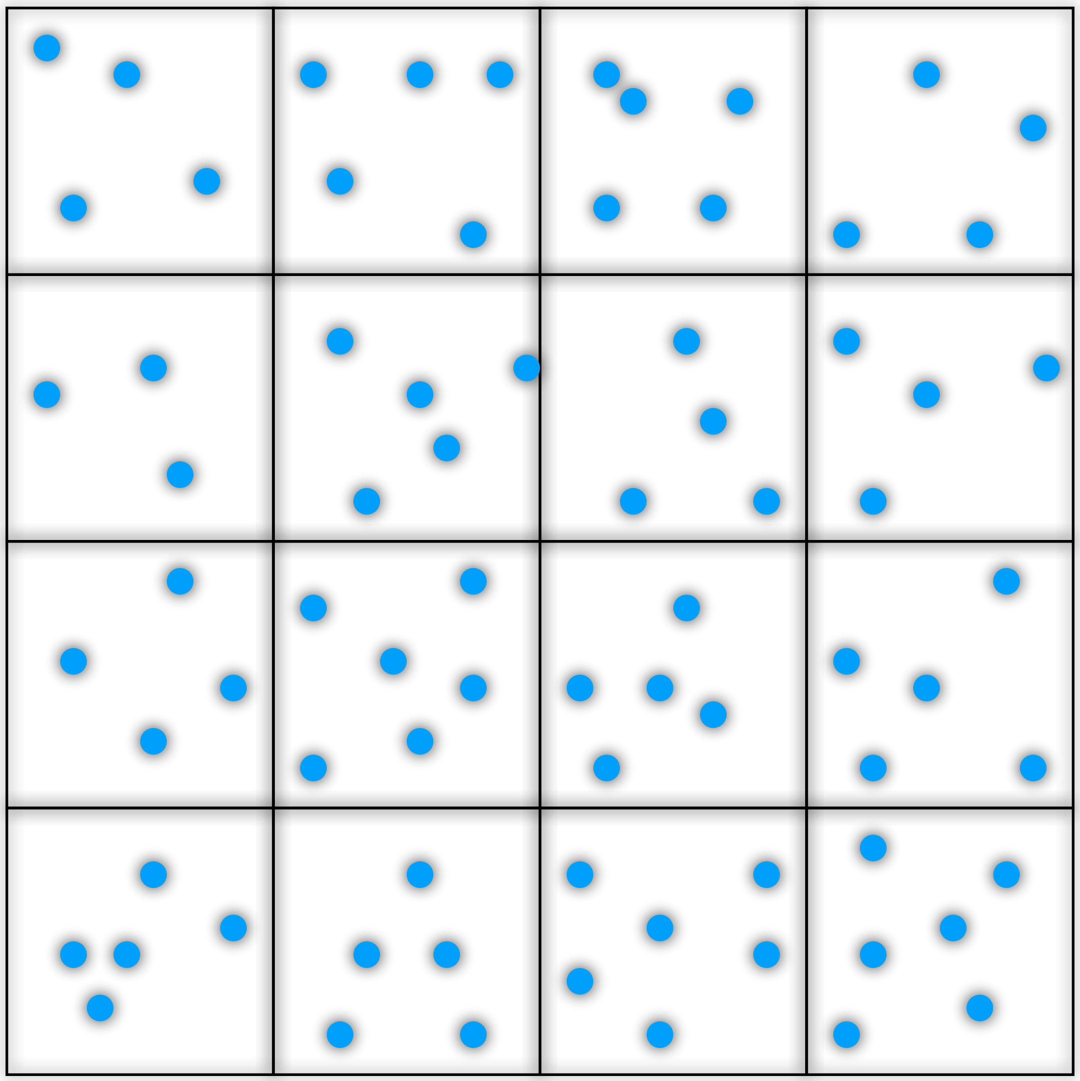
The softening ε avoids large-angle scattering and the formation of bound particle pairs (needed for collisionless dynamics)

N-body method: the gravity solver

The nonlinear regime of structure formation could possibly probe the largest deviation from ΛCDM: need of N-body!

Particle-Mesh



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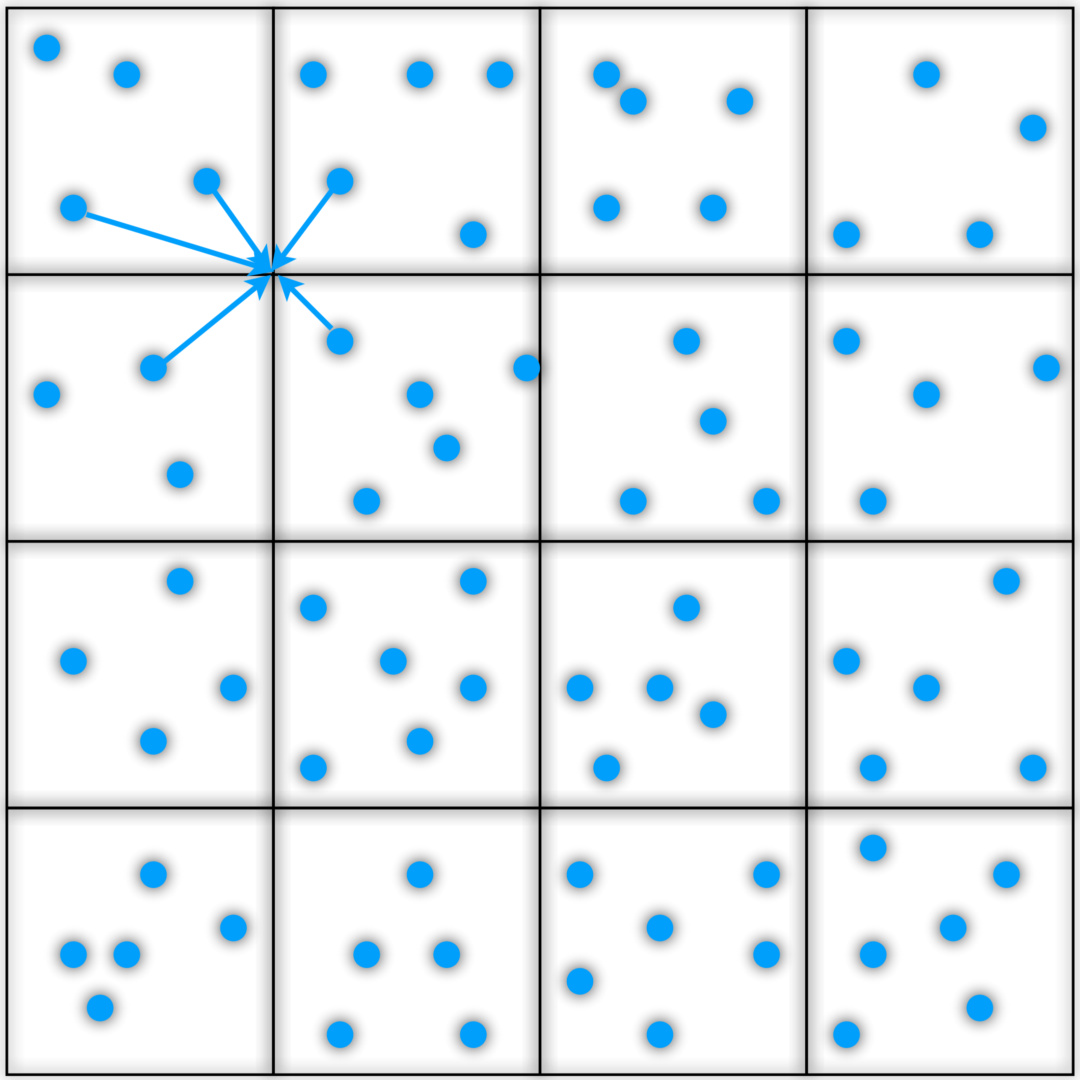

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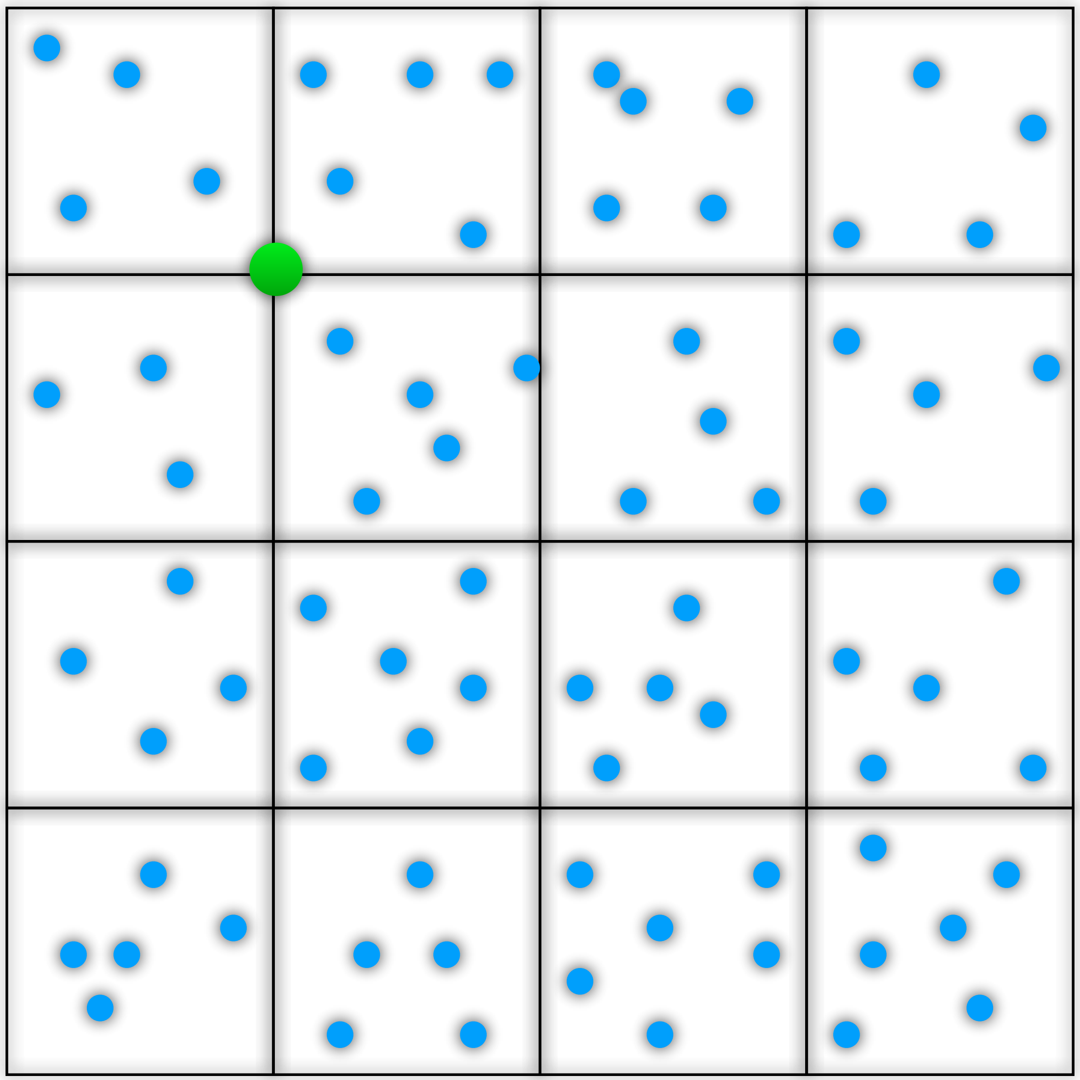

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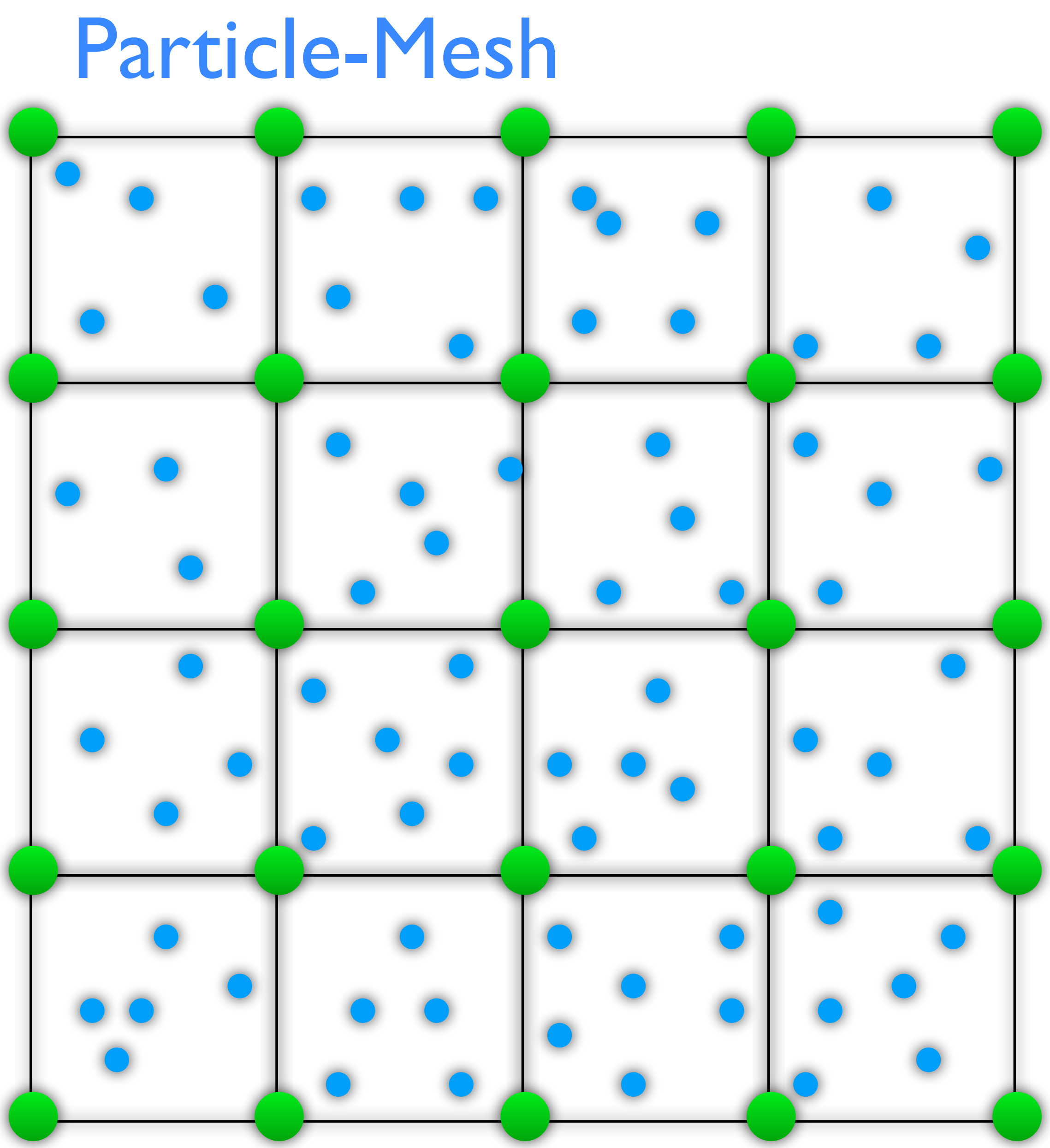

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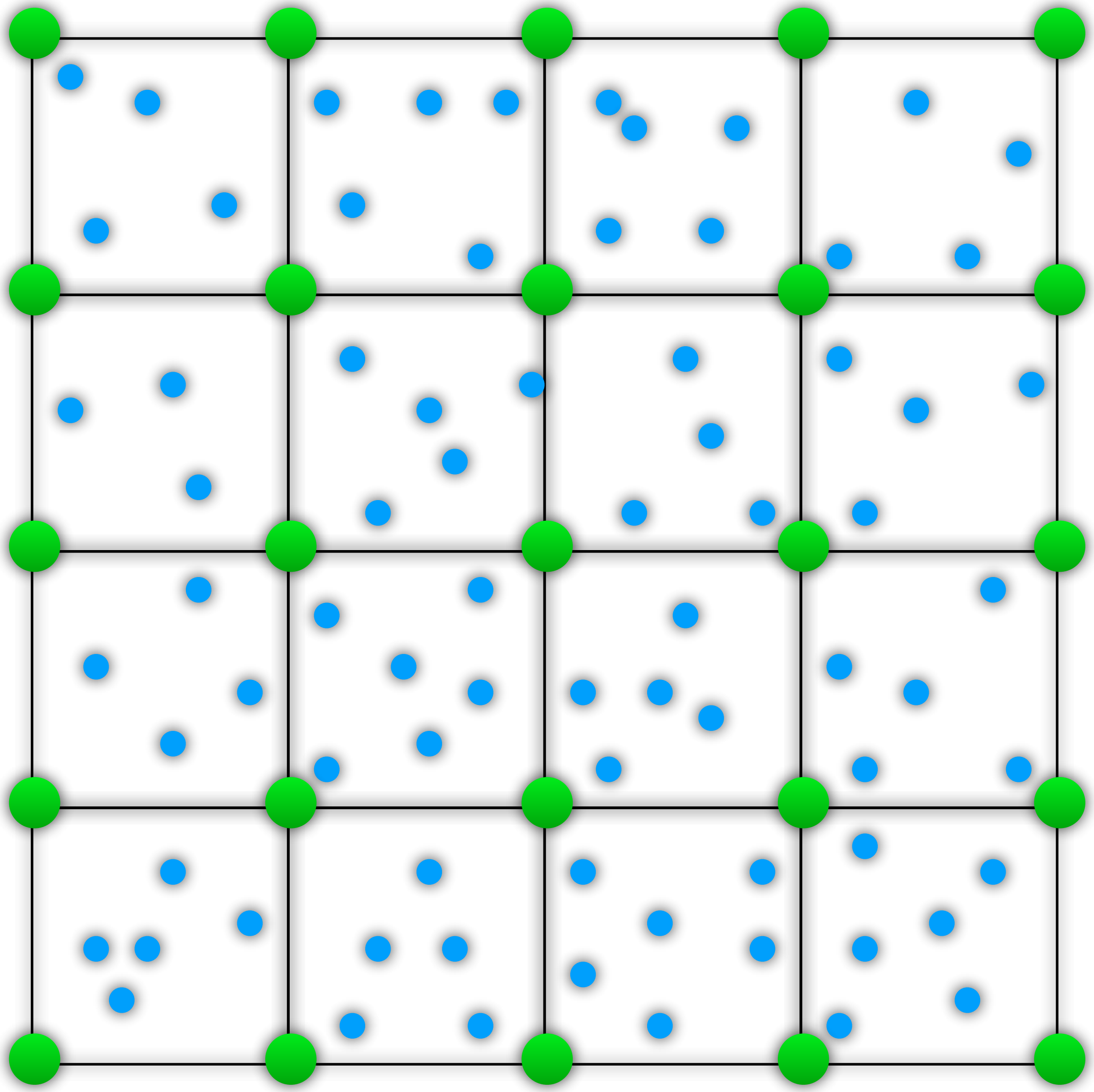

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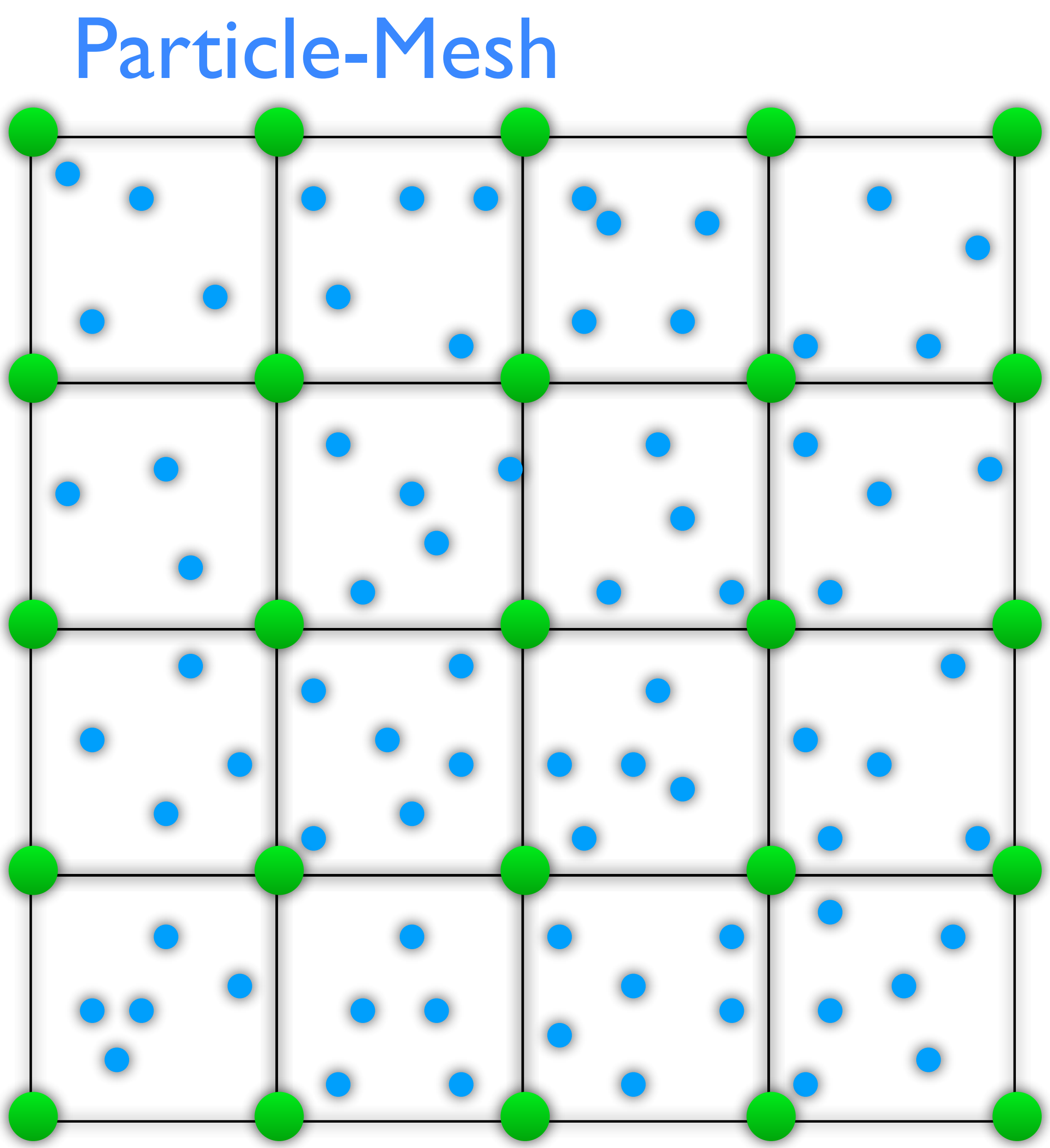

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3) Fourier transform back the potential to real space: potential on the grid

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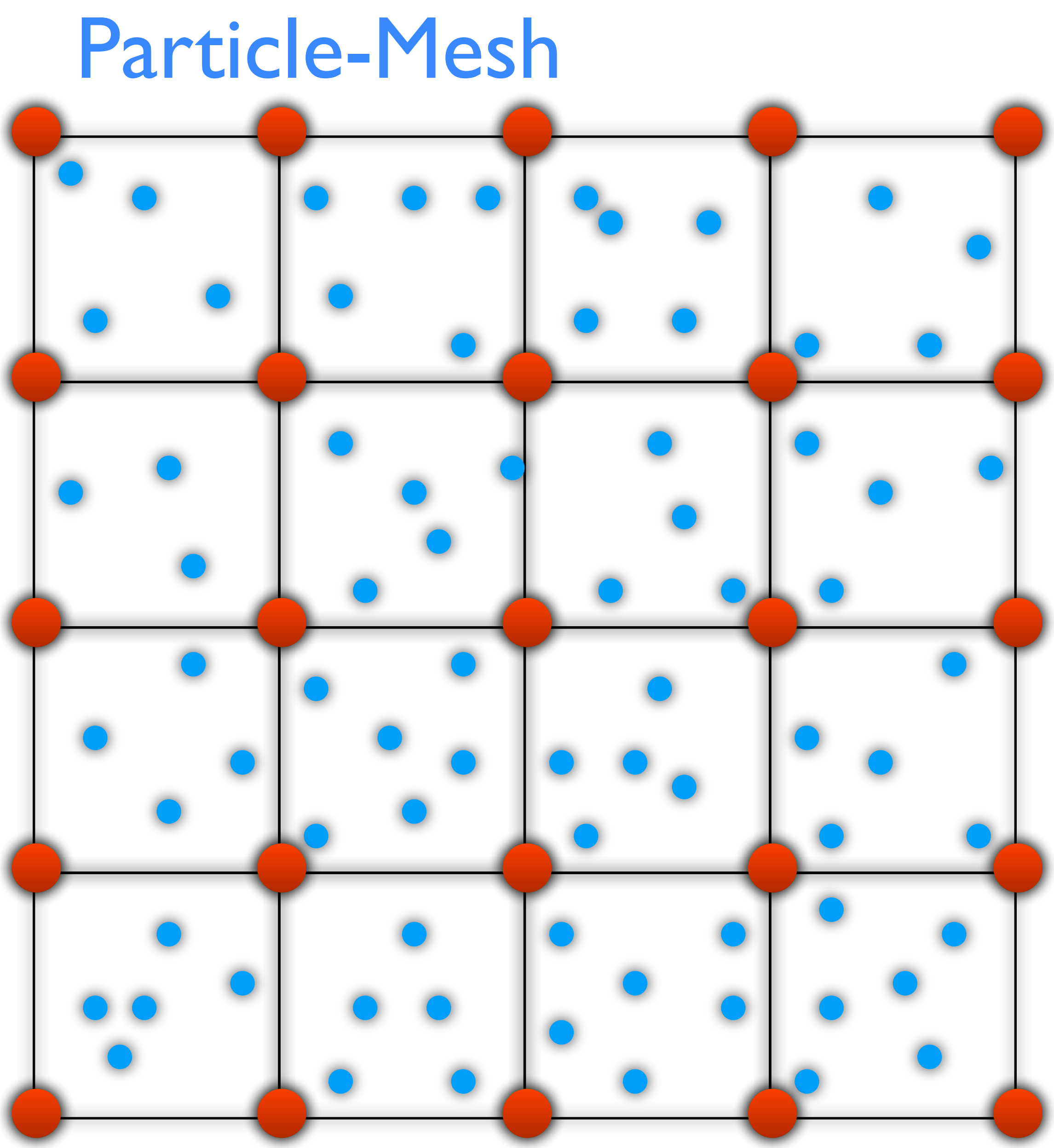
The nonlinear regime of structure formation could possibly probe the largest deviation from ΛCDM: need of N-body!



1) Assign mass to grid nodes, obtain density on the grid

2) In Fourier space, compute the gravitational potential with the Newtonian Green's function $1/k^2$

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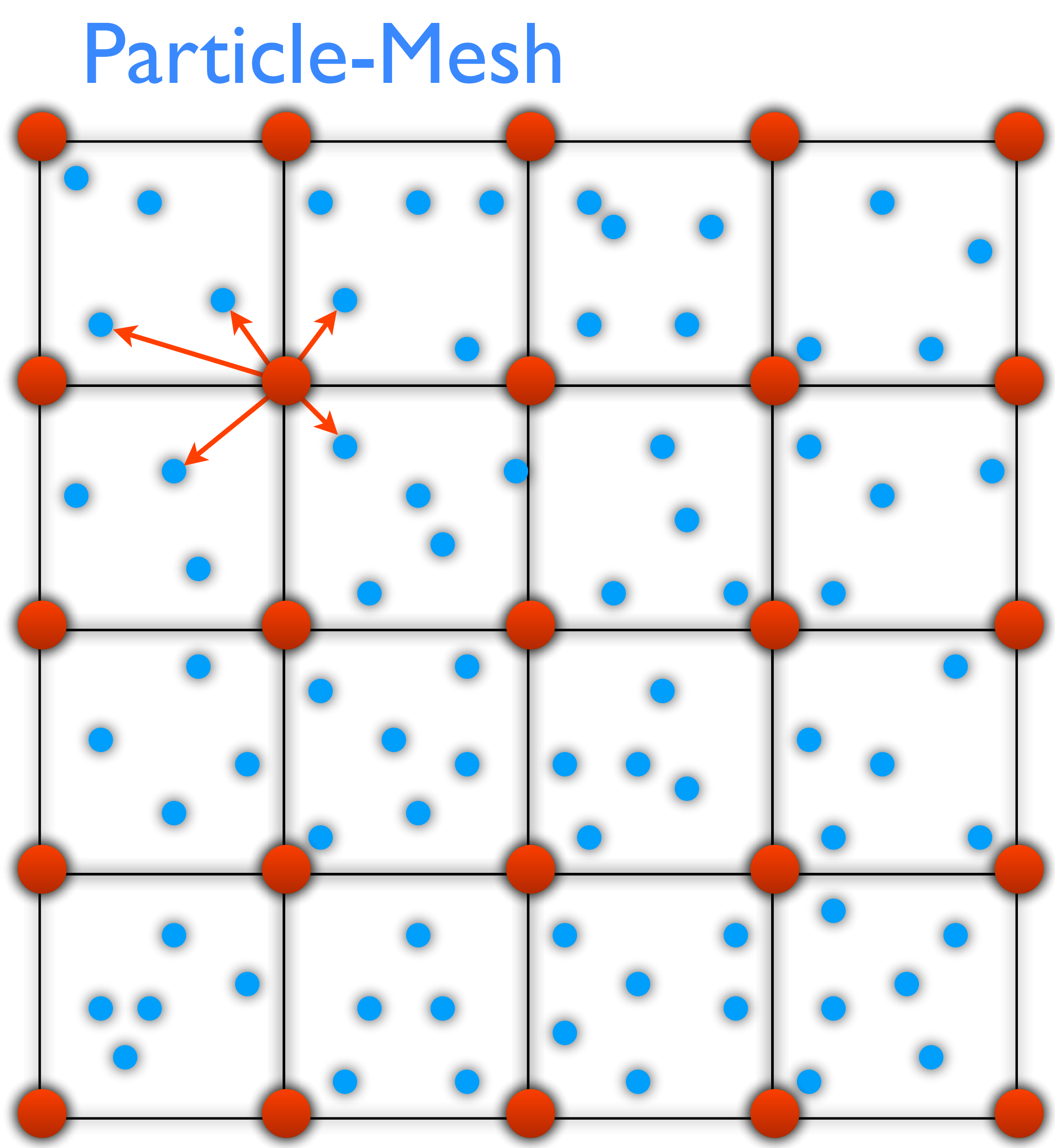

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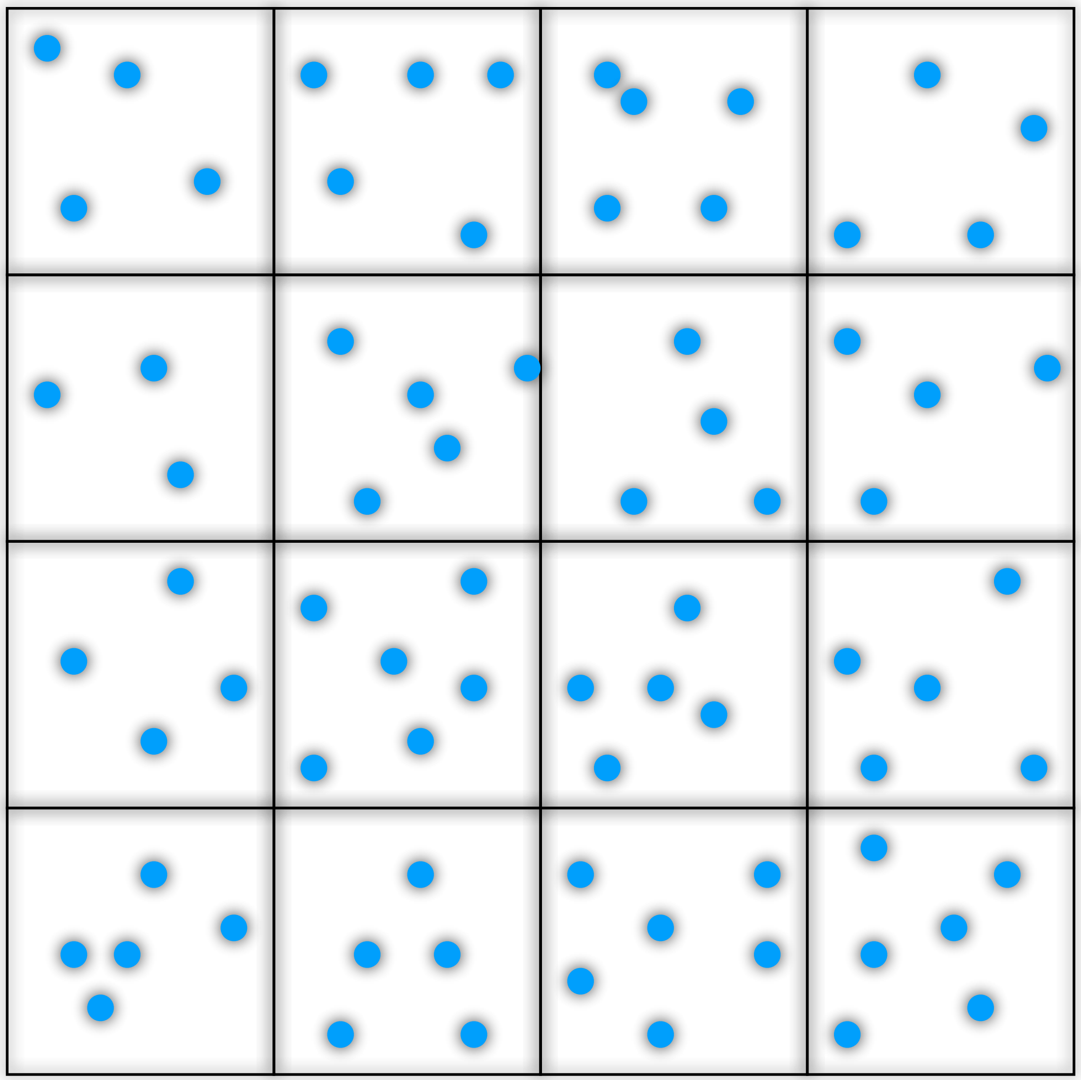
4) Compute the force on particles by finite differencing the gravitational potential

N-body method: the gravity solver

Particle-Particle (Tree) ⇒ N^2 problem (N logN problem)



Compute the direct gravitational interaction of a TARGET particle with another particle or a NODE (group of particles)

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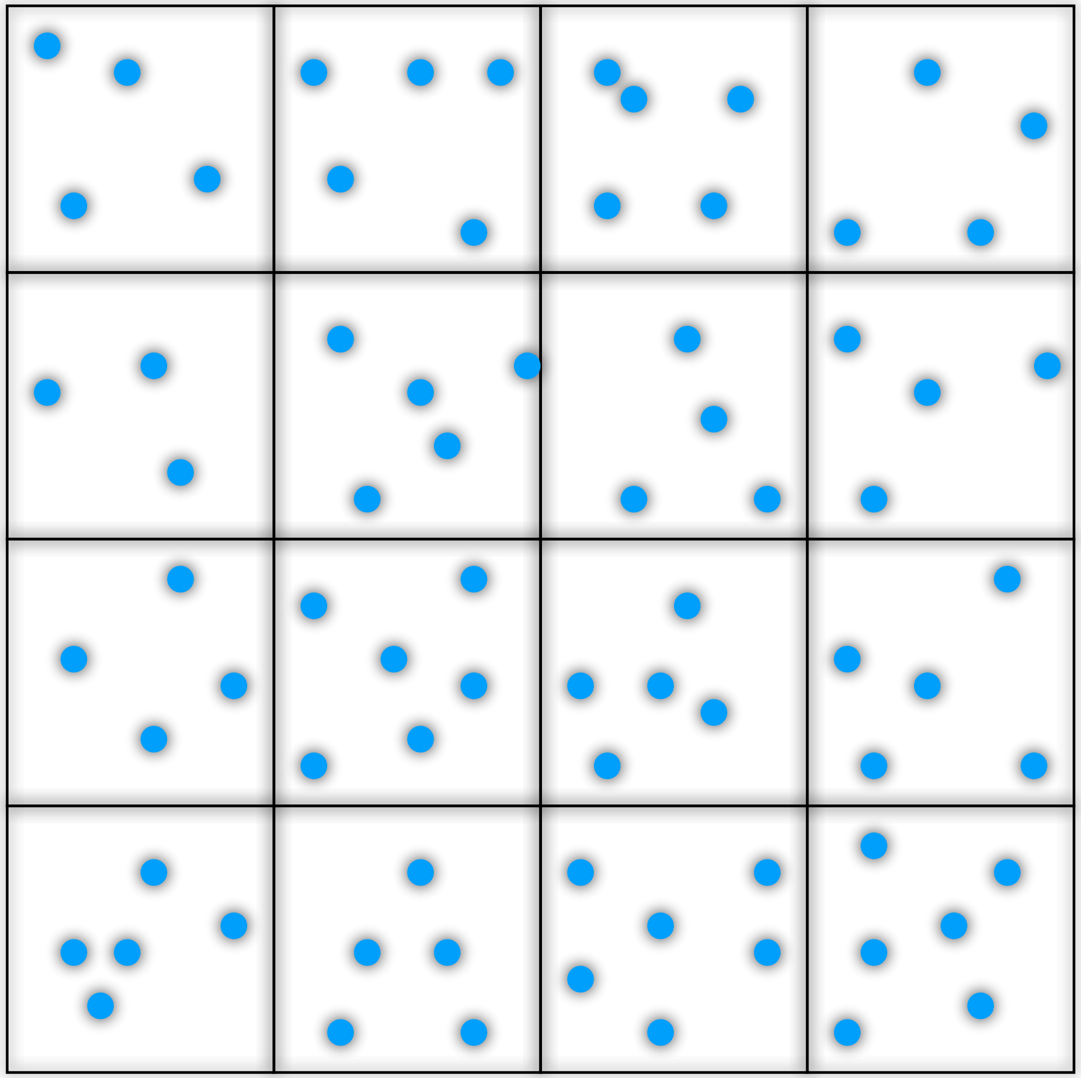

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A NODE is just a group of particles that are far enough (?) so that their gravitational potential is well (?) approximated by its monopole: a single particle in the center of mass, carrying the total mass of the node

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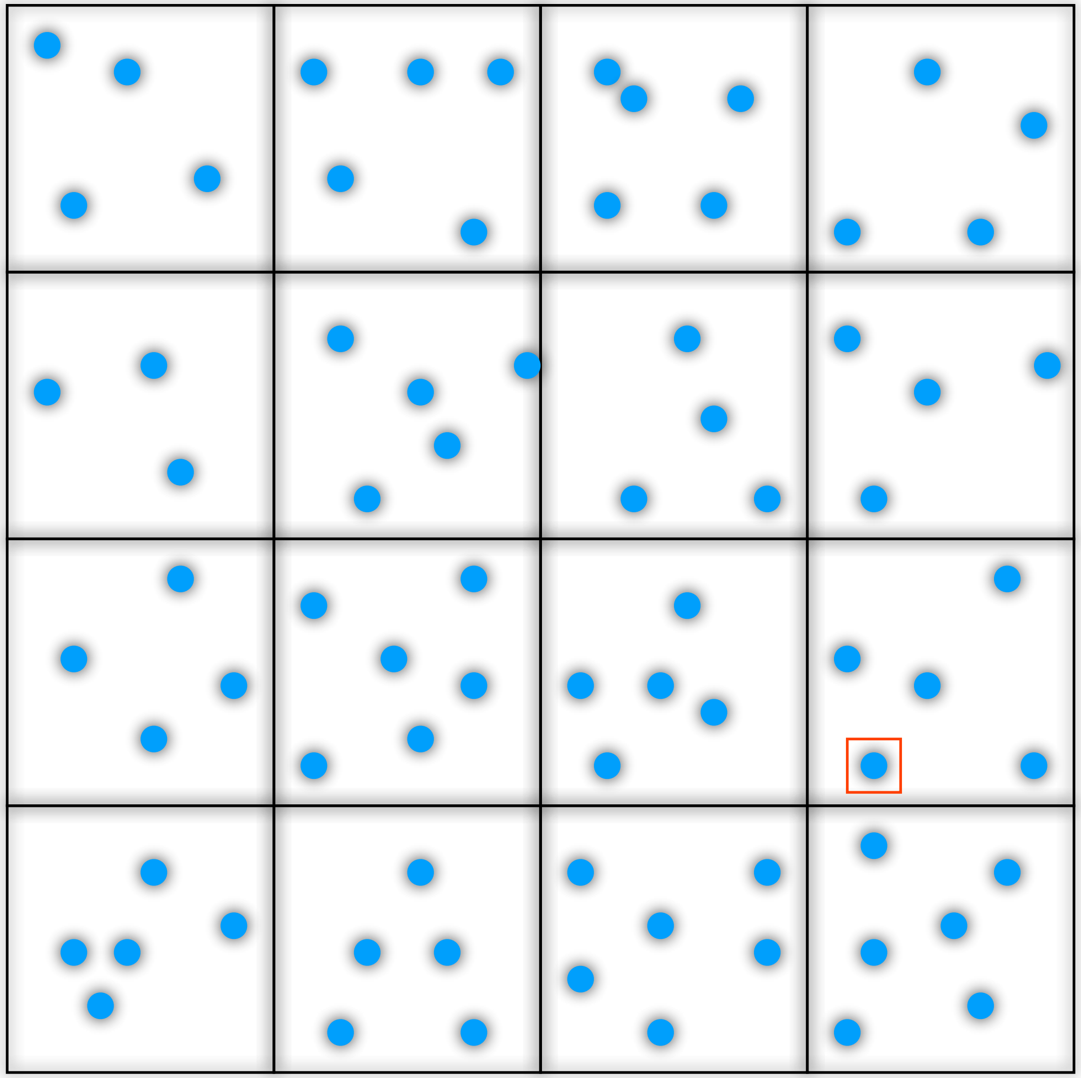

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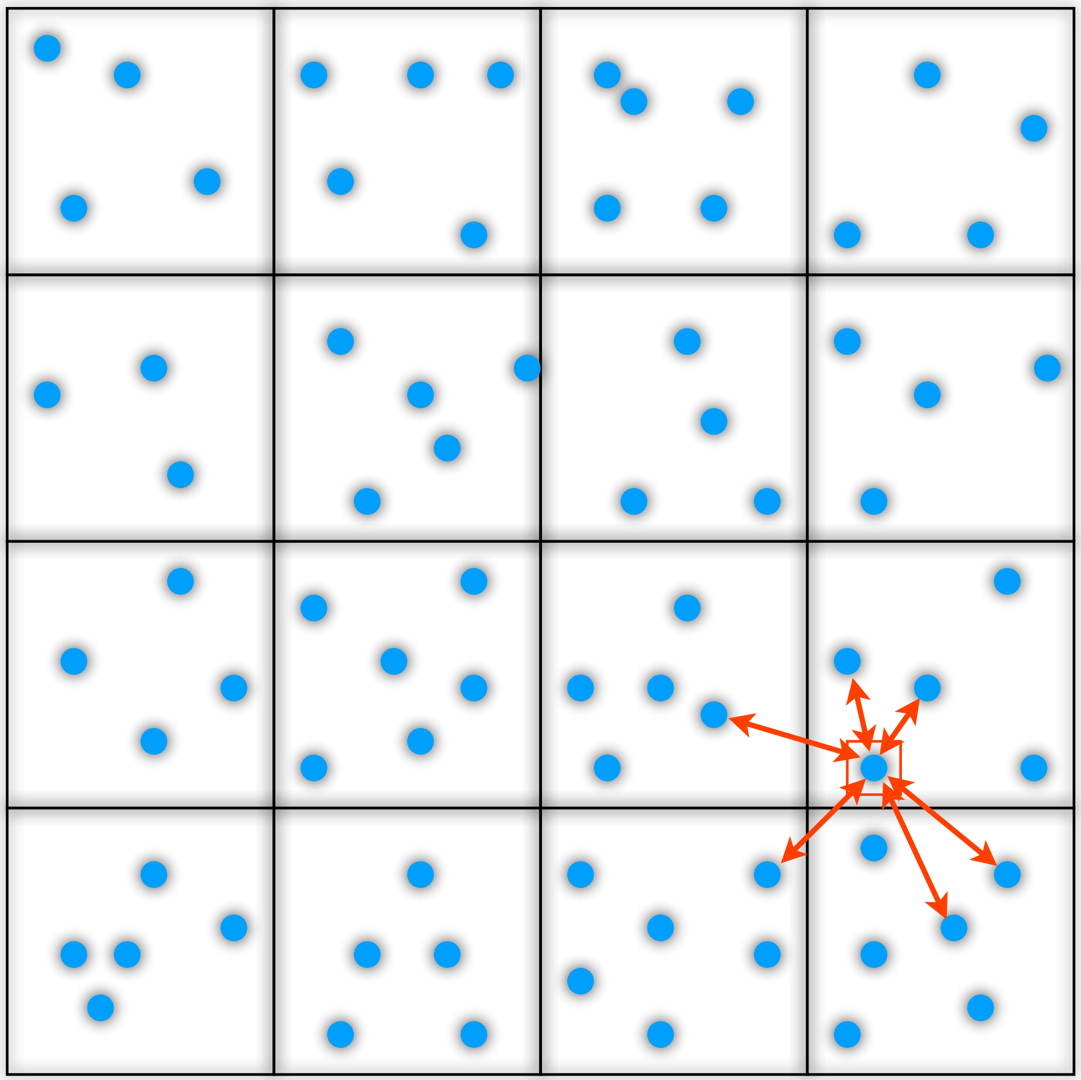

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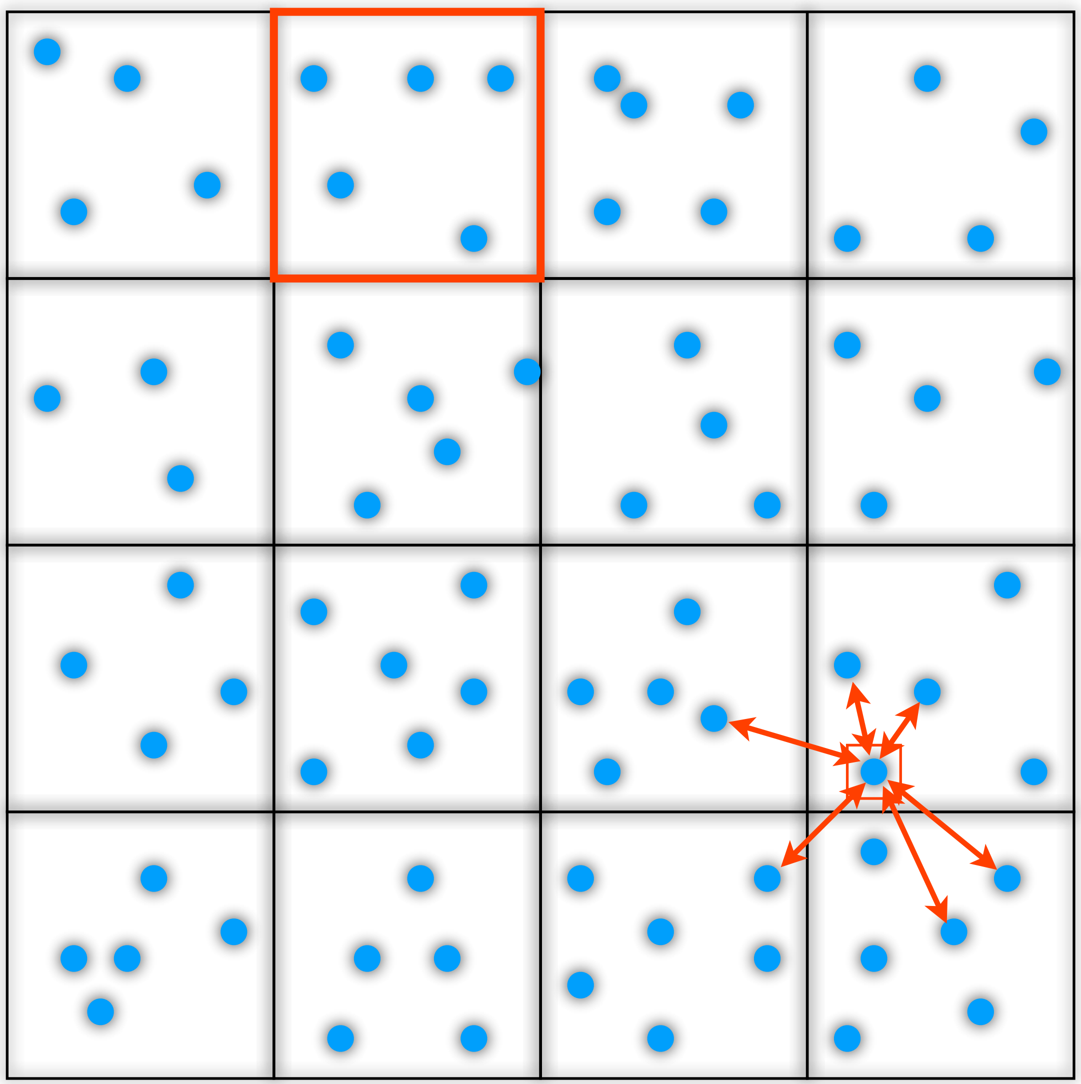

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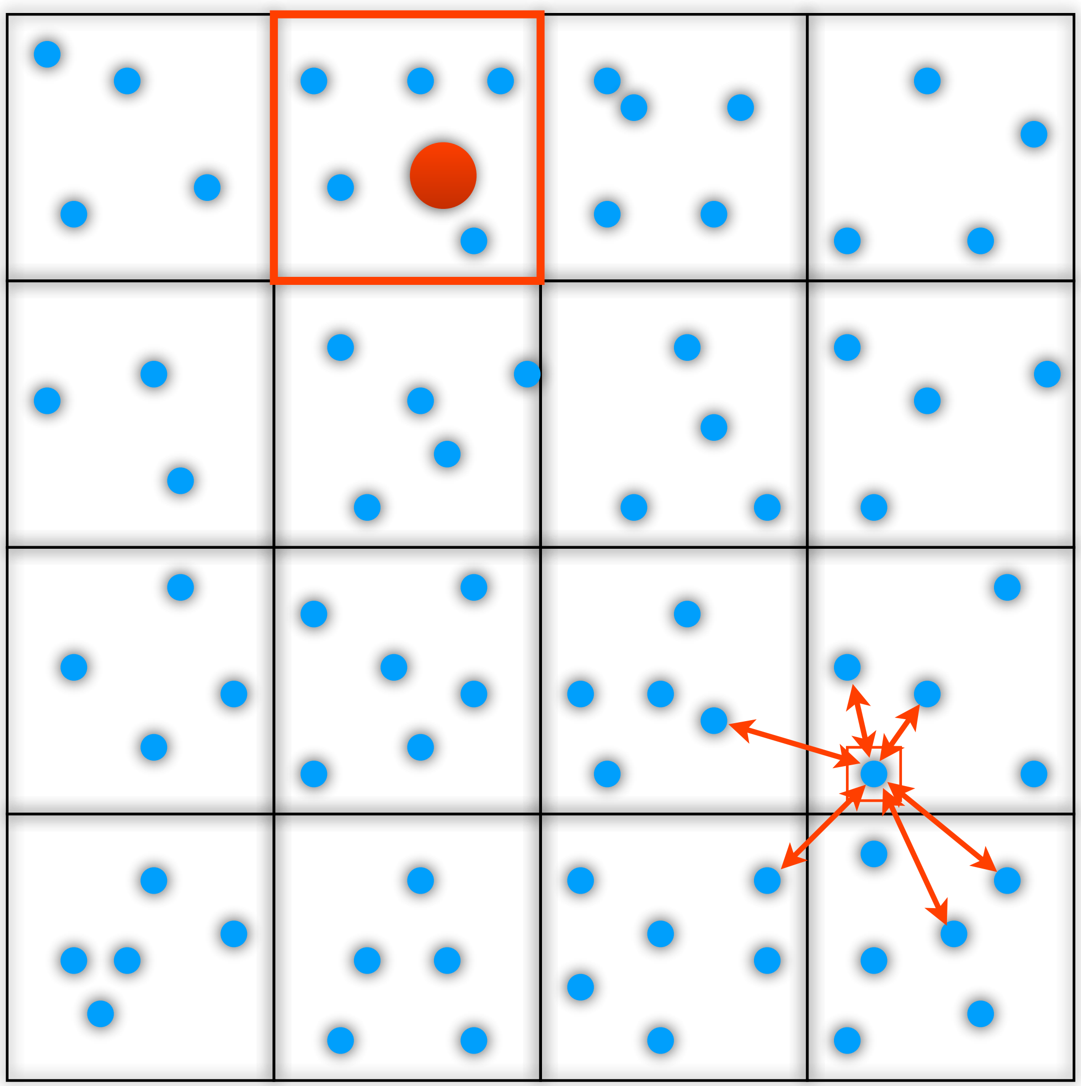

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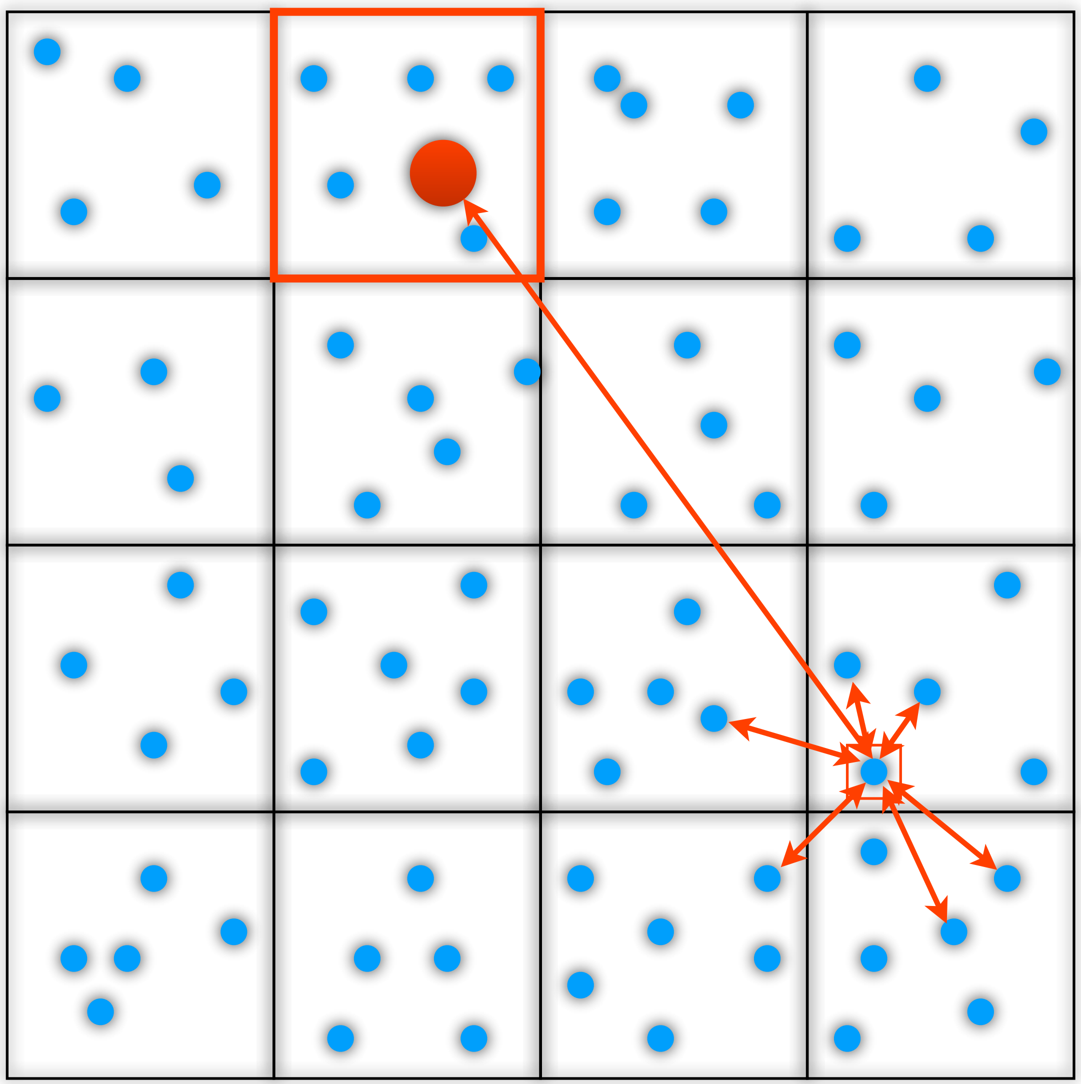

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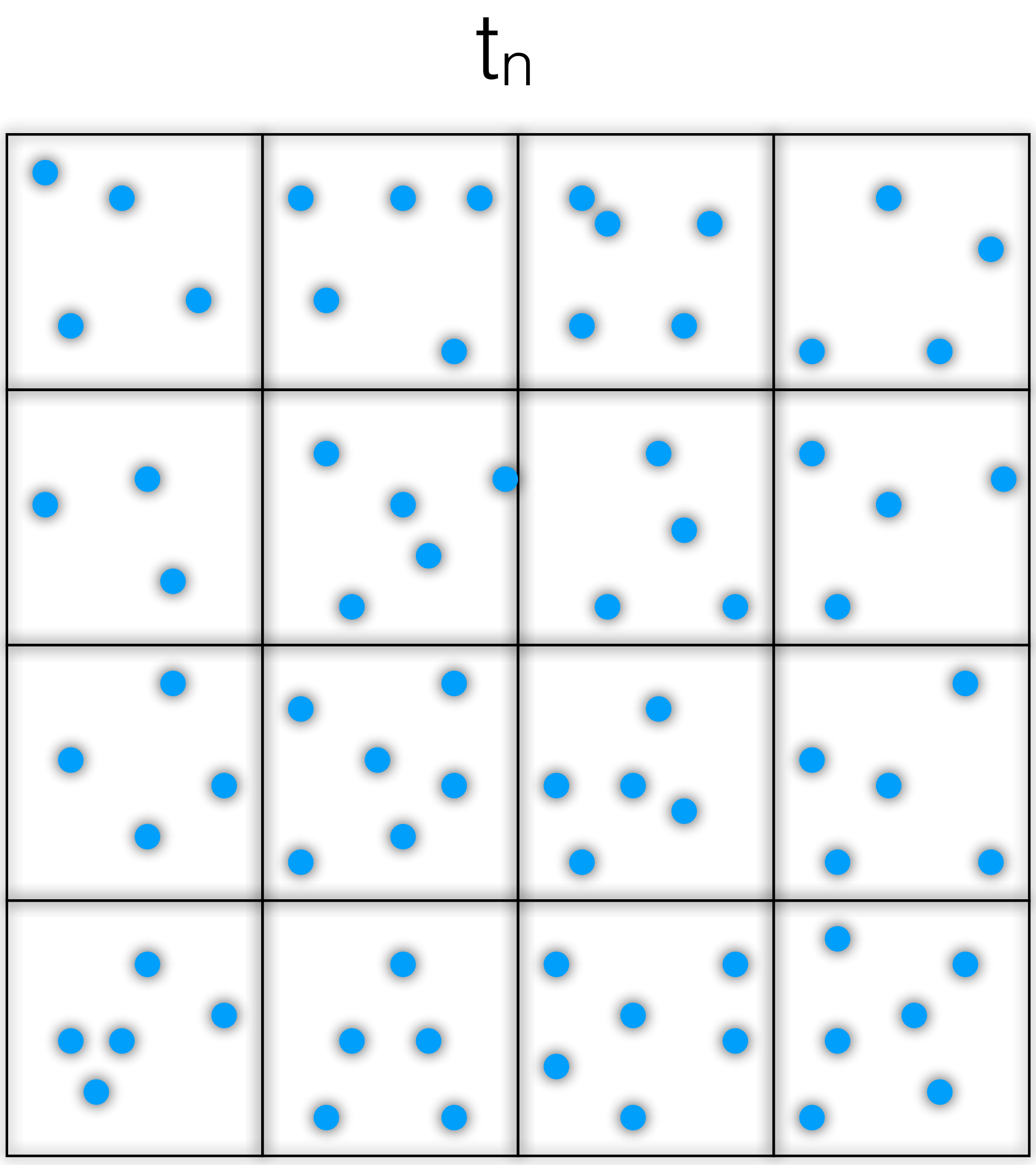
N-body method: the time integration

Once the force (hence the acceleration) on each particle is known, the system has to be moved **forward in time (positions and velocities)**

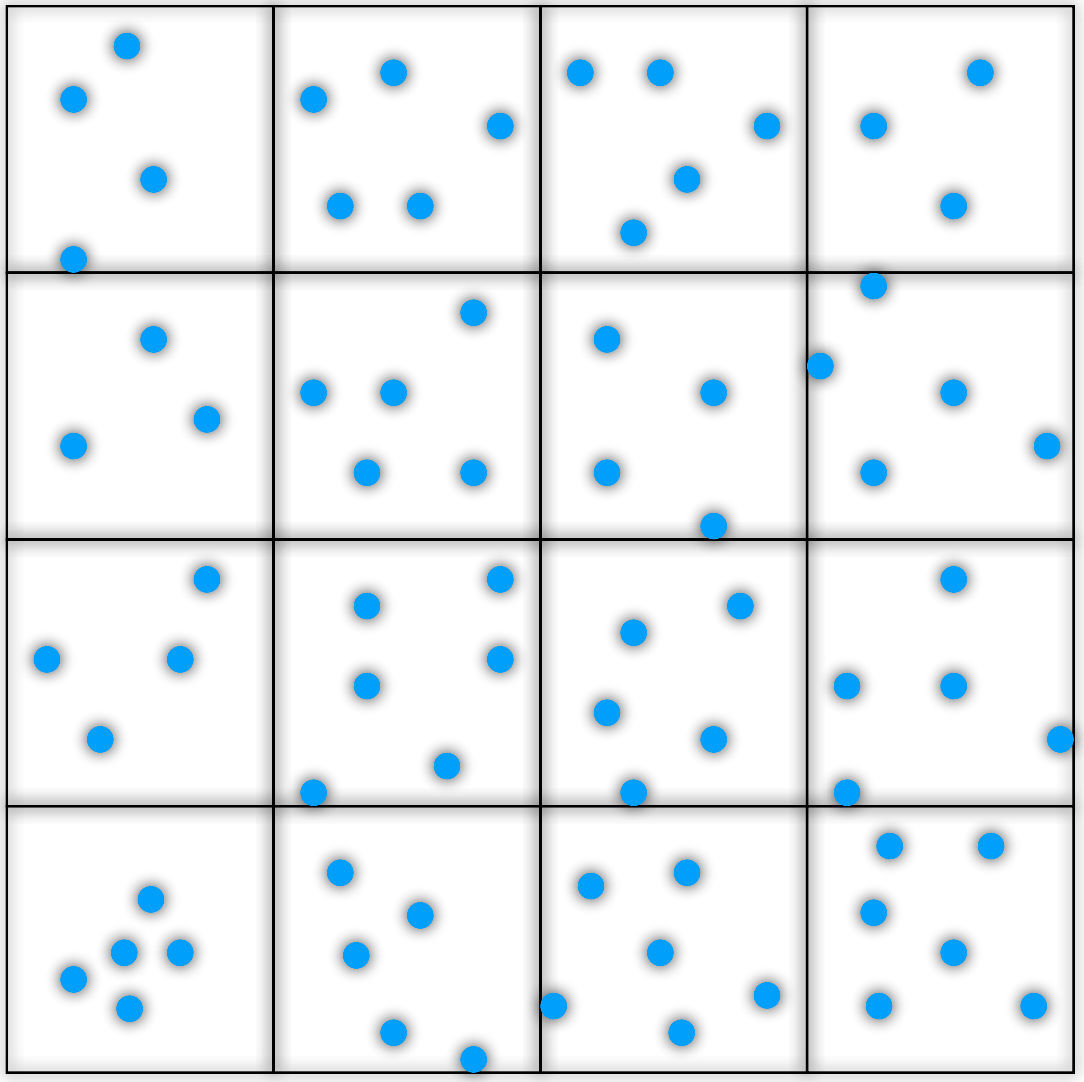


$$\ddot{\mathbf{x}}_i = -\nabla_i \Phi(\mathbf{x}_i) = f(\mathbf{x}_i)$$

N-BODY METHOD: THE TIME INTEGRATION

Once the force (hence the acceleration) on each particle is known, the system has to be moved **forward in time (positions and velocities)**

$t_n + \Delta t$



$$\ddot{\mathbf{x}}_i = -\nabla_i \Phi(\mathbf{x}_i) = f(\mathbf{x}_i)$$

Several possible time integration schemes (Euler, Runge-Kutta, mid-point), the most widely used is the **LEAPFROG:**

$$v_{n+\frac{1}{2}} = v_n + f(x_n)\frac{\Delta t}{2} \qquad \text{Kick}$$

$$x_{n+1} = x_n + v_{n+\frac{1}{2}}\frac{\Delta t}{2} \qquad \text{Drift}$$

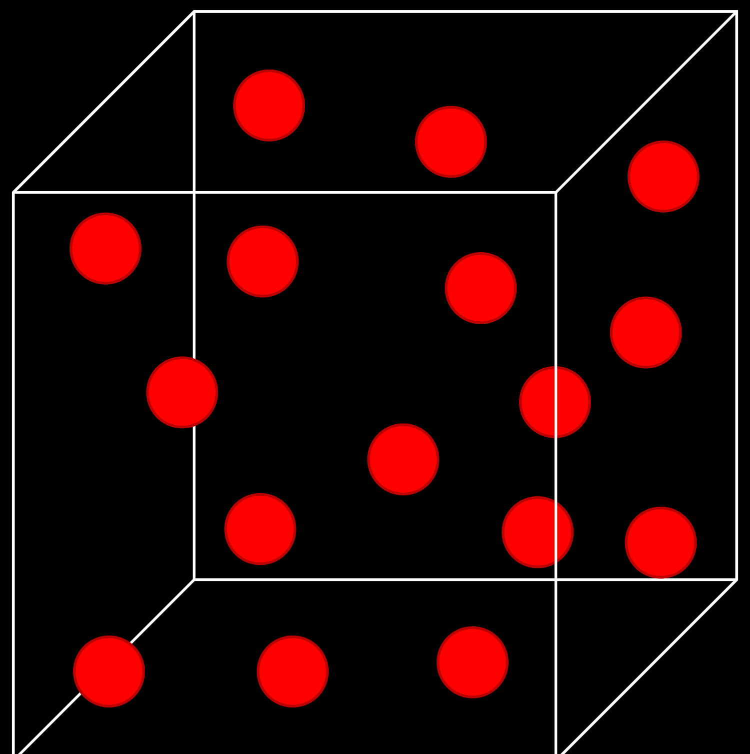
$$v_{n+1} = v_{n+\frac{1}{2}} + f(x_{n+1})\frac{\Delta t}{2} \qquad \text{Kick}$$

How to add hydrodynamics: the SPH method

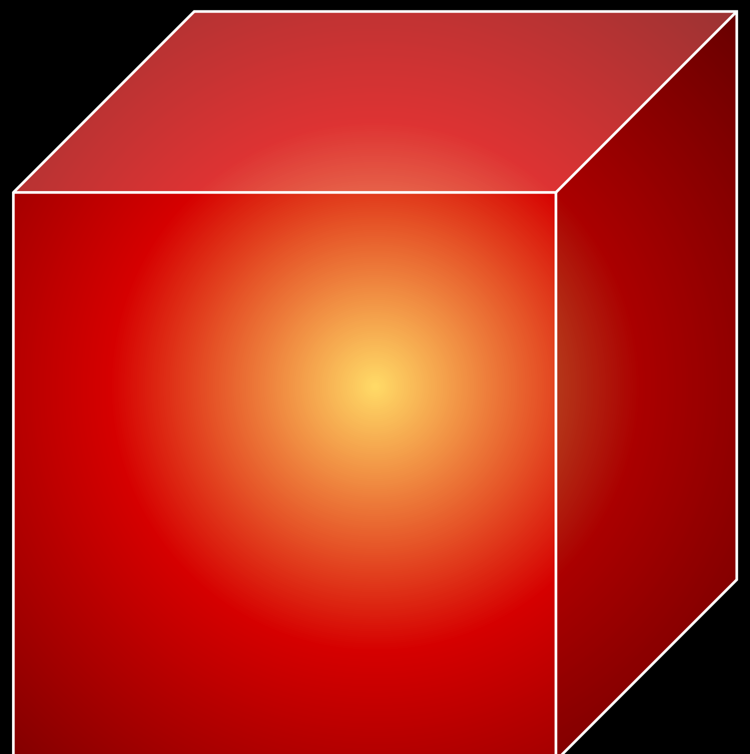
Credit: all slides with a black background are a ***courtesy of my PhD student Matteo Nori***

SPH implementation

$\{\vec{r}_i, o_i\}$



$o(\vec{r})$



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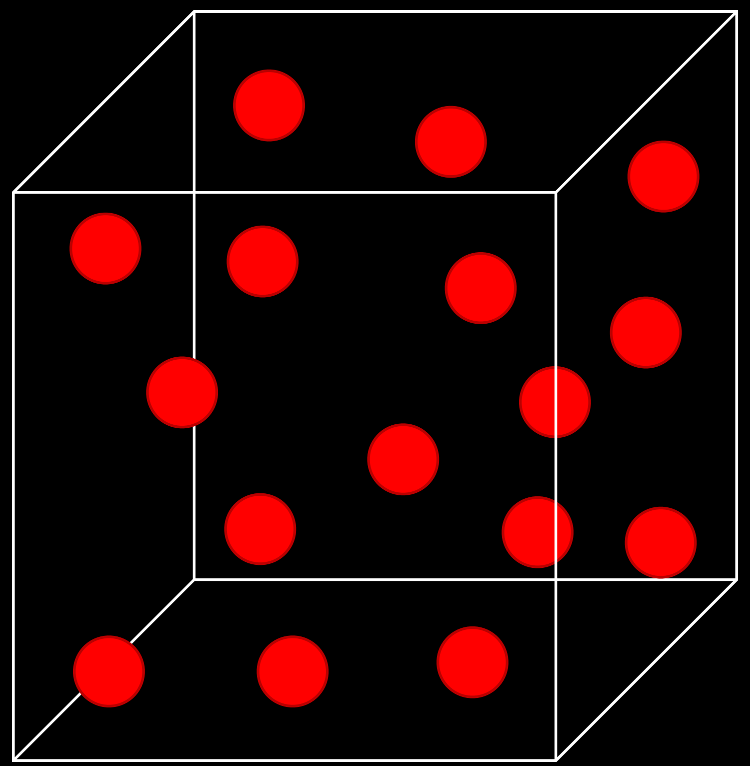
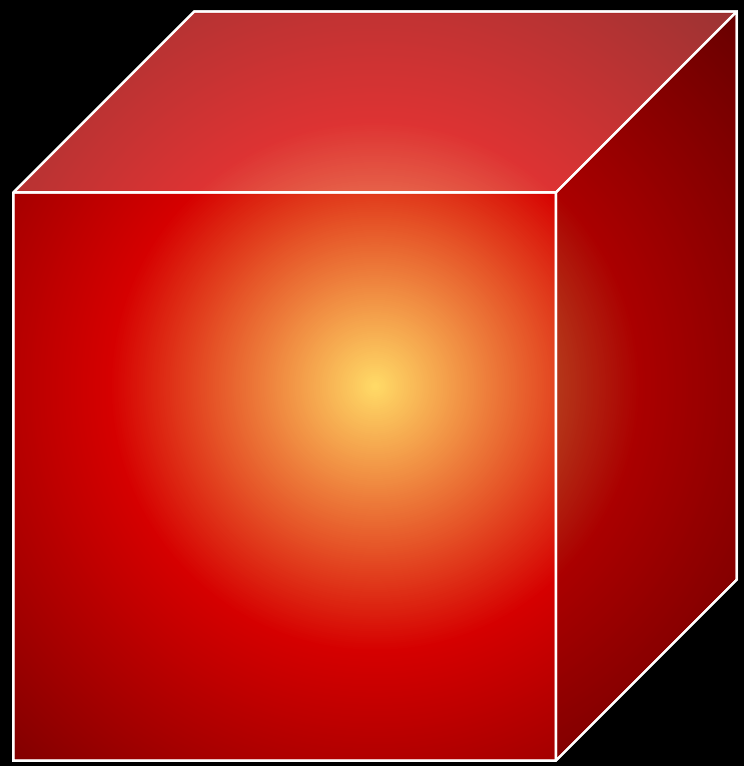


$$V_i\,\rho_i = M$$

$\boldsymbol{o}(\vec{\boldsymbol{r}})$



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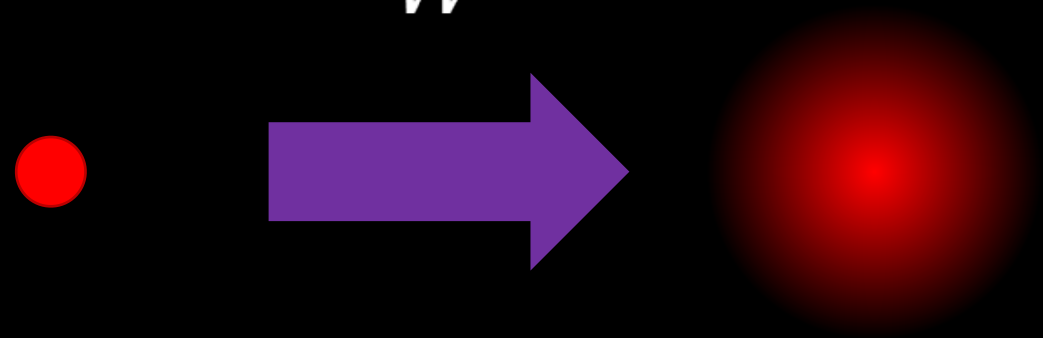
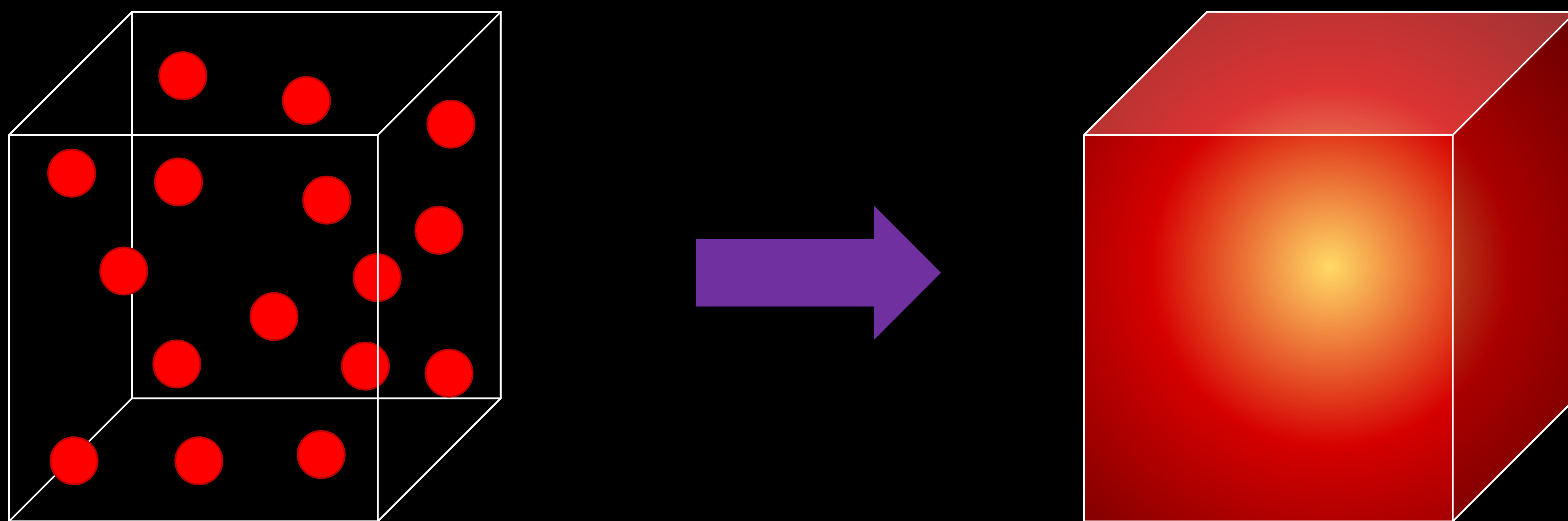


SPH implementation

$\{\vec{\boldsymbol{r}}_i, \boldsymbol{O}_i\}$ $\boldsymbol{O}(\vec{\boldsymbol{r}})$



$$O_i = \sum_{\mathrm{j}} m_j W_{ij} \frac{O_j}{\rho_j}$$

$$\vec{\nabla} O_i = \sum_{\mathrm{j}} m_j \vec{\nabla} W_{ij} \frac{O_j}{\rho_j}$$

The N-body code GADGET

GADGET IN PILLS

GADGET is a parallel N-body SPH code based on a TreePM algorithm that is widely used in the community:

- GADGET1 (Springel, Yoshida, White, 2001) 806 citations
- GADGET2 (Springel 2005) 2376 citations
- GADGET3 (not public)
- GADGET4 (coming soon?)

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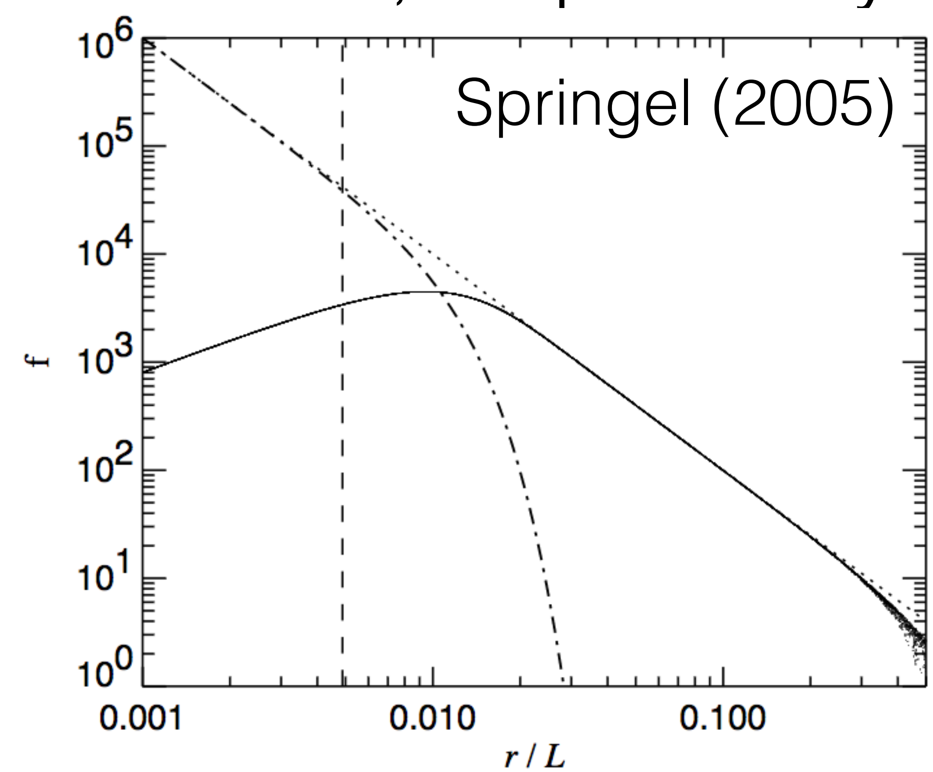
What does **TreePM** mean?

The gravitational potential is split into a long-range part and a short-range part, which are computed with PM and Tree, respectively

$$\phi_{\mathbf{k}} = \phi_{\mathbf{k}}^{\text{short}} + \phi_{\mathbf{k}}^{\text{long}}$$

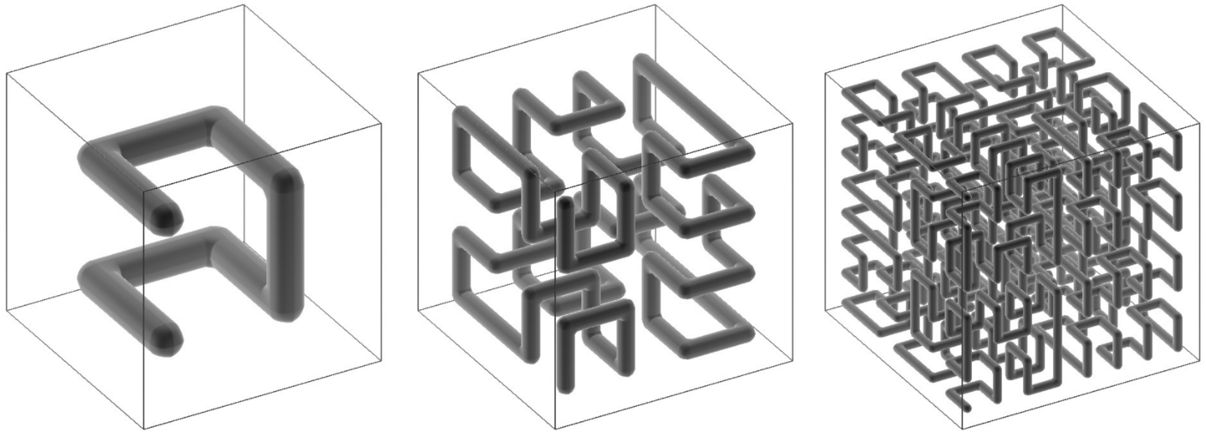
$$\phi_{\mathbf{k}}^{\text{long}} = \phi_{\mathbf{k}} \exp(-\mathbf{k}^2 r_s^2)$$ —> PM

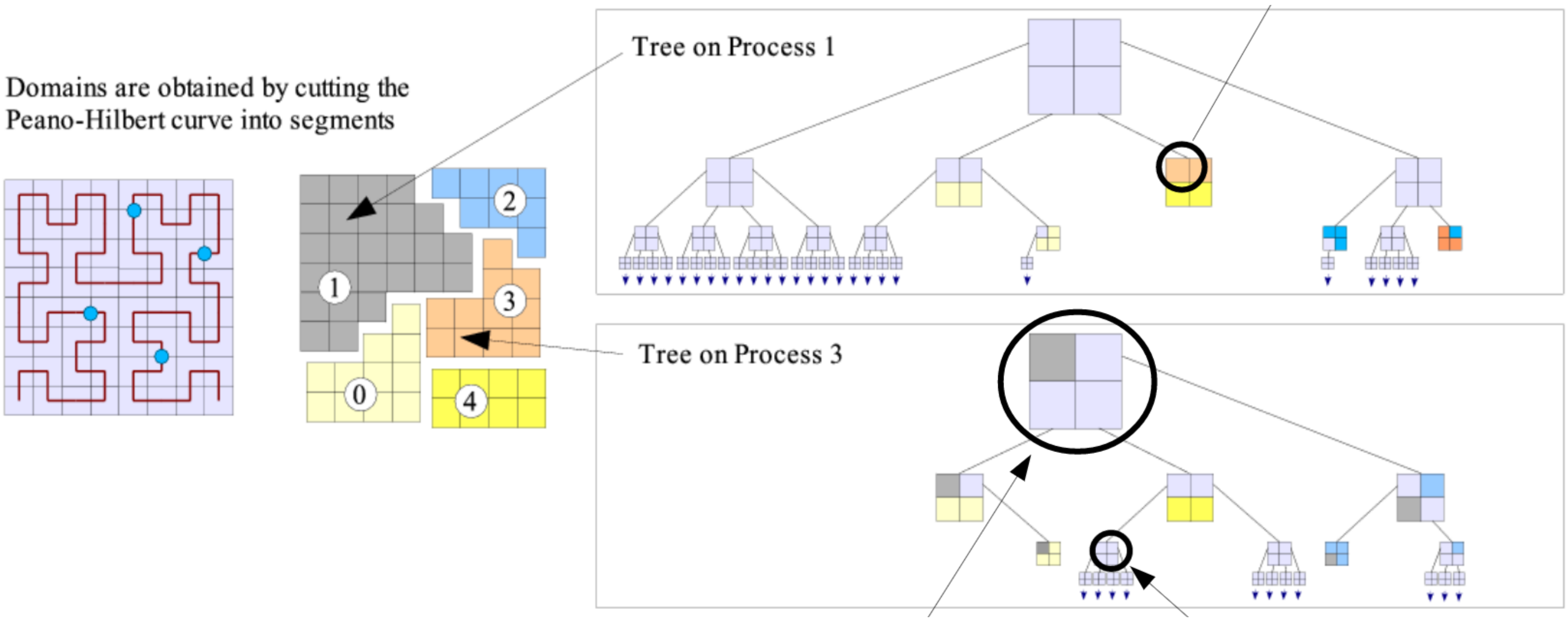
$$\phi_{\mathbf{k}}^{\text{short}} = \phi_{\mathbf{k}} \left[1 - \exp(-\mathbf{k}^2 r_s^2)\right]$$ —> Tree



GADGET IN PILLS

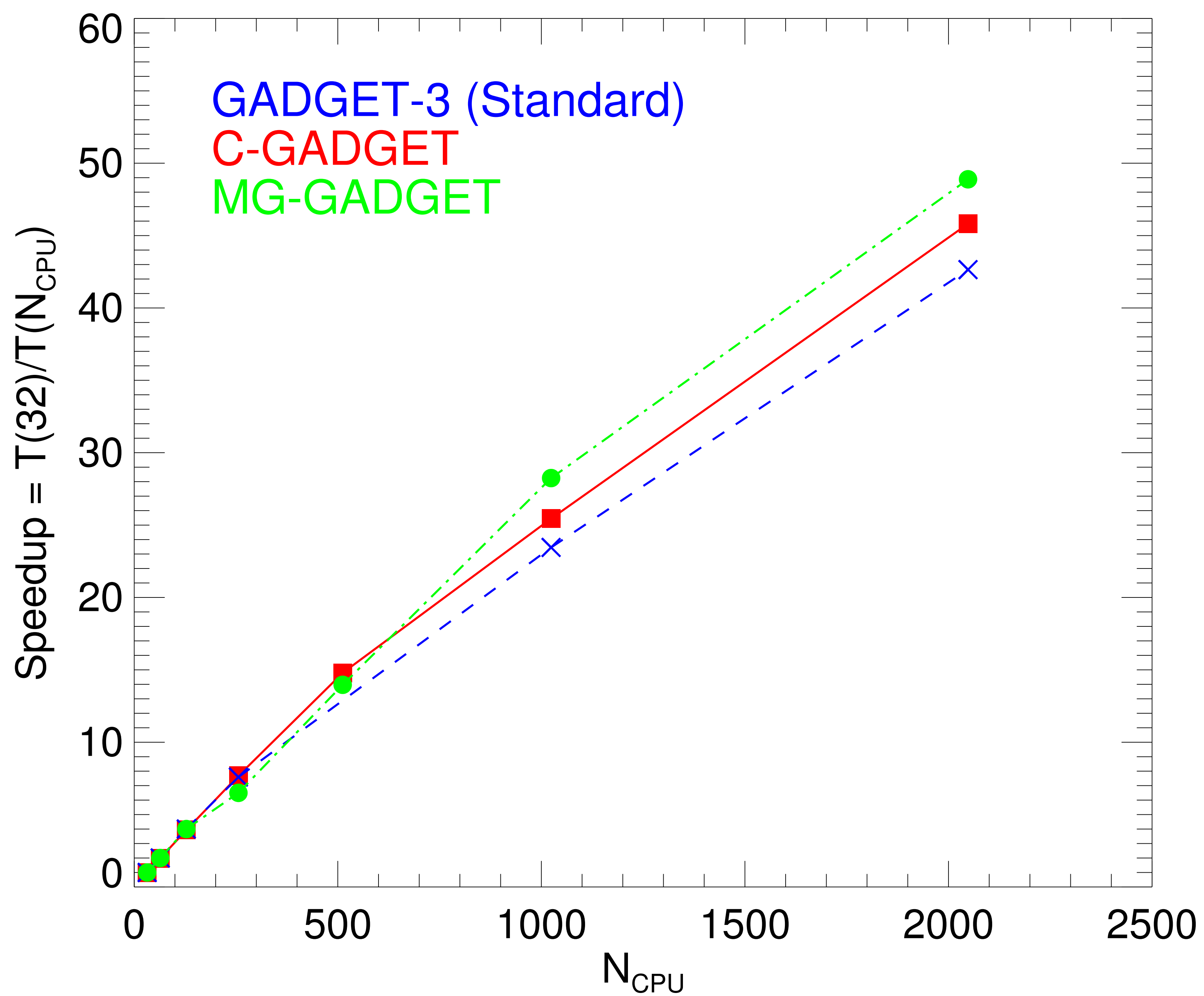
Language: **ANSI-C**
Parallelization: **MPI** (pthreads, hybrid MPI/OpenMP)
Domain decomposition: space-filling Peano-Hilbert curve





GADGET IN PILLS

Scalability: tested up to 8192 cores with MPI implementation
tested up to 12000 cores with hybrid MPI/OpenMP



ACCURACY OF N-BODY SIMULATIONS

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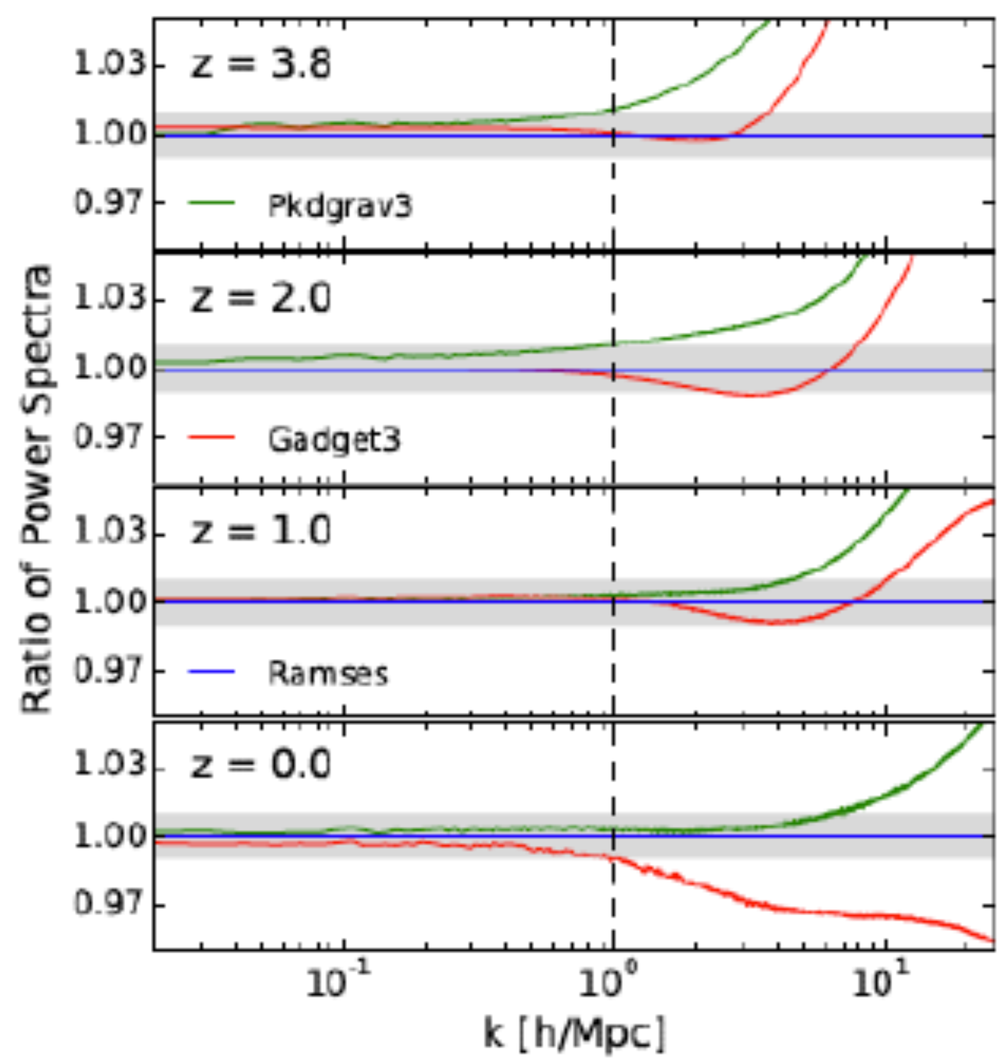
Code comparison: Euclid "calibration" simulations

Codes: **Ramses, Pkdgrav3, and Gadget3**
Parameters: **1 Gpc/h, 2048^3** (M_p=10^9 Msun/h)

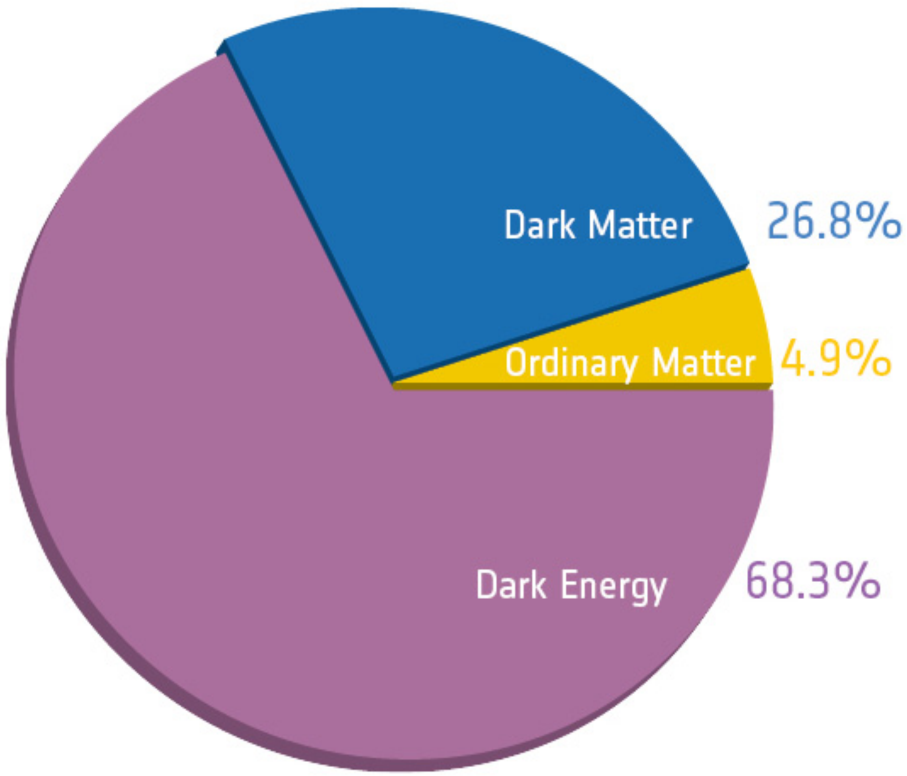
- ✓ **All codes agree within 1% up to k = 1 h/Mpc** (for all z), and ~3 % up to k = 10 h/Mpc (z<1)
- ✓ **Box-size L > 500 h/Mpc** is required for 1% accuracy in the P(k)
- ✓ **Particle mass** must be at least **10^9 h/Msun** for 1% accurate power up **to k = 10 h/Mpc**. [or 8x10^{10} h/Msun, for k<1 h/Mpc]
- ✓ **Higher-order Lagrangian displacements** required for ICs (**2LPT**)



Schneider et al. arXiv: 1503.05920

Non-standard cosmologies

The standard cosmological model



The Universe after Planck:
6 parameters to fit all data

$$\Omega_{\mathrm{b}} = 0.049 \qquad \Omega_{\mathrm{CDM}} = 0.265$$

$$\Omega_{\Lambda} = 0.6844 \qquad \sigma_8 = 0.831$$

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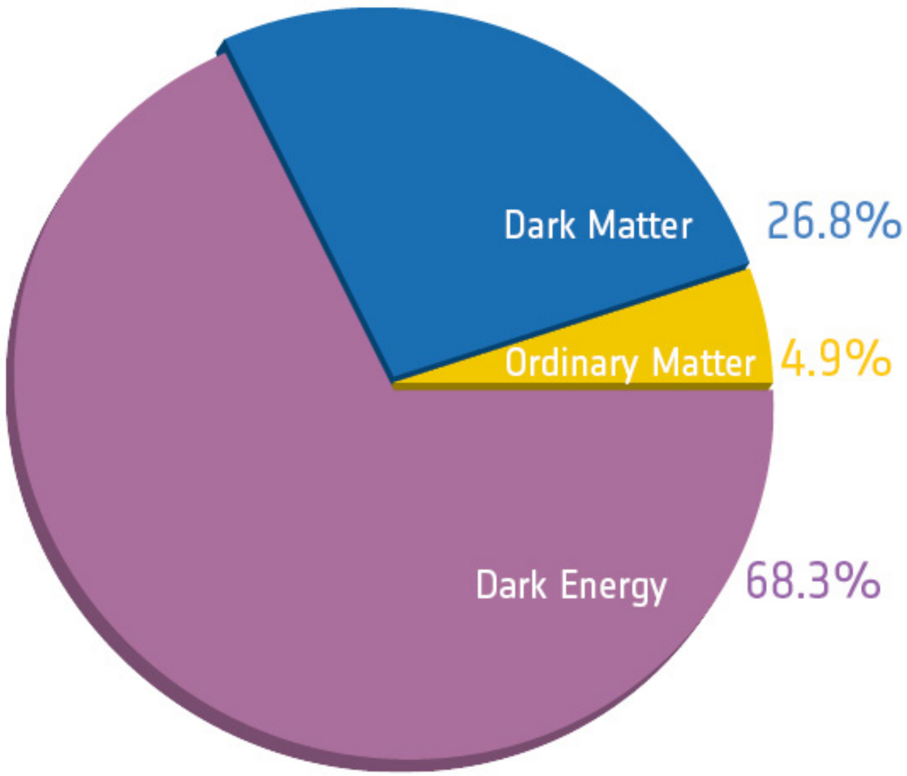

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Standard ΛCDM cosmology is based on a series of assumptions:

- Cosmological Principle (homogeneity & isotropy);
- Gaussian and Adiabatic initial conditions;
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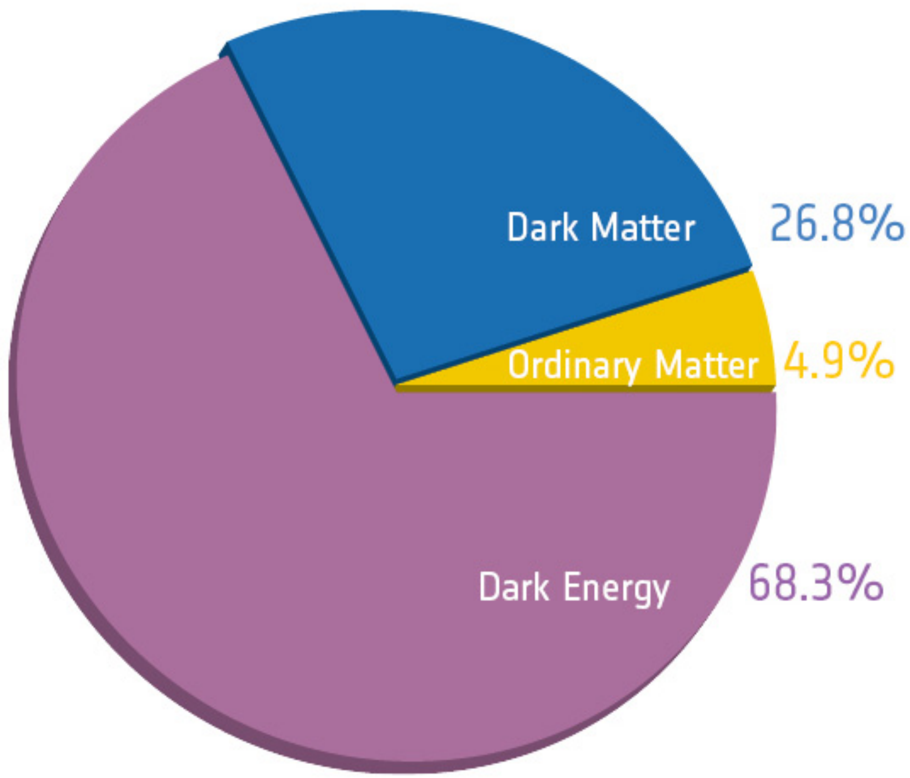
Testing the assumptions of the SM



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DE is not a cosmological constant but a dynamical d.o.f. with perturbations and interactions (e.g. to CDM or massive neutrinos)

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Background:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_M a^{-3} + (1-\Omega_M)\exp\left(-3\int_1^a \frac{1+w(a')}{a'}da'\right)$$

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Structure formation:

$$\nabla^2\Phi_g = -4\pi G\,(\delta\rho_{\mathrm{M}}+\delta\rho_\phi)$$

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$$\delta\rho_\phi \propto \delta\phi$$

Field equations:

$$\nabla^2\delta\phi = -\frac{dV}{d\phi}(\delta\phi) + 4\pi G\beta(\phi)\delta\rho_M$$

$$\nabla^2\delta\phi \approx \beta(\phi)\delta\rho_M \Rightarrow \delta\phi \approx \beta(\phi)\Phi_g$$

if we assume a non-universal interaction that leaves baryons uncoupled

non-standard models: Dark Energy Fields

[MB, v. pettorino, G. Robbers, v. springel, MNRAS 403 (2010)]

Interacting DE: two families of particles with independent couplings

$$\vec{a}_1 = -\vec{\nabla}\Phi_N - 2\beta_1^2\vec{\nabla}\Phi_1 - 2\beta_1\beta_2\vec{\nabla}\Phi_2 + \beta_1\dot{\phi}\vec{v}_1$$

$$\vec{a}_2 = -\vec{\nabla}\Phi_N - 2\beta_2^2\vec{\nabla}\Phi_2 - 2\beta_1\beta_2\vec{\nabla}\Phi_1 + \beta_2\dot{\phi}\vec{v}_2$$

Different phenomenology for different types of coupled particles and strength of coupling

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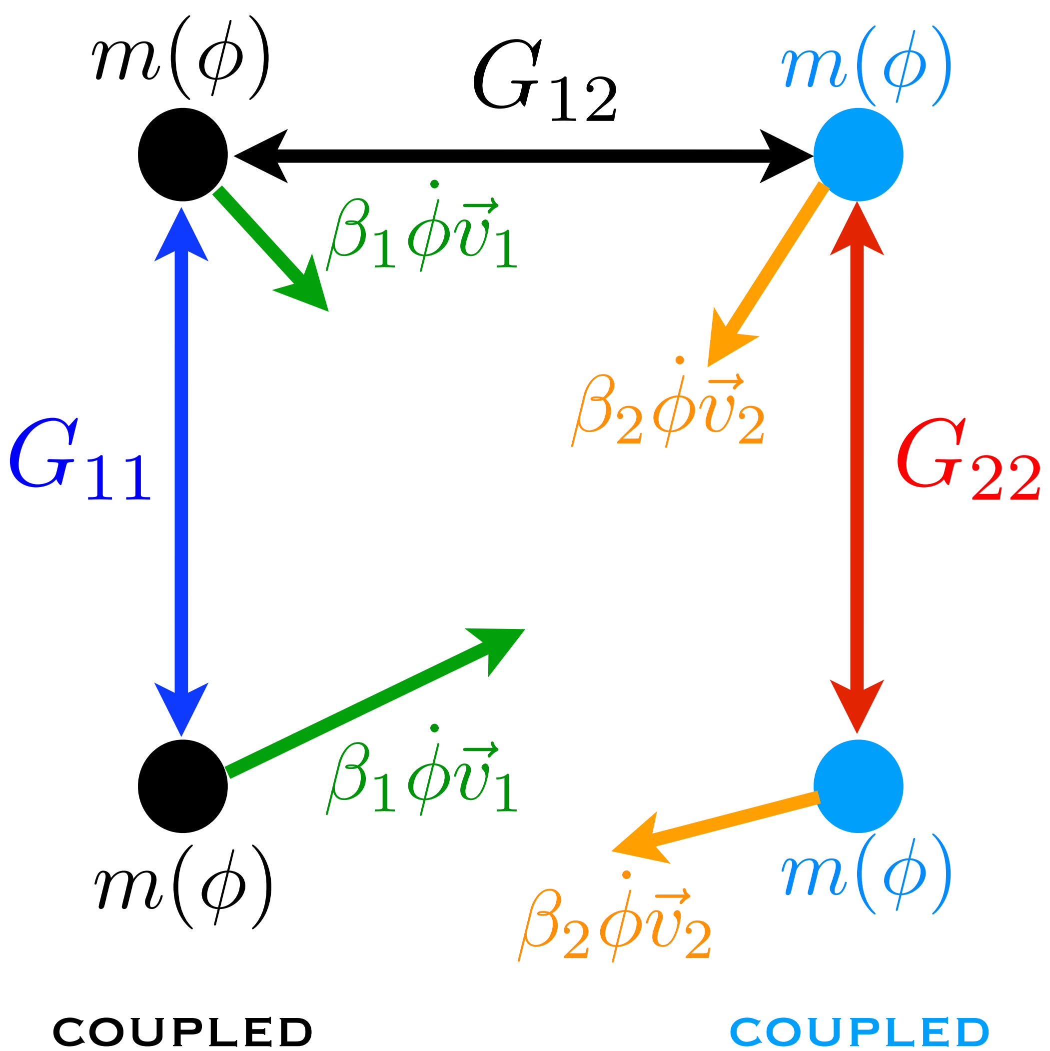

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case 1: CDM is coupled, baryons are uncoupled

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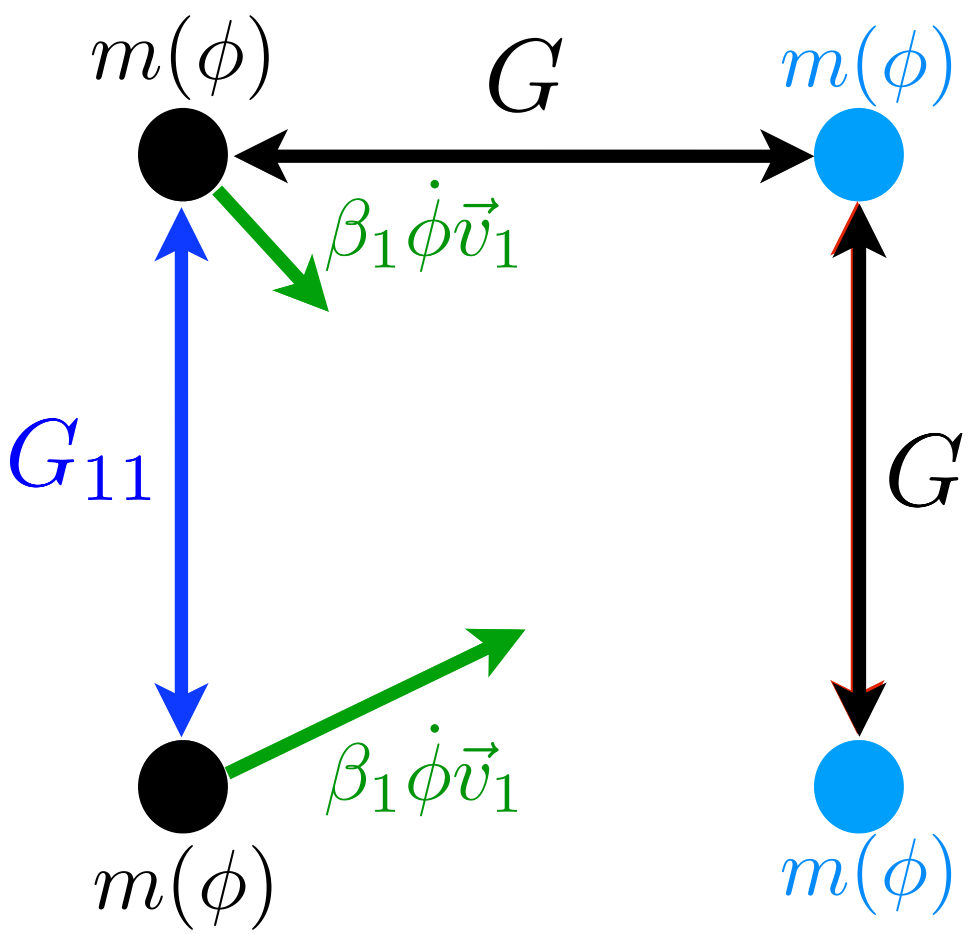
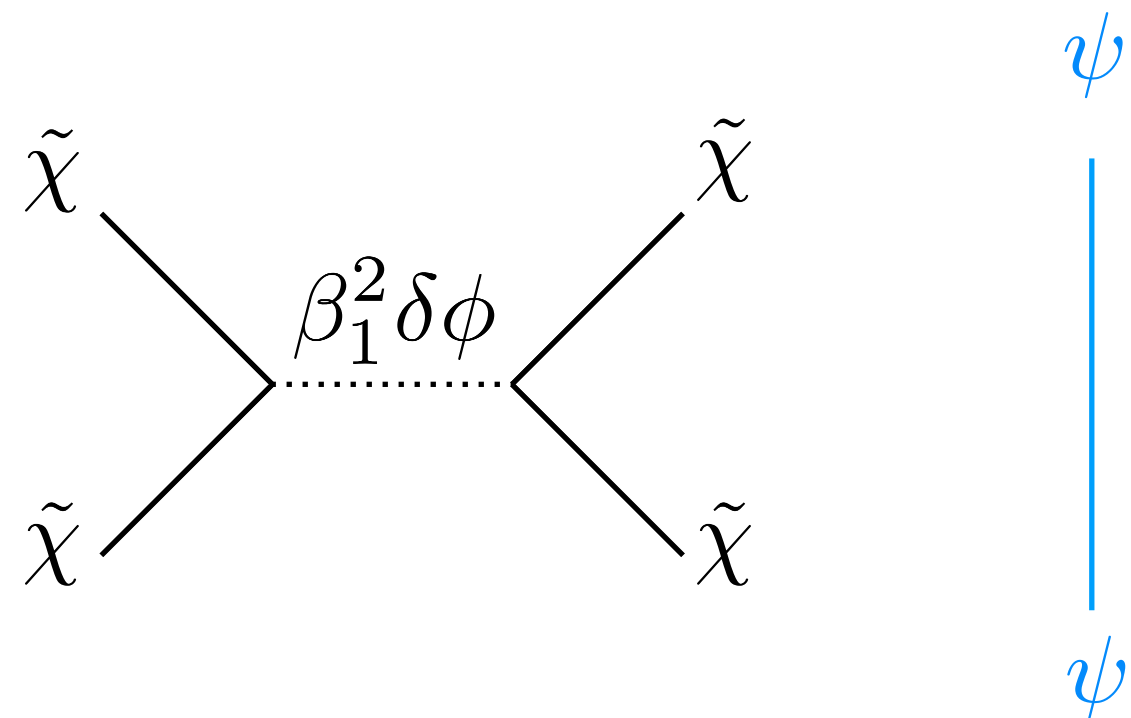


COUPLED UNCOUPLED



Just a cartoon!
Not a quantum theory!

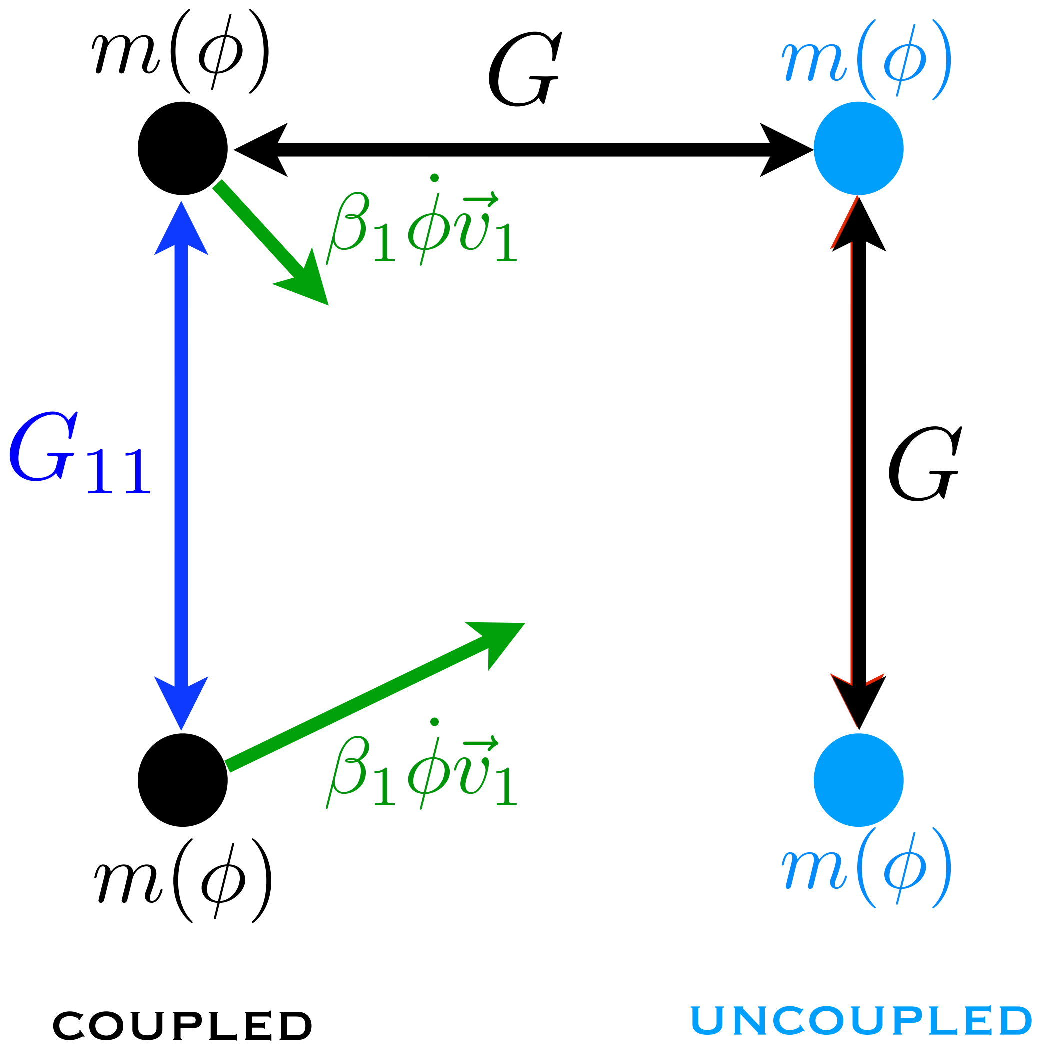
NON-STANDARD MODELS: DARK ENERGY FIELDS

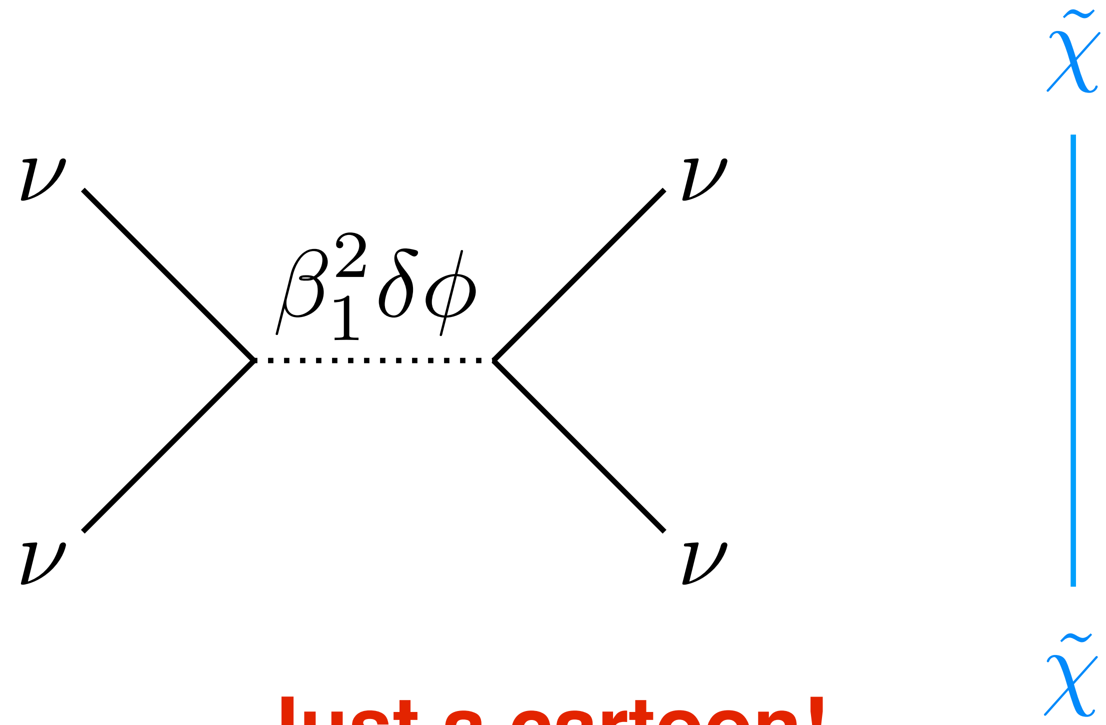
[MB, V. PETTORINO, G. ROBBERS, V. SPRINGEL, MNRAS 403 (2010)]

case 2: CDM and baryons are uncoupled, neutrinos are coupled

$$\vec{a}_1 = -\vec{\nabla}\Phi_N - 2\beta_1^2\vec{\nabla}\Phi_1 \quad + \beta_1\dot{\phi}\vec{v}_1$$

$$\vec{a}_2 = -\vec{\nabla}\Phi_N$$





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NON-STANDARD MODELS: DARK ENERGY FIELDS

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case 3: a CDM doublet with opposite coupling constants

$$\vec{a}_1 = -\vec{\nabla}\Phi_N - 2\beta_1^2\vec{\nabla}\Phi_1 - 2\beta_1\beta_2\vec{\nabla}\Phi_2 + \beta_1\dot{\phi}\vec{v}_1$$

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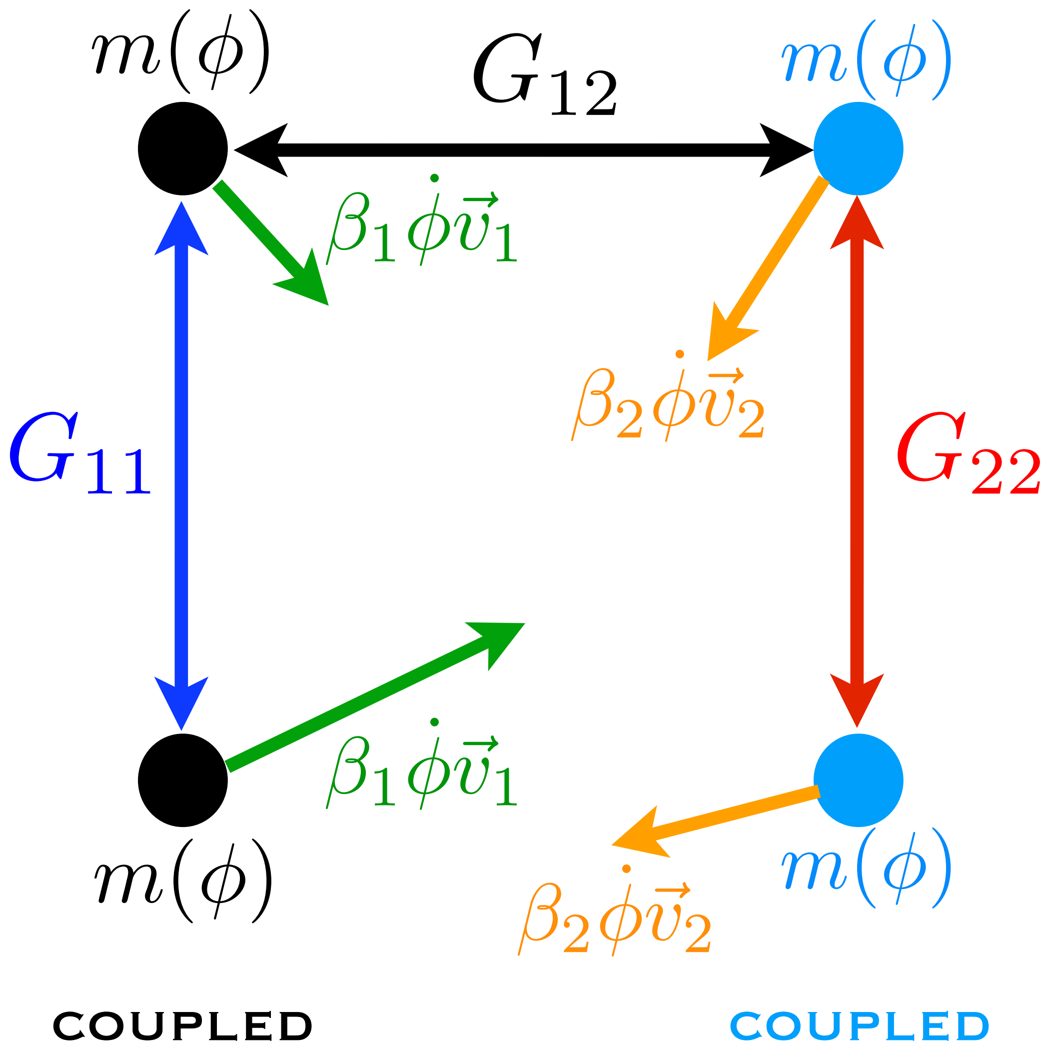
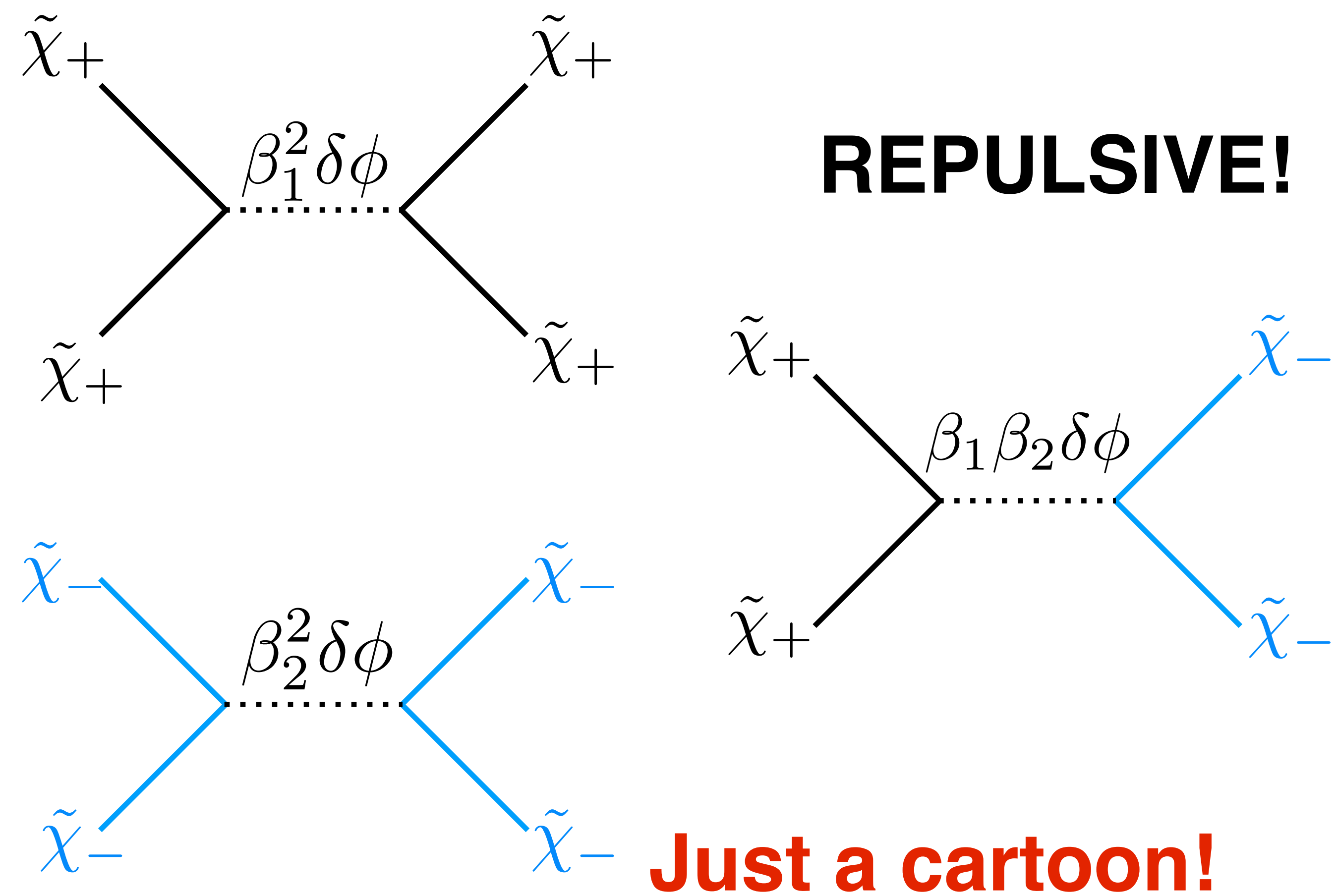
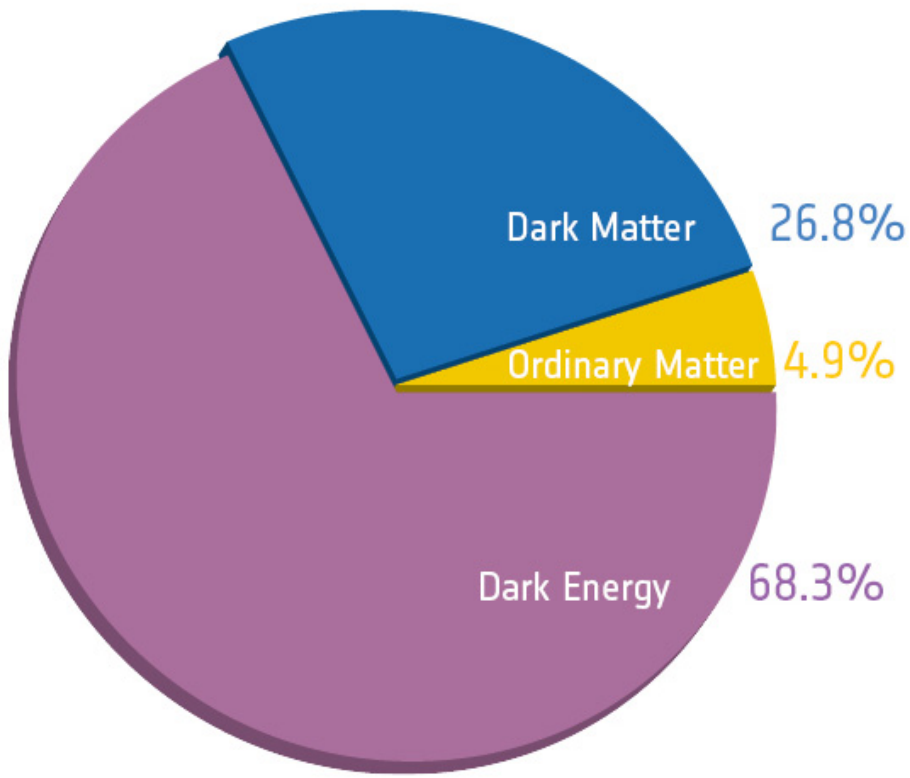




REPULSIVE!

Just a cartoon!
Not a quantum theory!

Testing the assumptions of the SM



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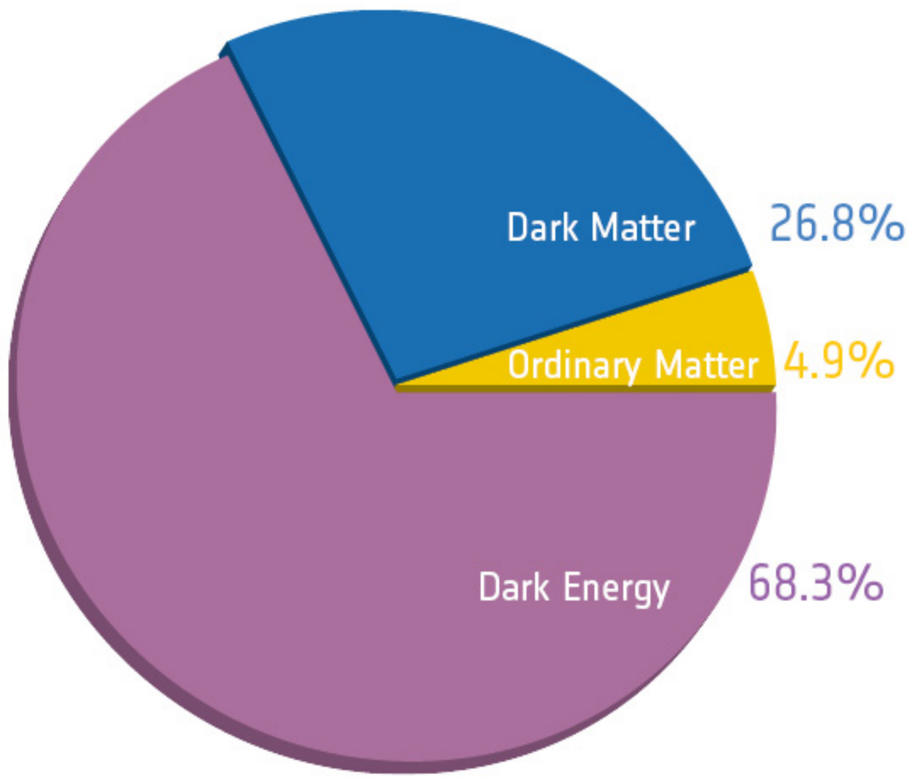

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non-standard models: Modified Gravity

Gravity does not follow GR at large scales: $R \rightarrow R + f(R)$
GR recovered in our local environment (screening mechanism)

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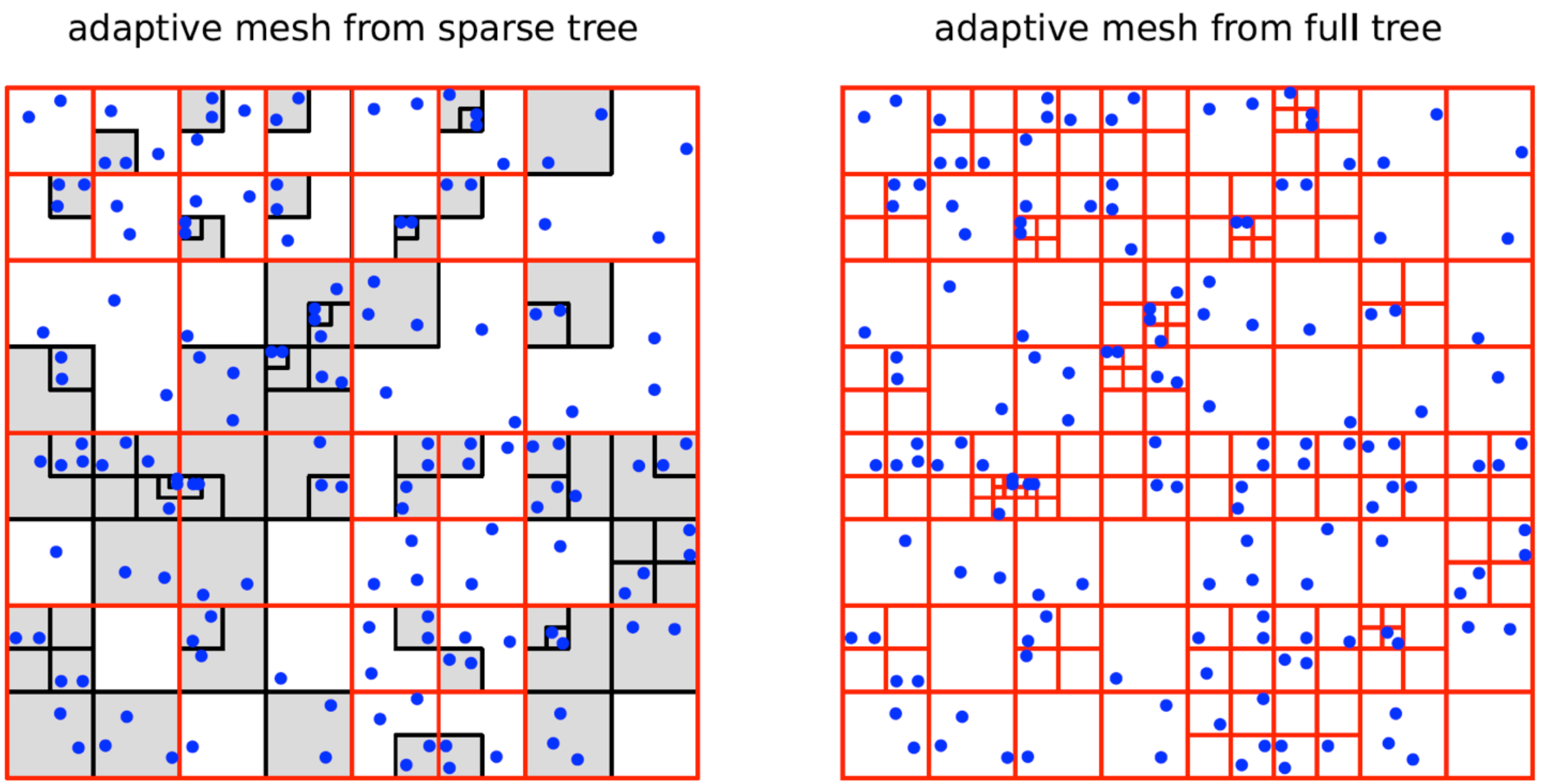
Can be solved with an iterative NGS relaxation scheme on an AMR grid obtained from the Gadget gravitational tree



MG-GADGET

Puchwein, MB, Springel 2013

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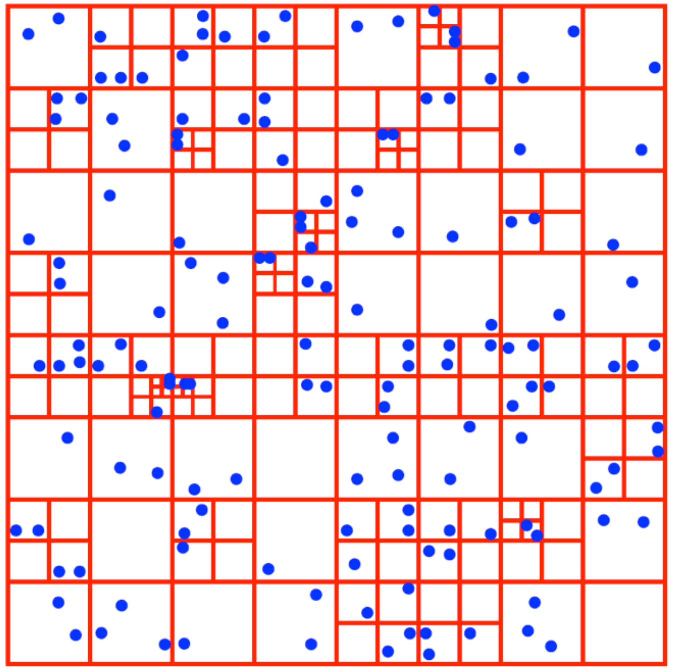

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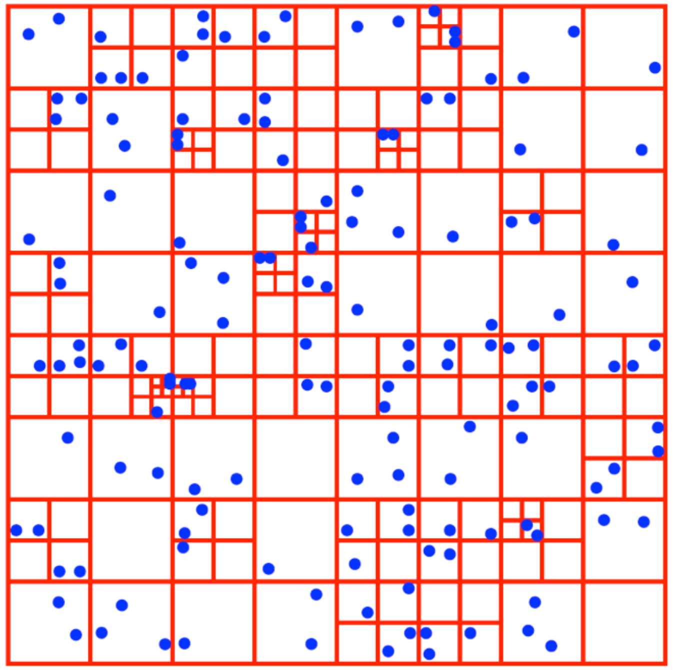
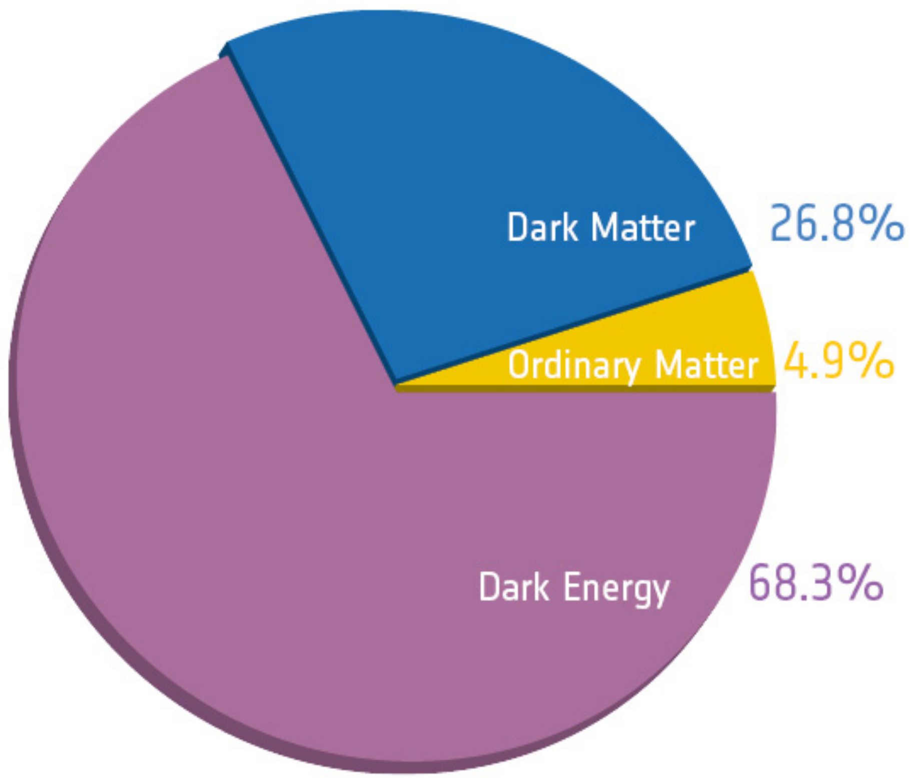
adaptive mesh from full tree



Structure formation:

$$\nabla^2 \Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R$$

Testing the assumptions of the SM



The Universe after Planck: 6 parameters to fit all data

$$\Omega_{\mathrm{b}} = 0.049 \qquad \Omega_{\mathrm{CDM}} = 0.265$$

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$$n_s = 0.9645 \qquad \tau = 0.079$$

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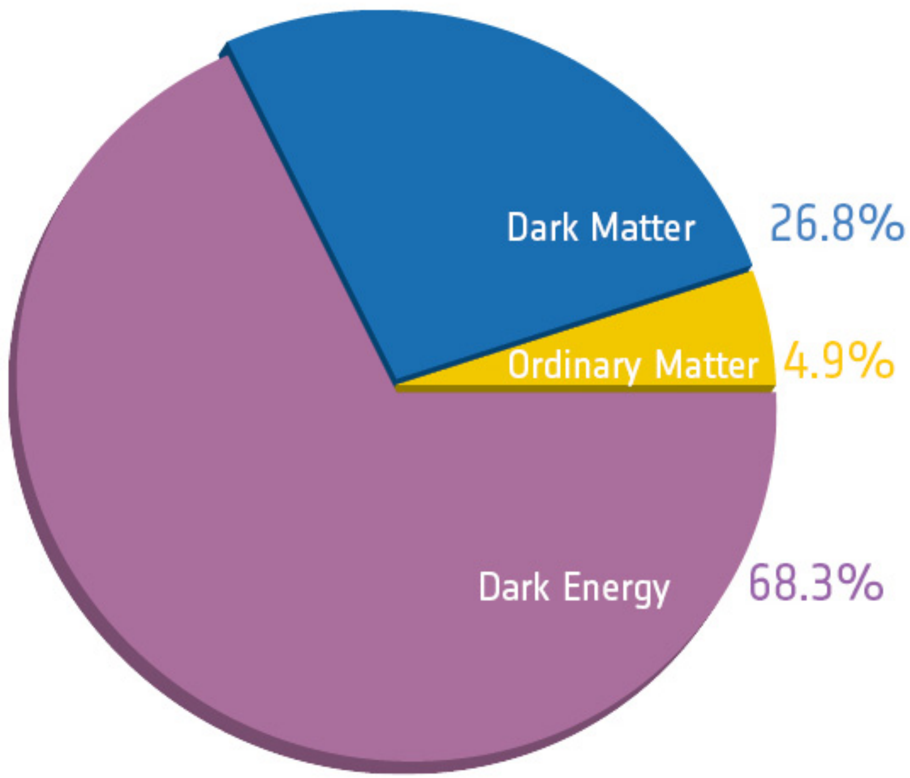

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- Thermal velocities obey a Fermi-Dirac distribution with mean:

$$\bar{v}_{th} \sim 160(1+z)\frac{\mathrm{eV}}{m_\nu} \qquad k_{\mathrm{fs}}(z) = 0.82\frac{H(z)}{H_0}\frac{1}{(1+z)^2}\frac{m_\nu}{\mathrm{eV}}\frac{h}{\mathrm{Mpc}}$$

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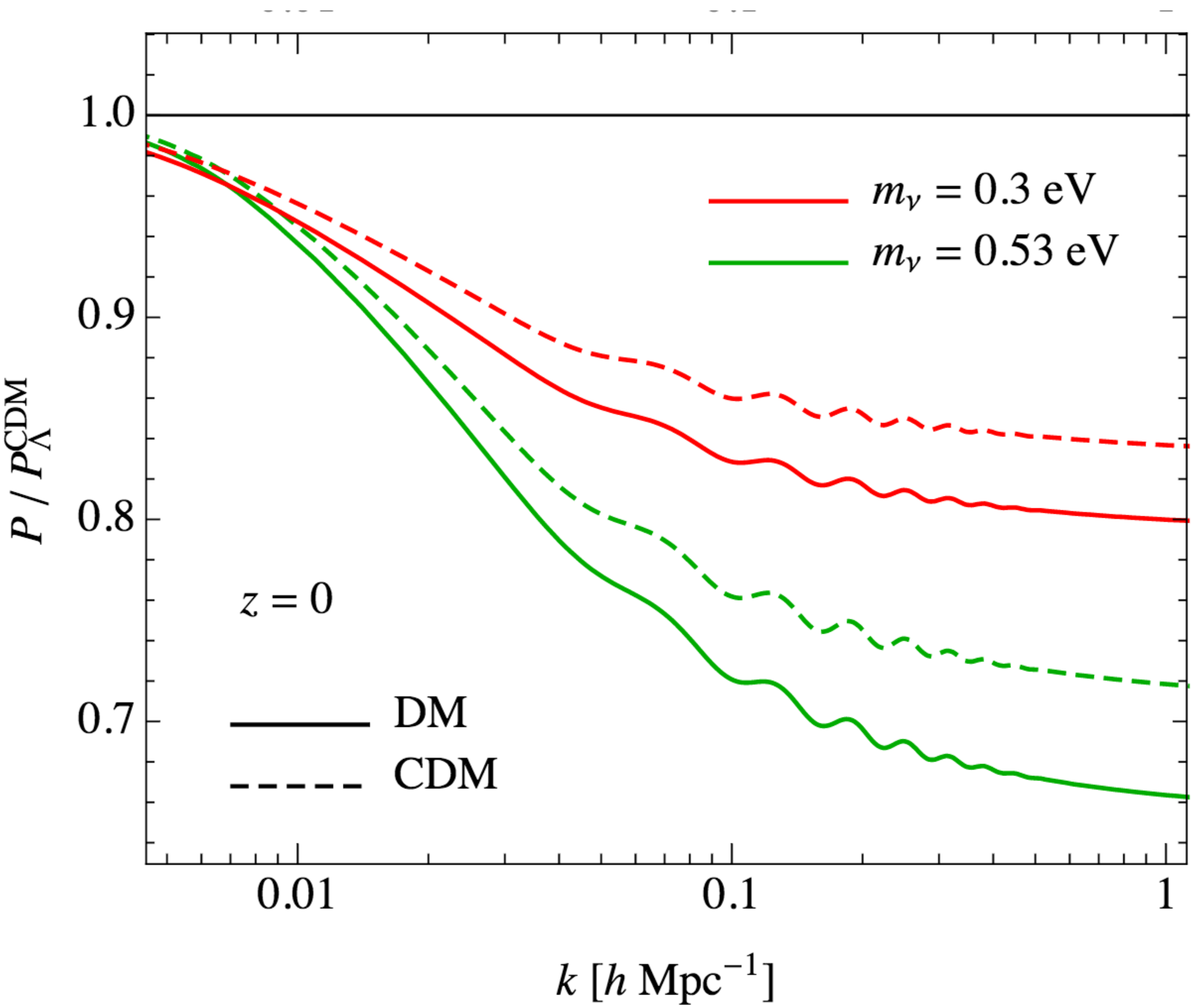

Free streaming suppresses structure at small scales

Castorina et al. 2016

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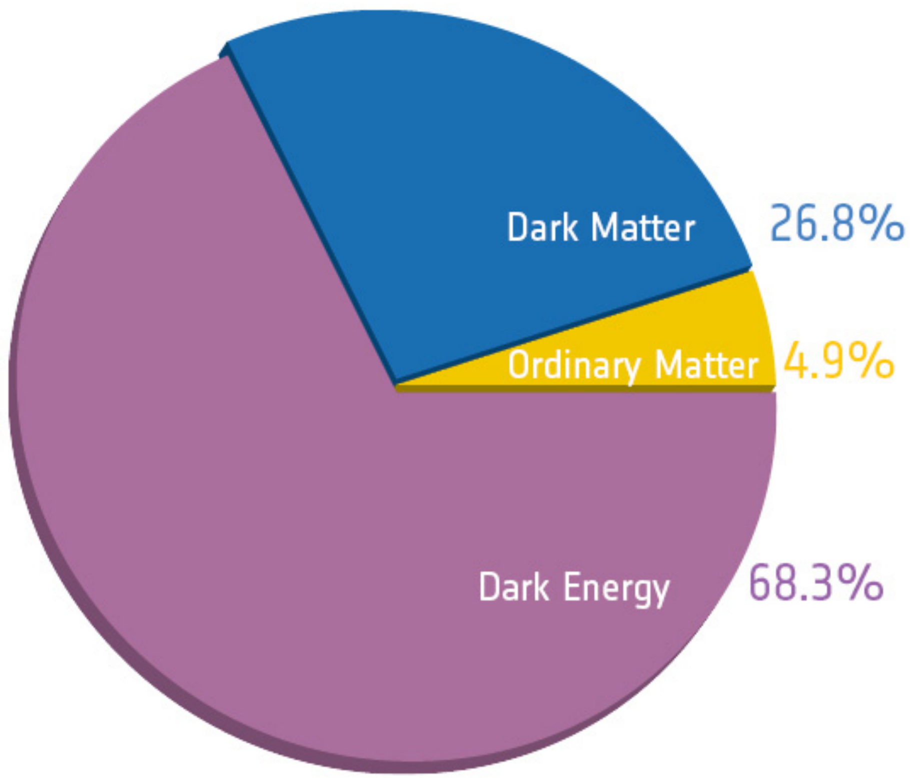

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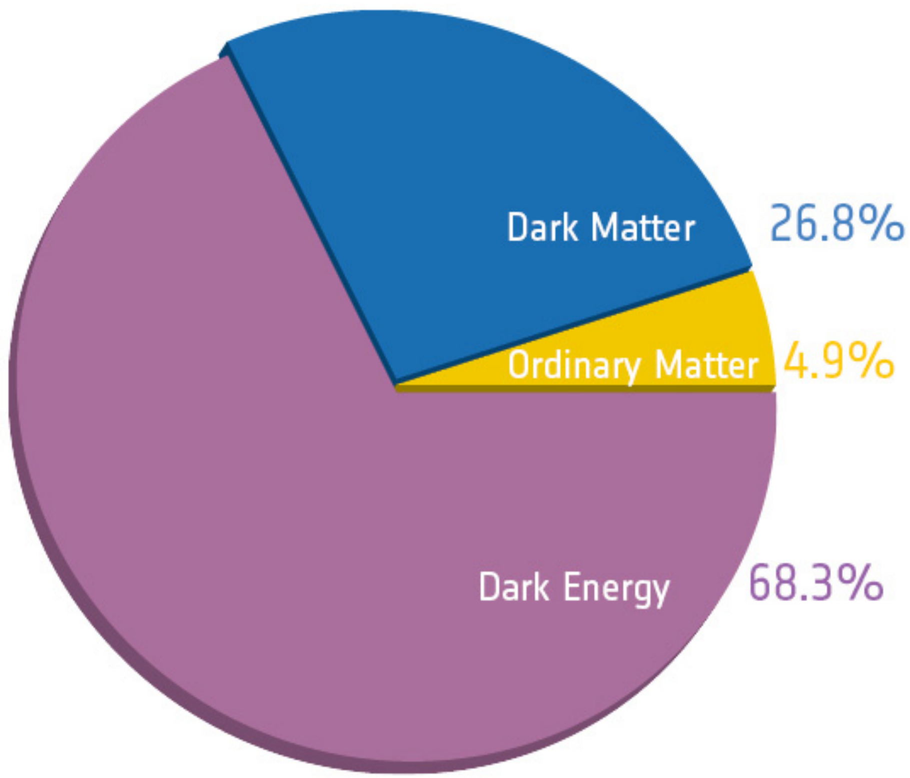

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$$\hat{\psi}(\vec{r},t) = \sqrt{Nm\boldsymbol{\rho}(\vec{r},t)}\,e^{i\theta(\vec{r},t)}$$

$$\vec{\mathbf{v}} = \frac{\hbar}{m}\vec{\nabla}\,\theta$$

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KINETIC

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KINETIC GRAVITY SELF-INTERACTION

$$\partial_t\vec{\mathrm{v}} + (\vec{\mathrm{v}}\cdot\vec{\nabla})\vec{\mathrm{v}} = \frac{\hbar^2}{2m^2}\vec{\nabla}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) - \vec{\nabla}\Phi + g\vec{\nabla}\rho$$

Non-standard cosmological simulations

NUMERICAL TOOLS FOR NON-STANDARD SIMULATIONS

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Dark Energy models
(no screening)

Modified Gravity models
(with screening)

Primordial Running
non-Gaussianity

Massive neutrinos

Axion Dark Matter

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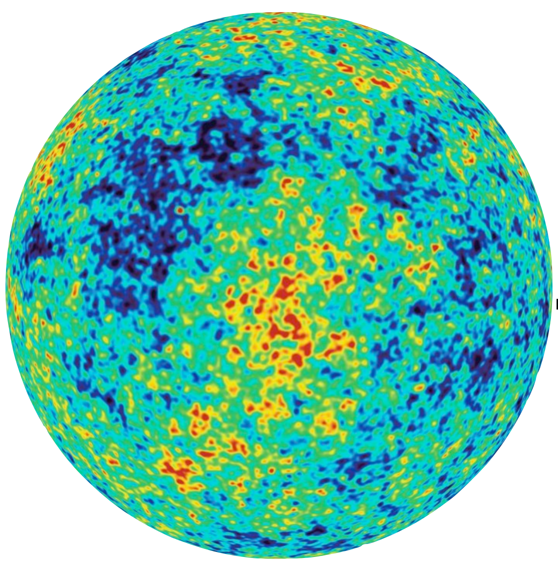
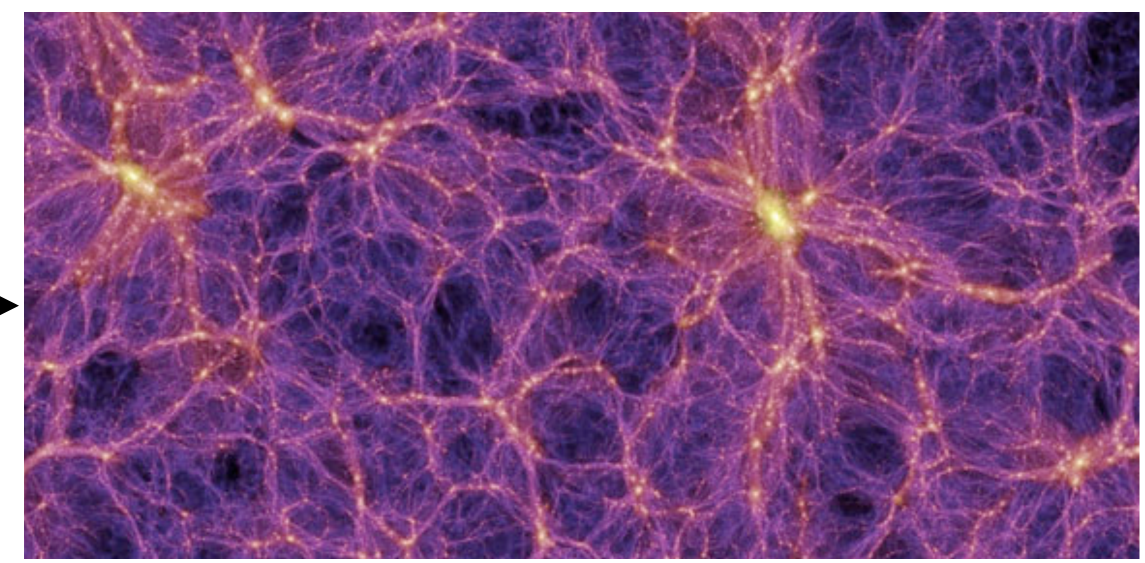
Primordial Running non-Gaussianity →	PNGRUN	Wagner et al. 2010-2012
Massive neutrinos →	NU-GADGET	Viel, Haehnelt, Springel 2010
Axion Dark Matter →	AX-GADGET	Nori & Baldi in prep.

FROM INFLATION TO GALAXIES... HOW?





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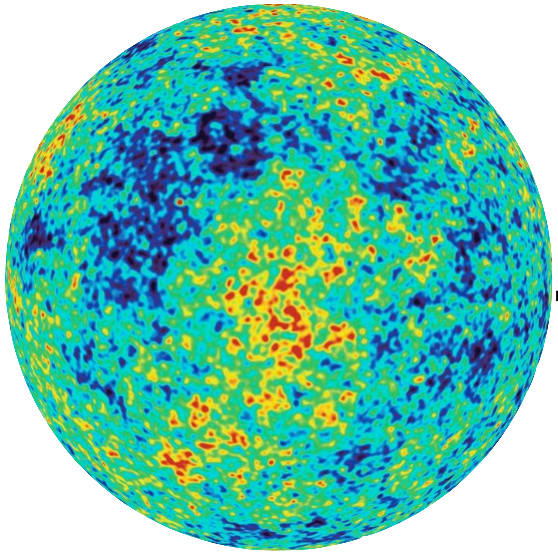
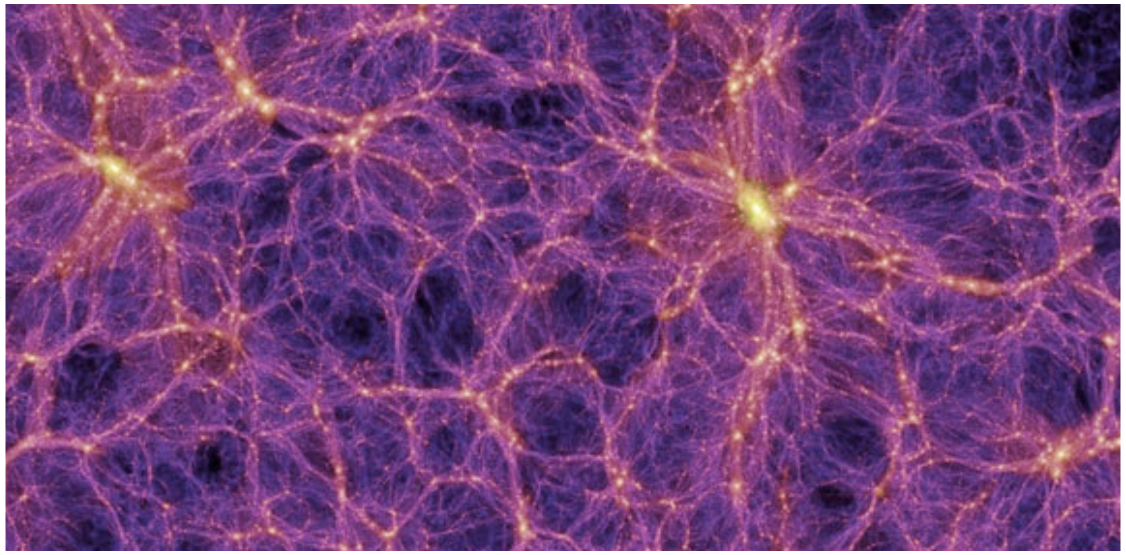


$\xrightarrow{\Lambda CDM}$



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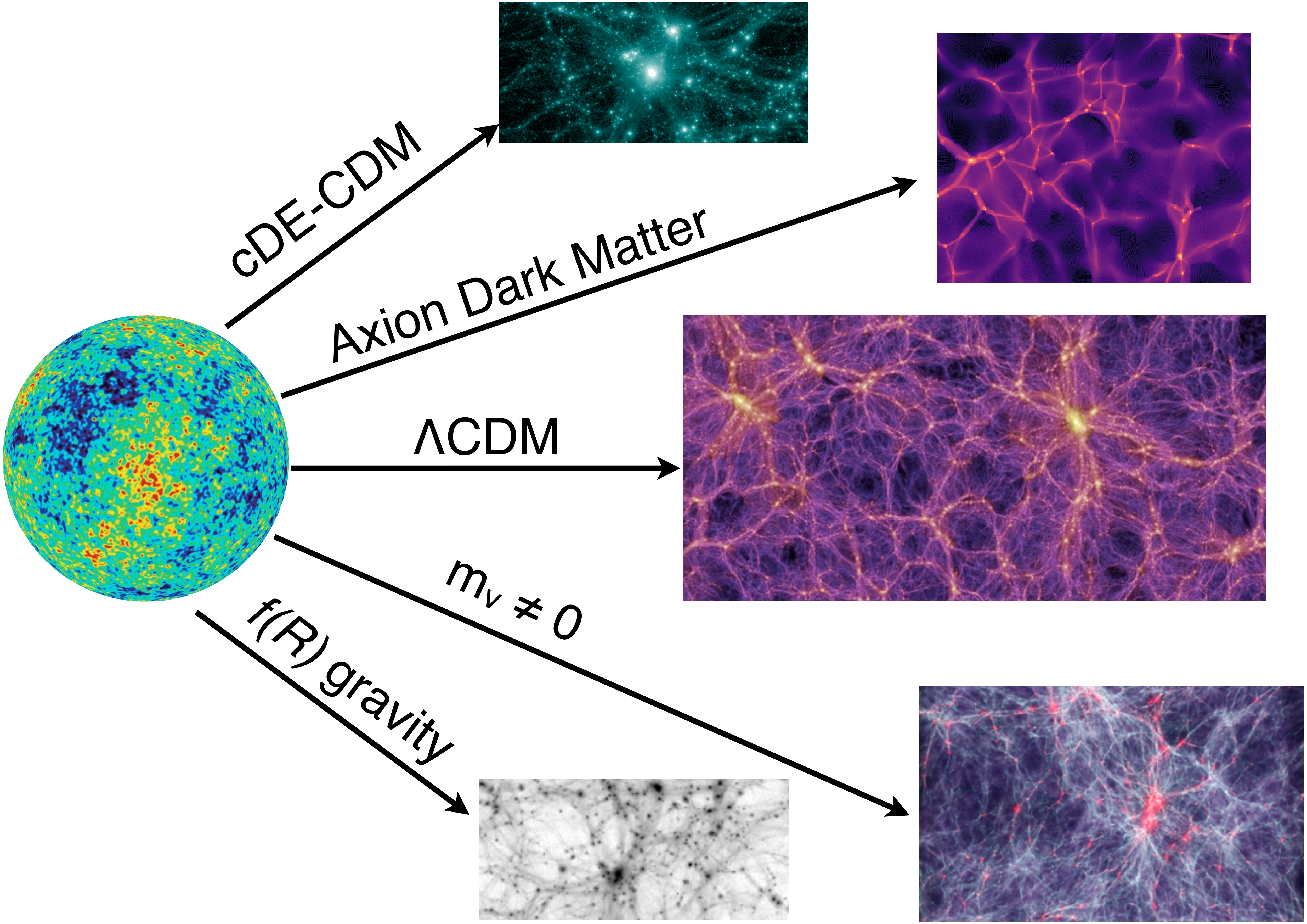

WHY DO WE NEED SIMULATIONS FOR NON-STANDARD COSMOLOGIES?

INTERACTING DE: BREAK DEGENERACIES

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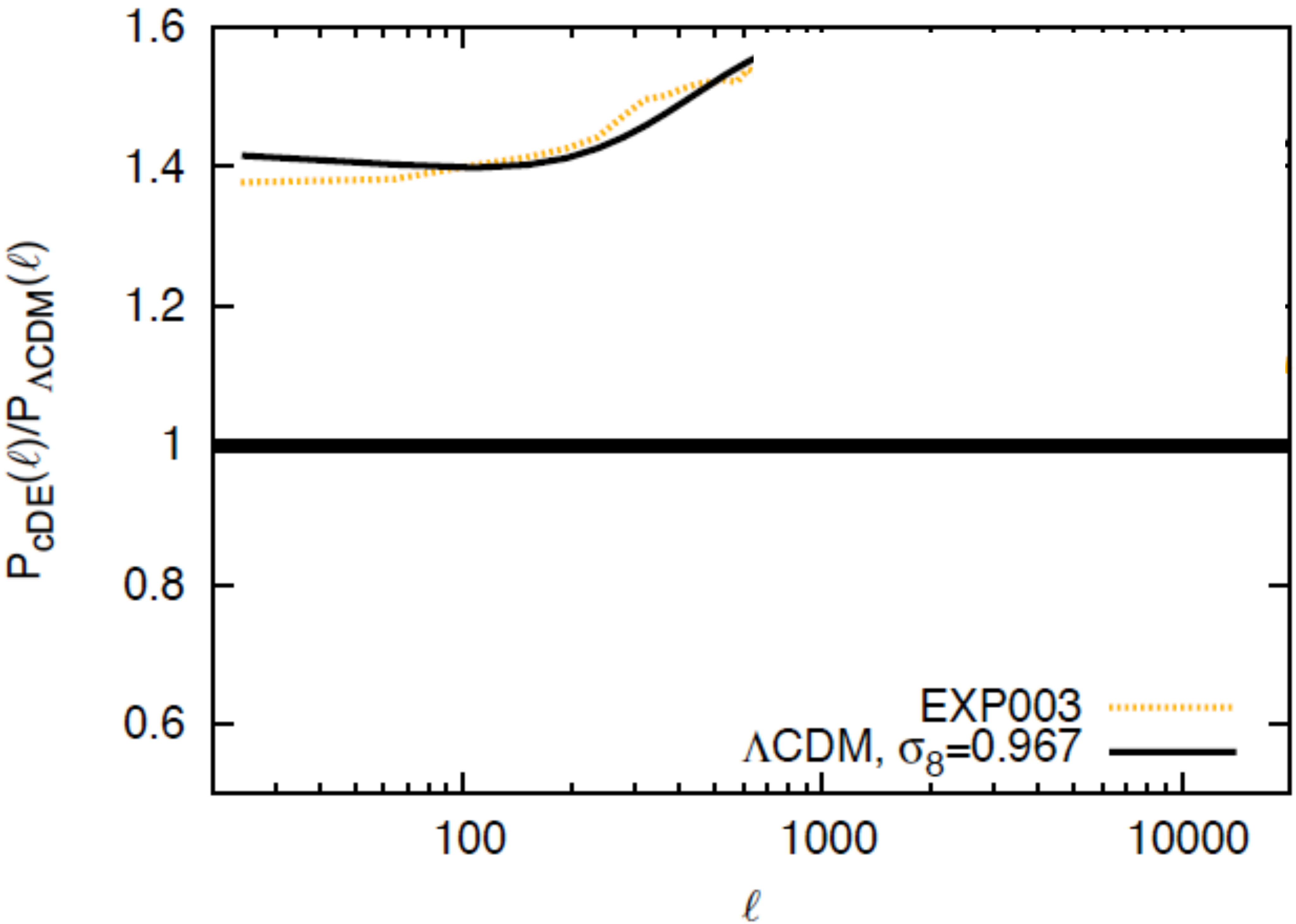
Lensing power spectrum extracted from N-body simulations with a ray-tracing technique (Pace, MB, et al. 2014)



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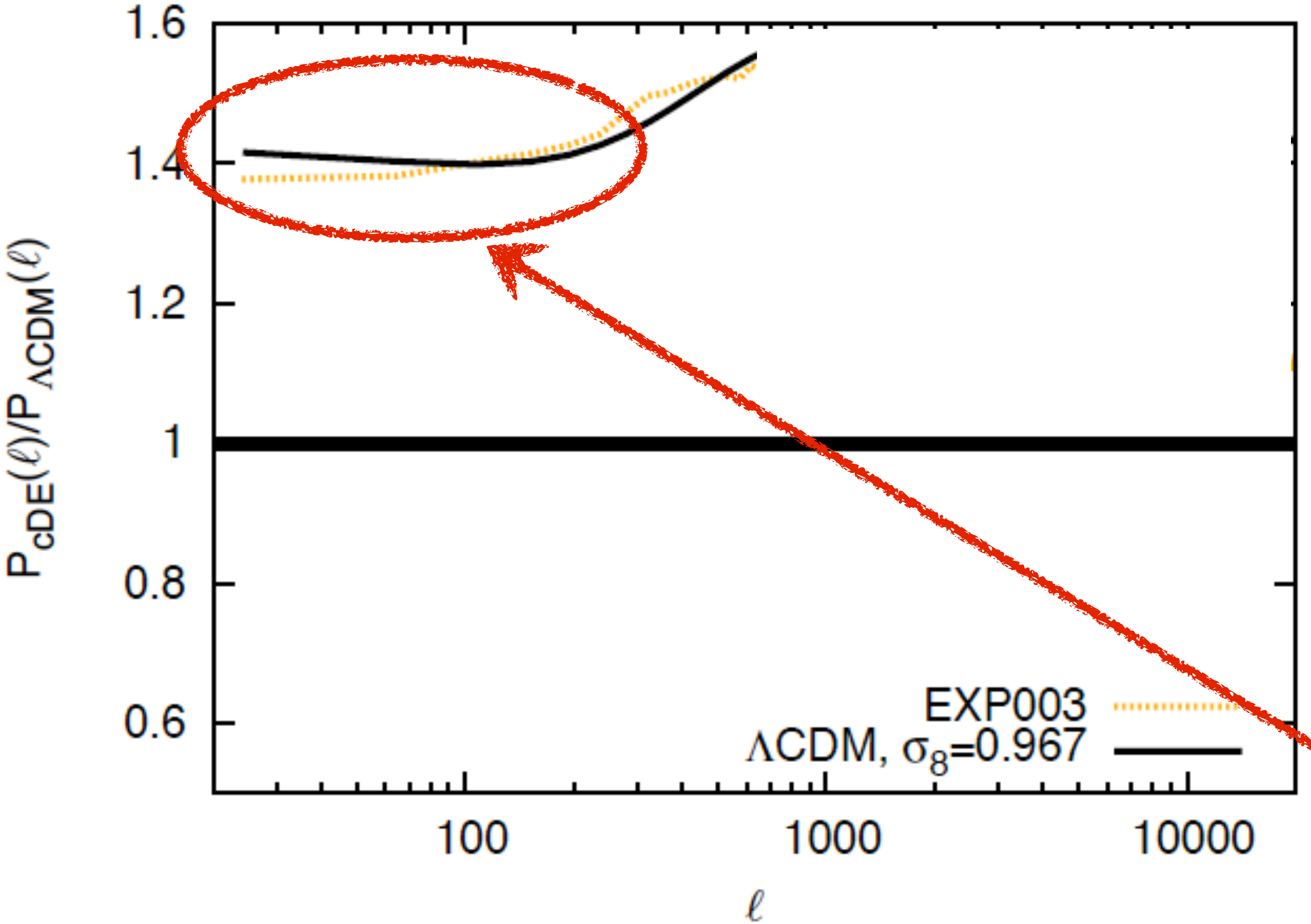

degeneracy with standard ΛCDM parameters

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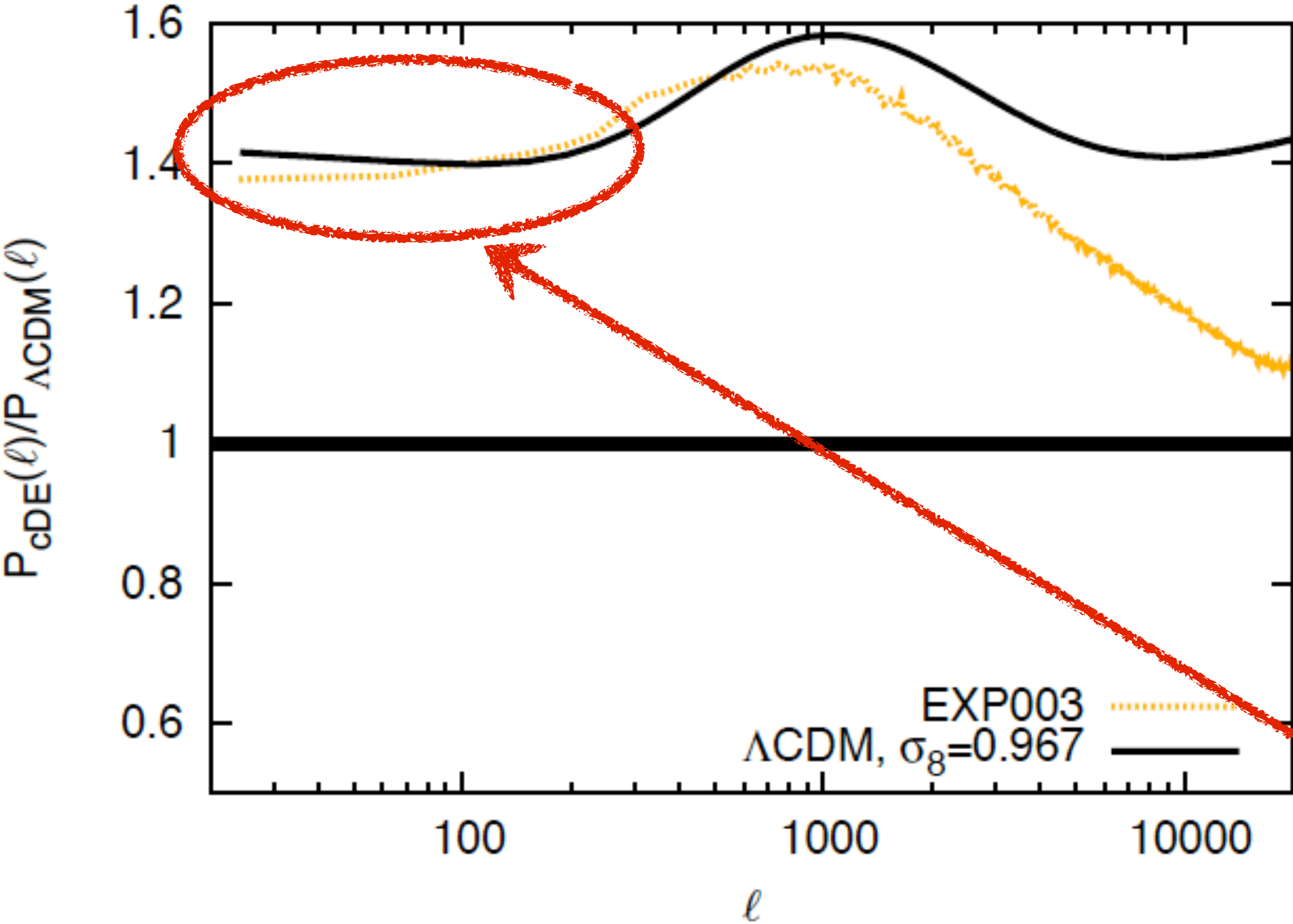

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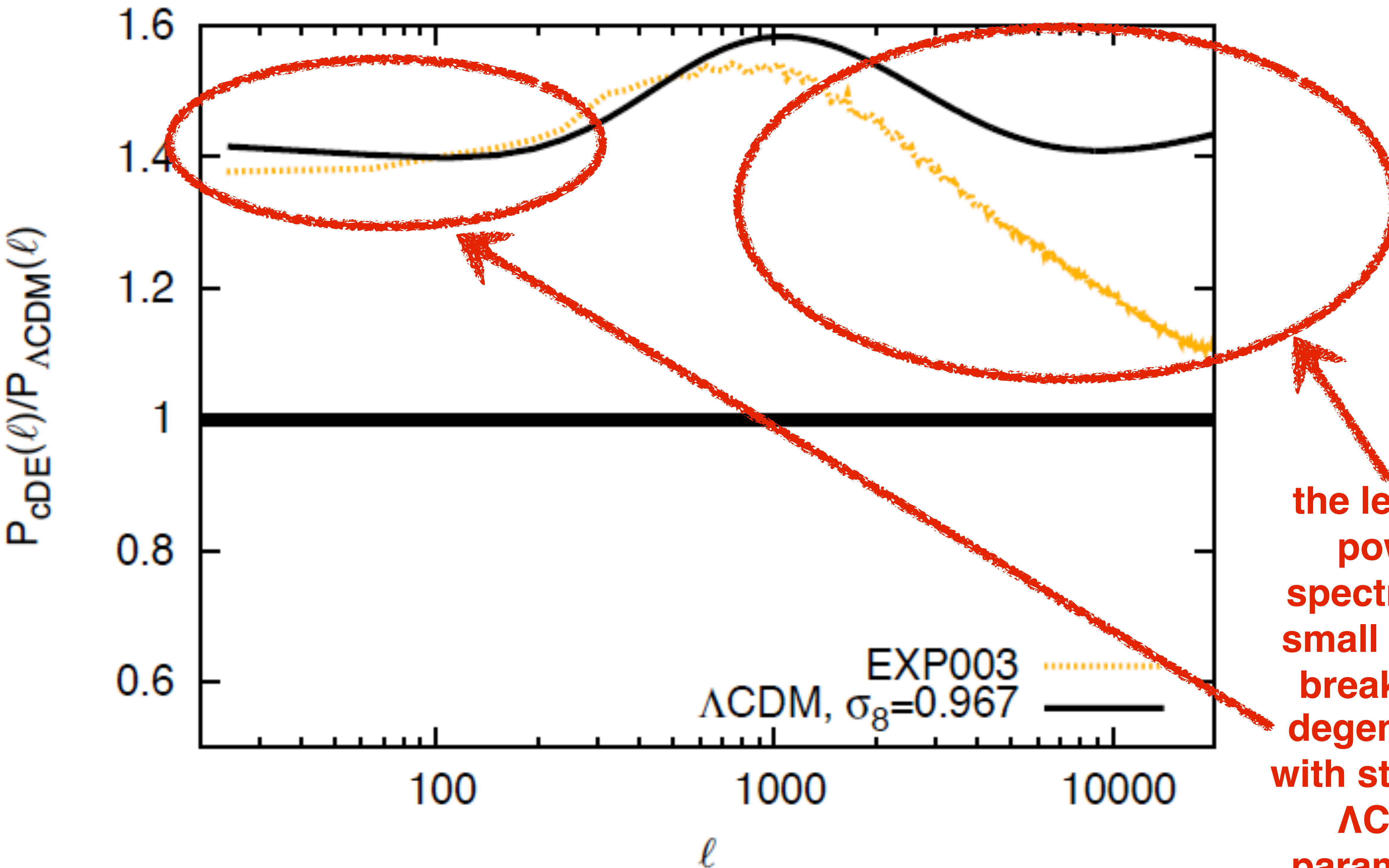
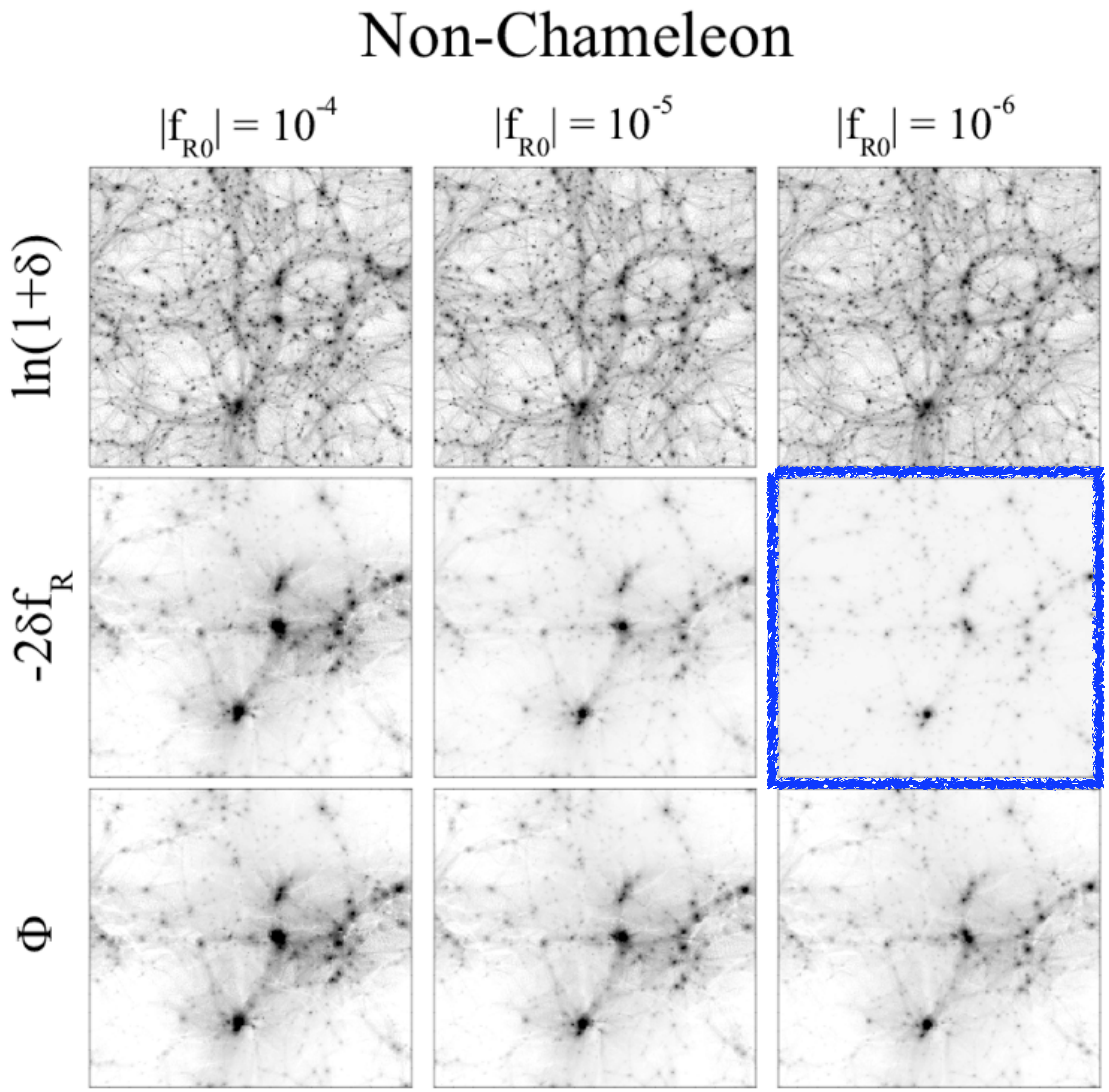


the lensing power spectrum at small scales breaks the degeneracy with standard ΛCDM parameters

MODIFIED GRAVITY: CAPTURE THE SCREENING



without full non-linear screening

Zhao et al. 2011

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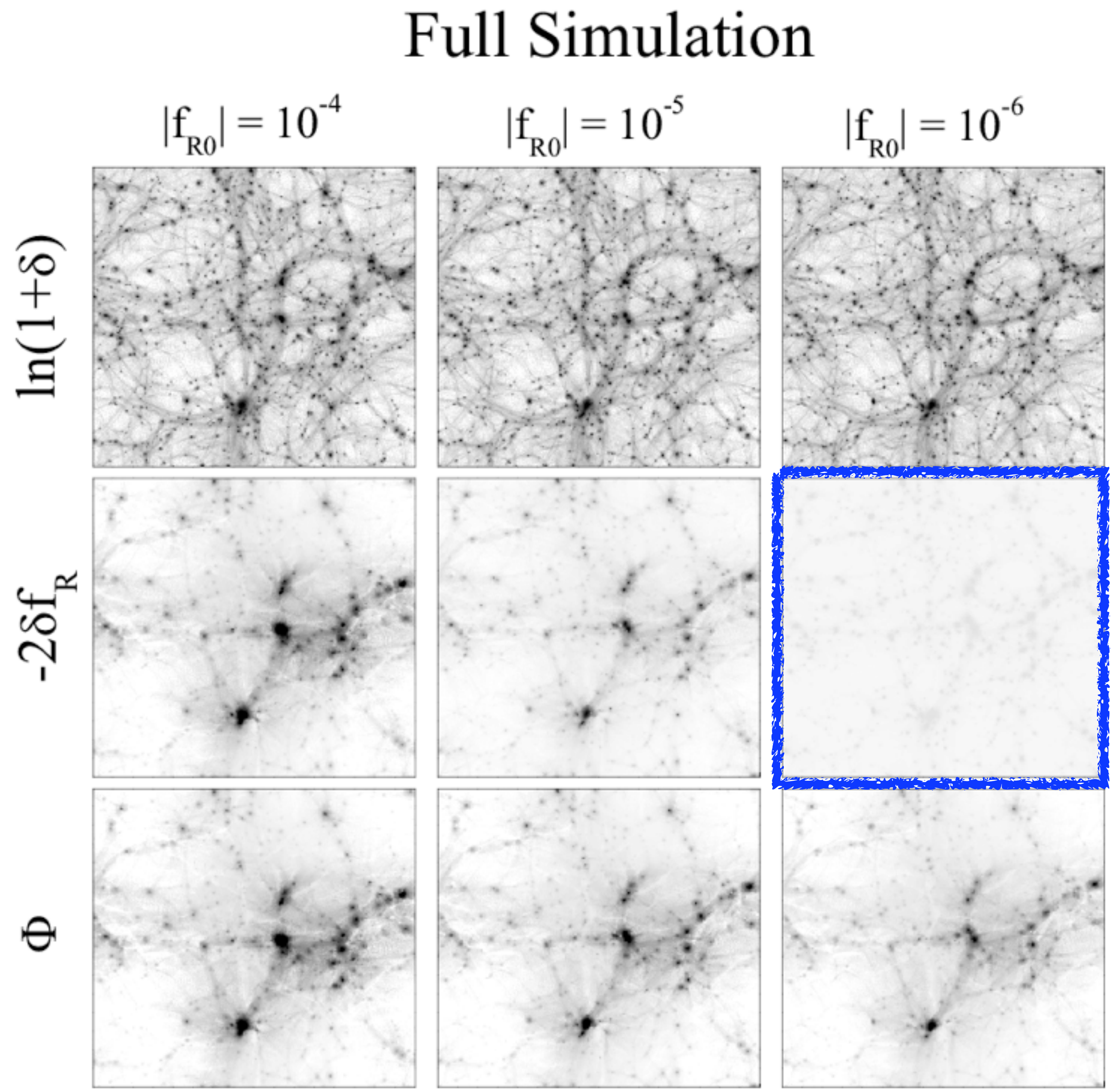

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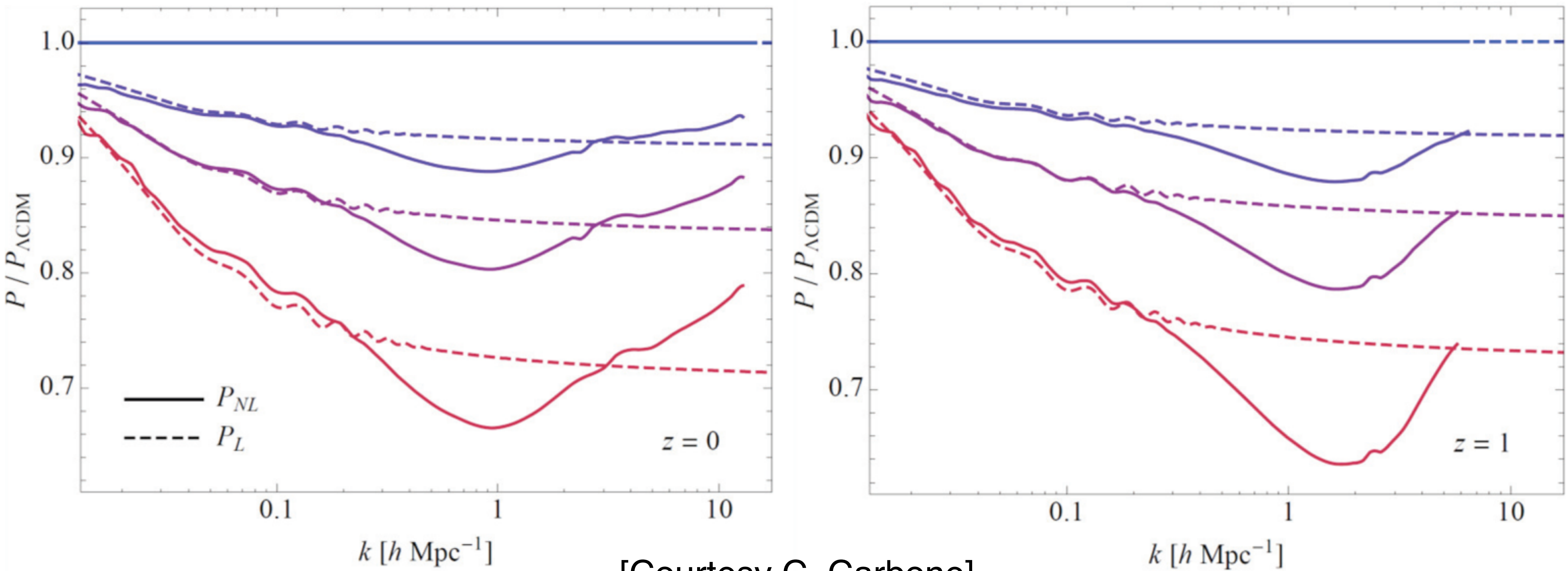
MASSIVE NEUTRINOS: CORRECTLY PREDICT FEATURES

m_ν = 0.17; 0.3; 0.53 eV

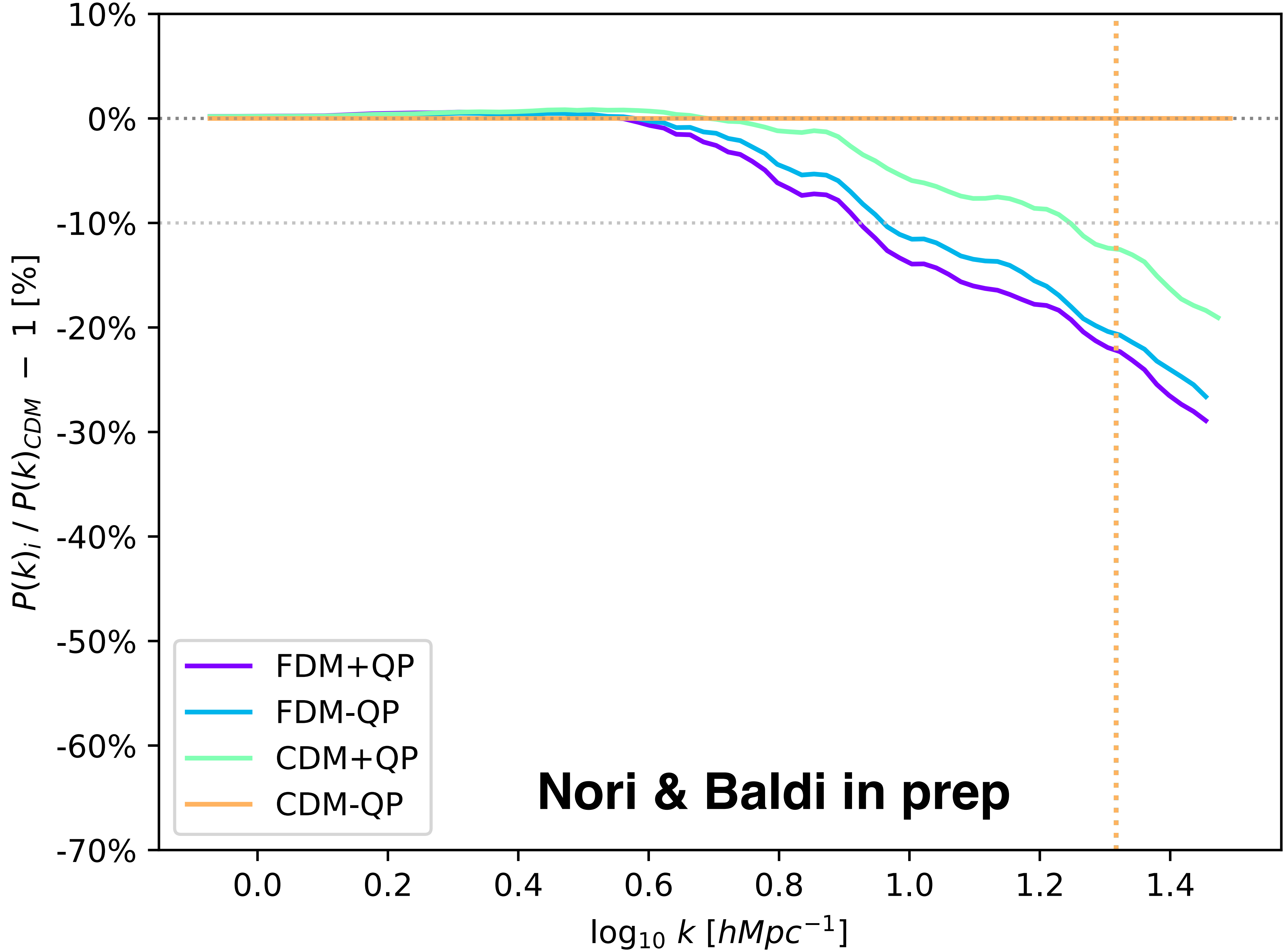


[Courtesy C. Carbone]

[see also Bird, Viel & Haehnelt 2012]

Nonlinear suppression of the matter P(k) ~15% larger than linear predictions at k~1-2 h/Mpc (critical range of scales for WL surveys)

AXION DM: CORRECTLY PREDICT FEATURES



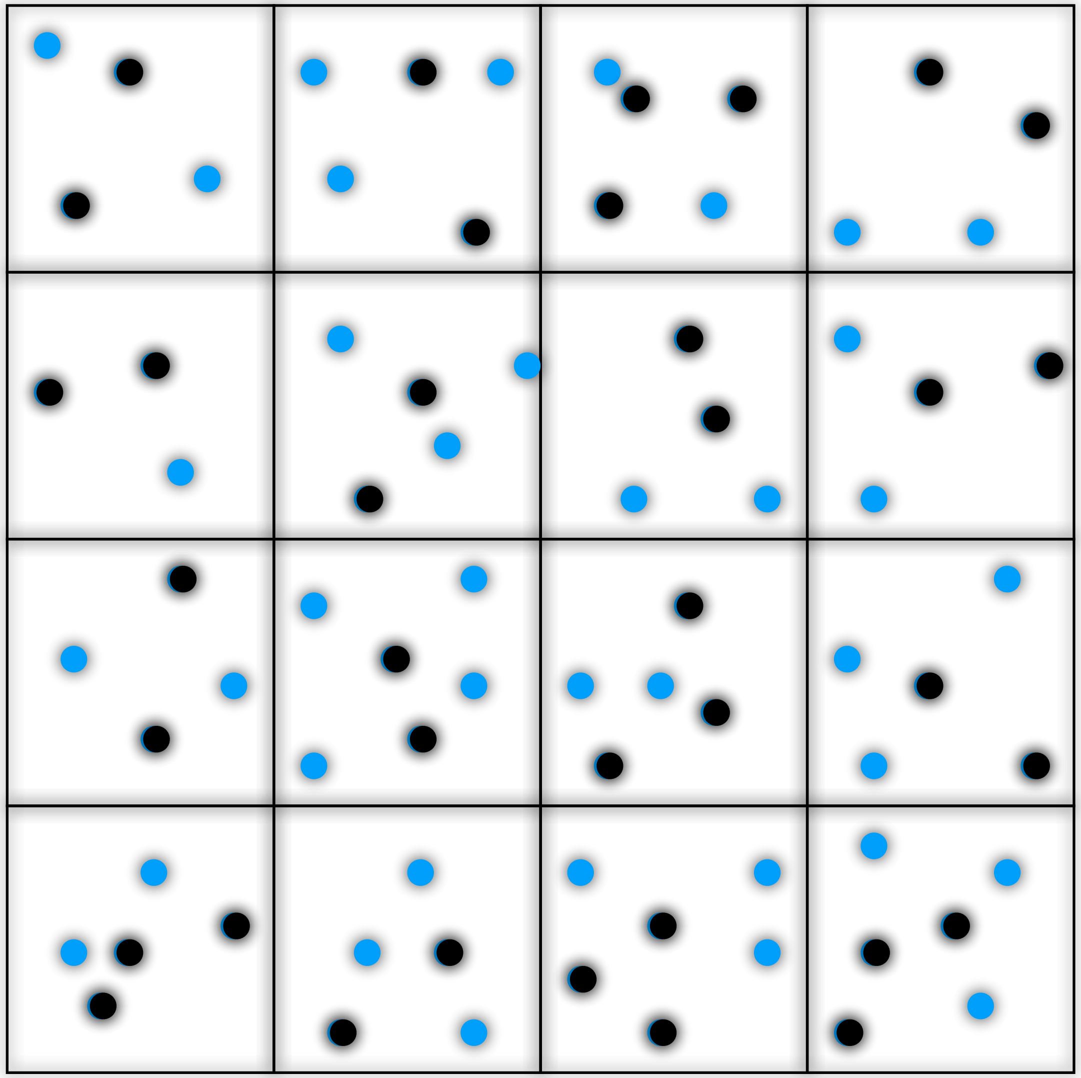
Main numerical implementations

Dark Energy

N-body algorithms for interacting Dark Energy

In interacting DE the coupling determines two different gravitational forces for dark matter and baryons:

Particle-Mesh (for interacting DE)



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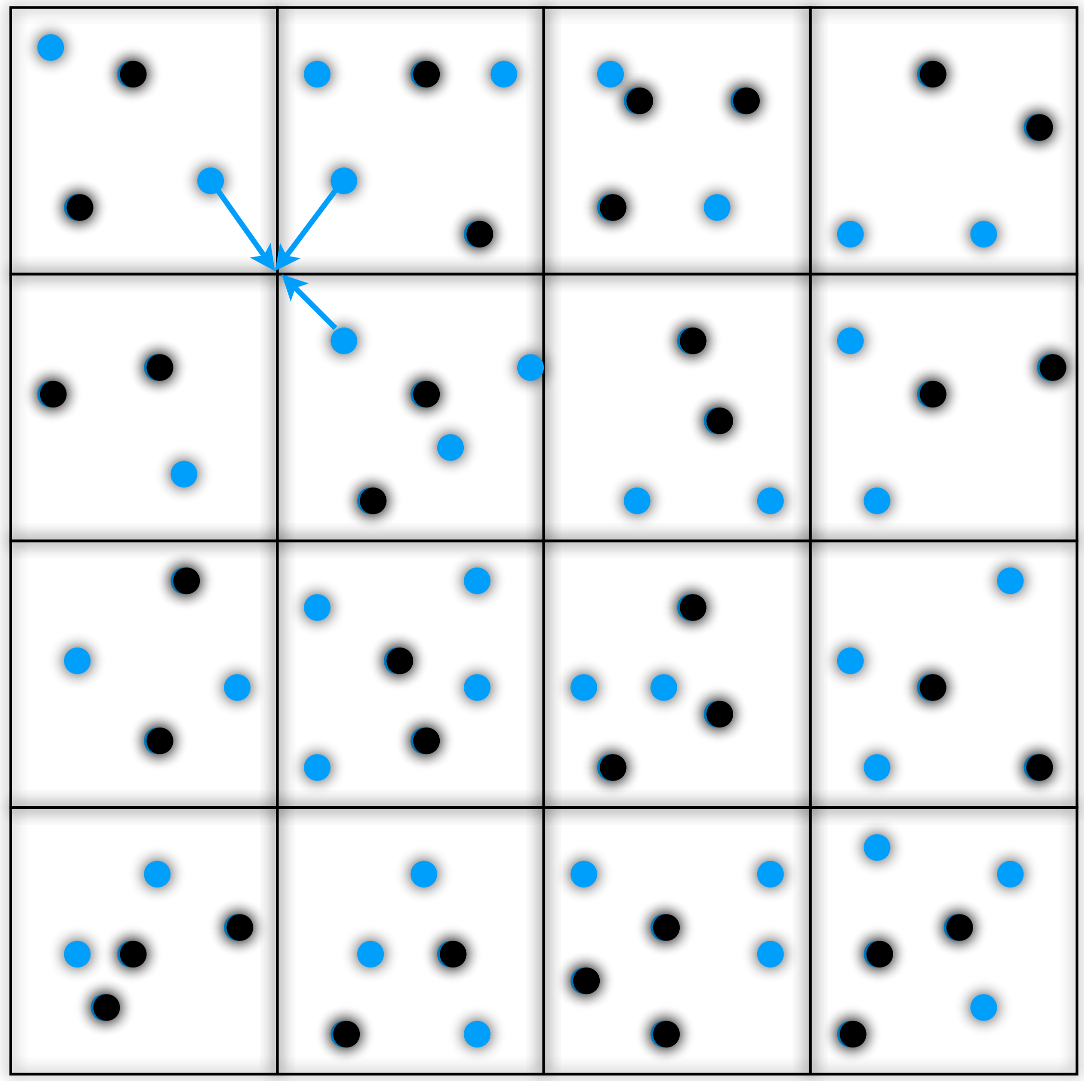

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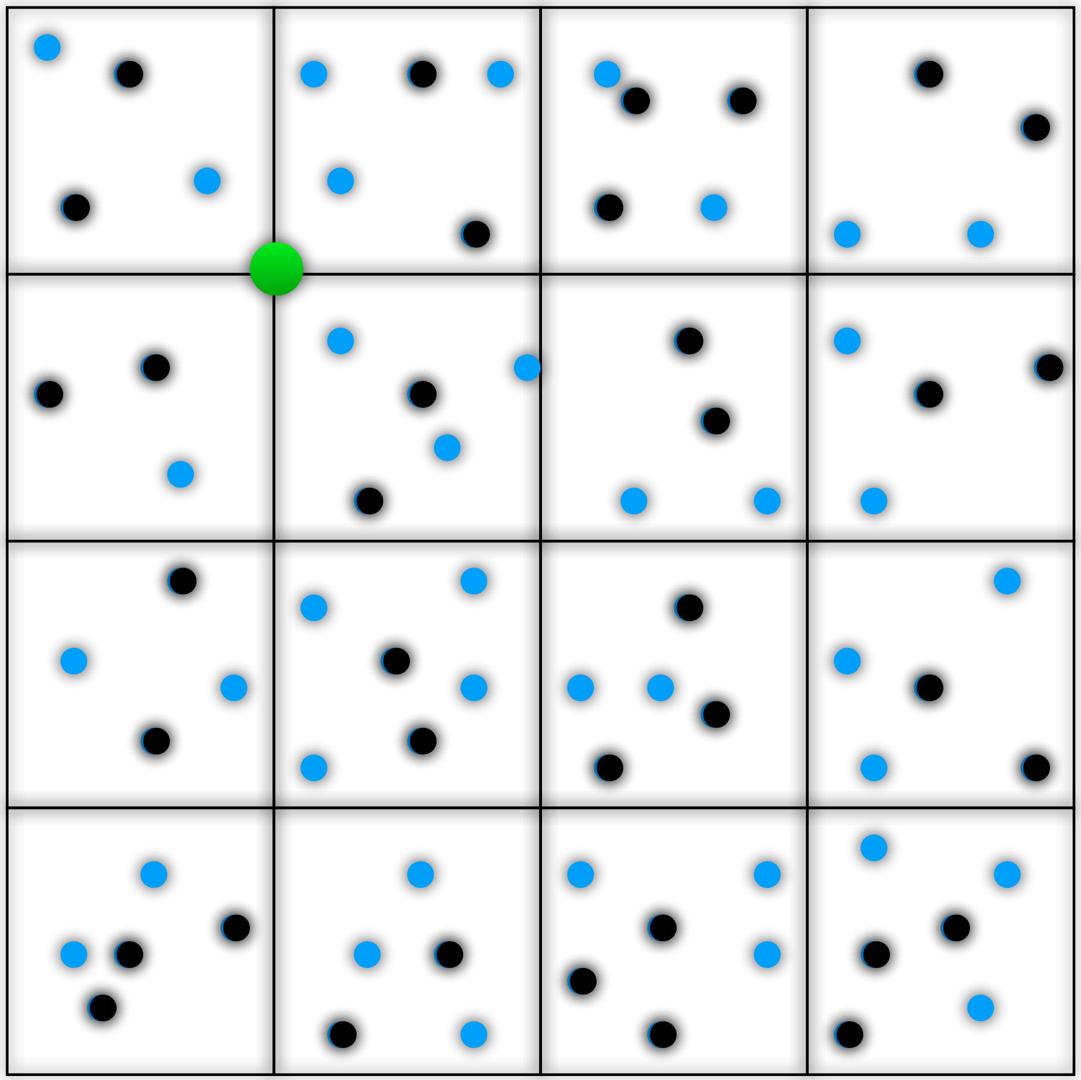

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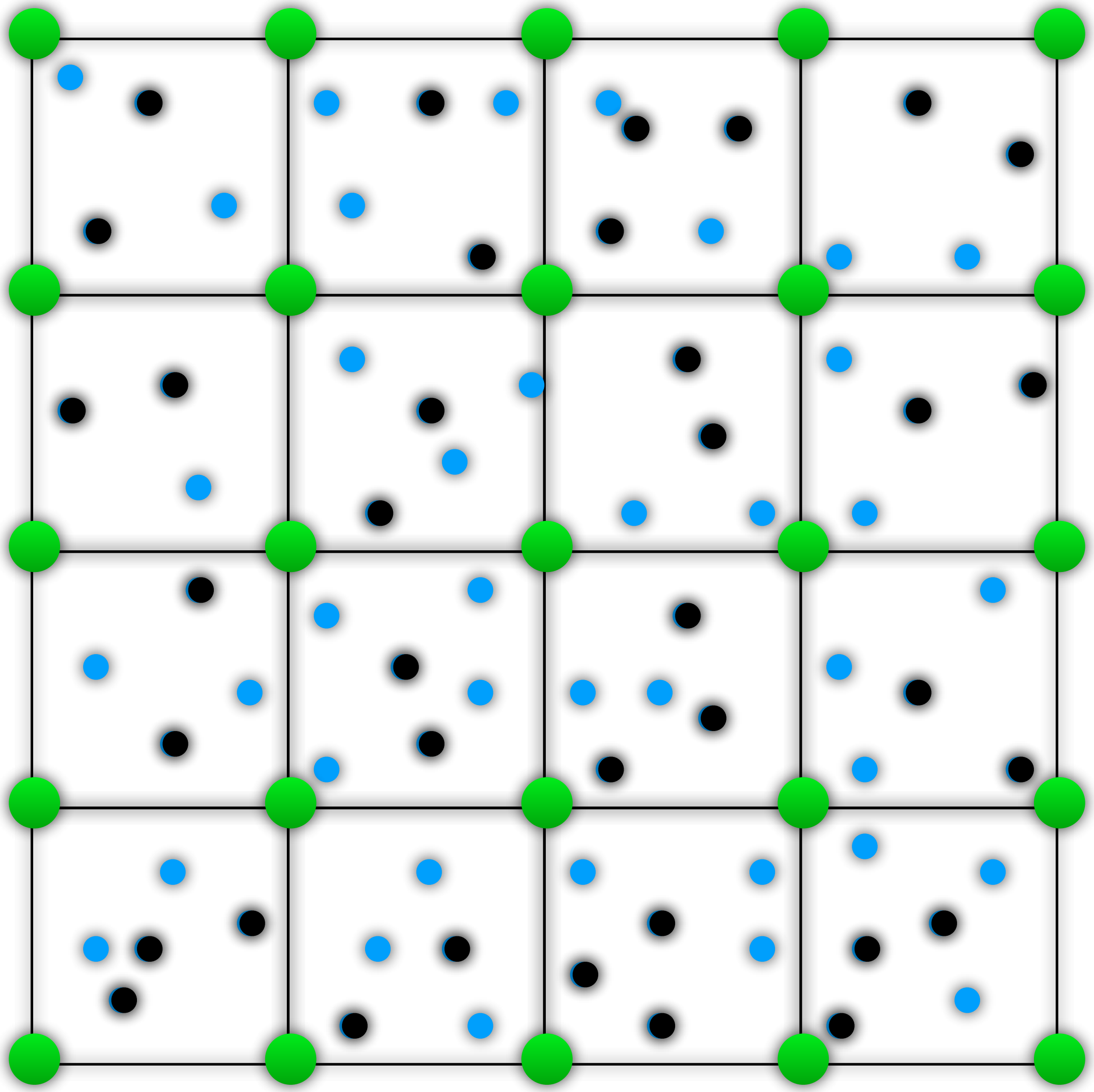

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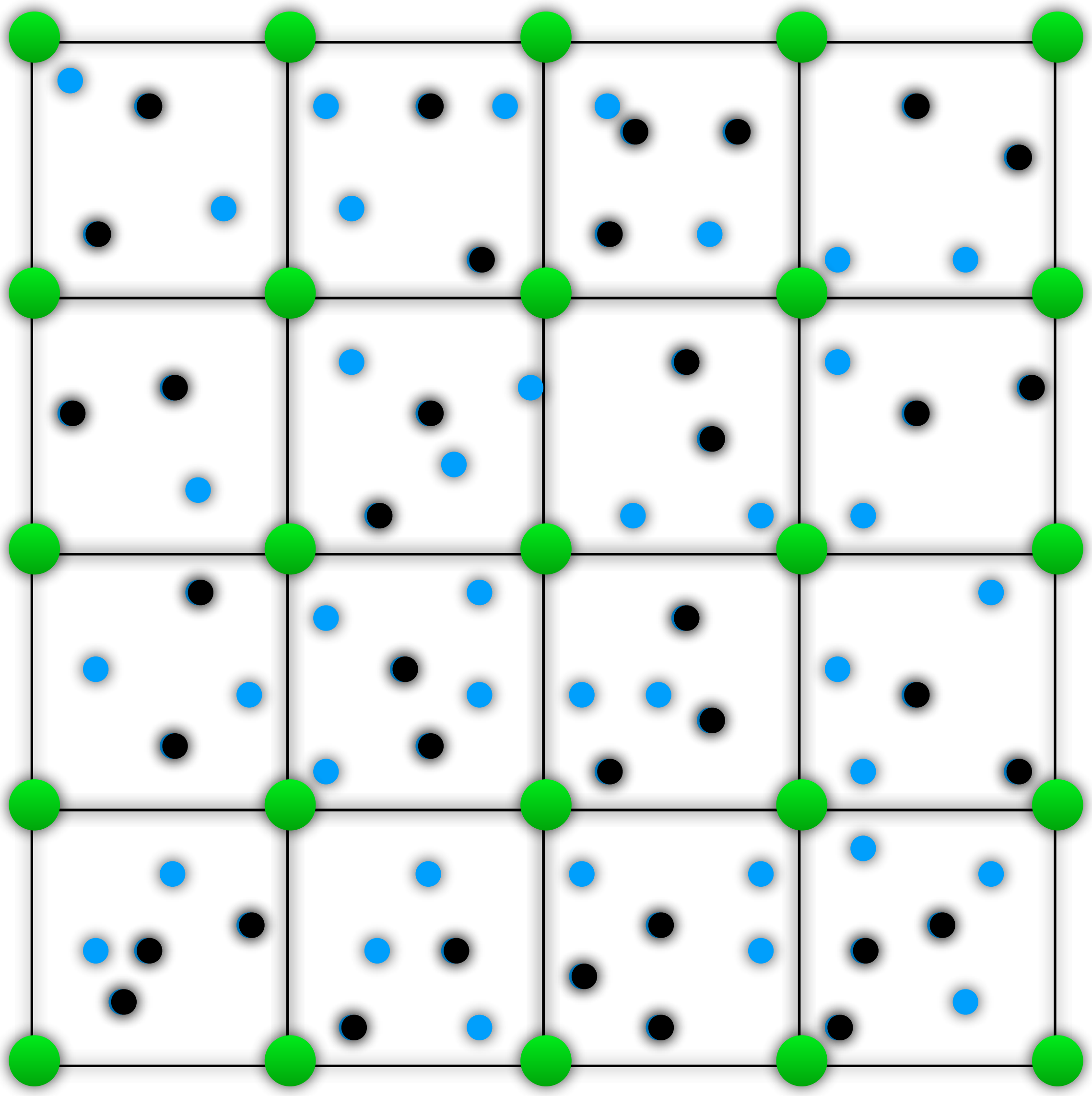

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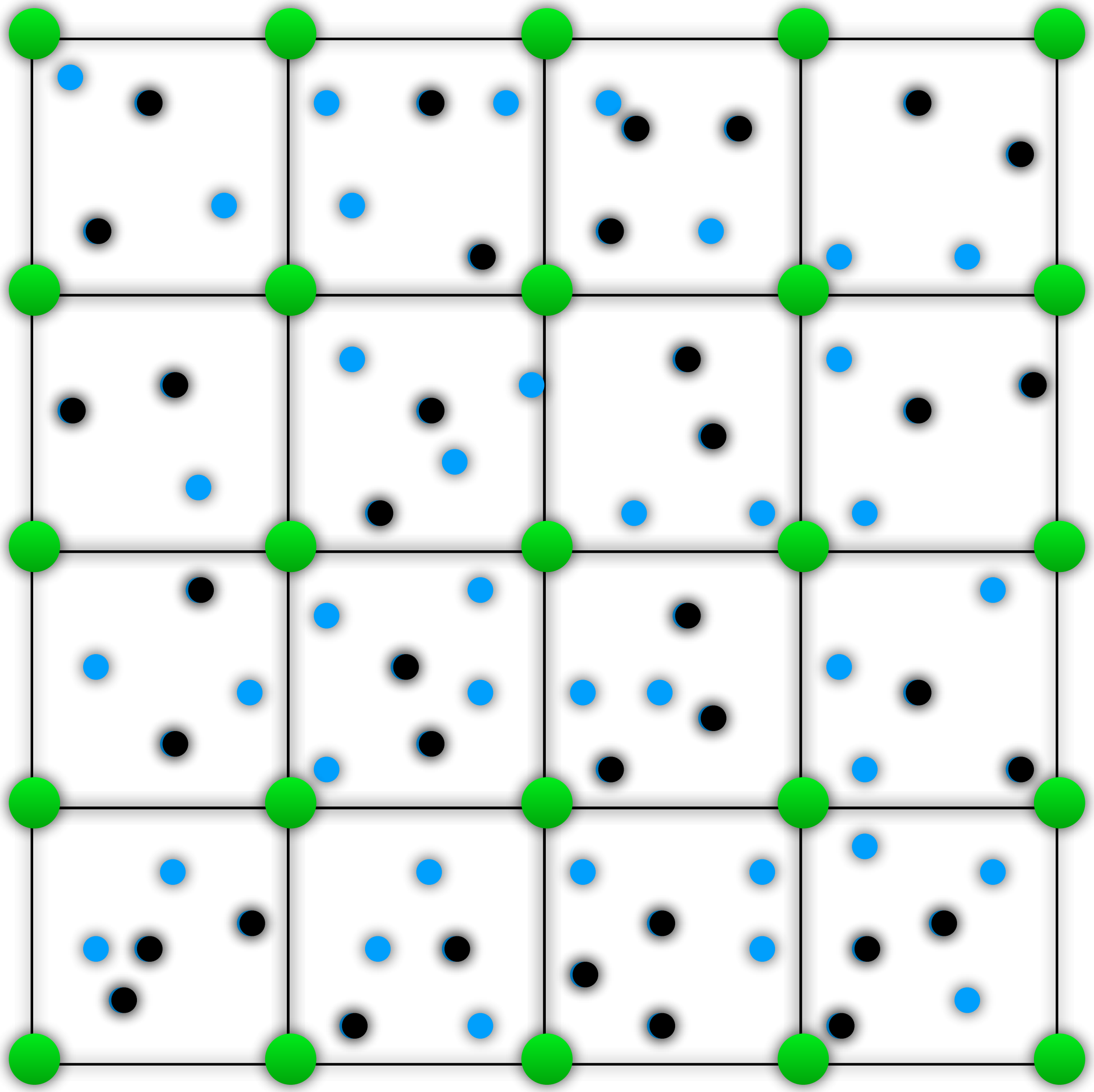

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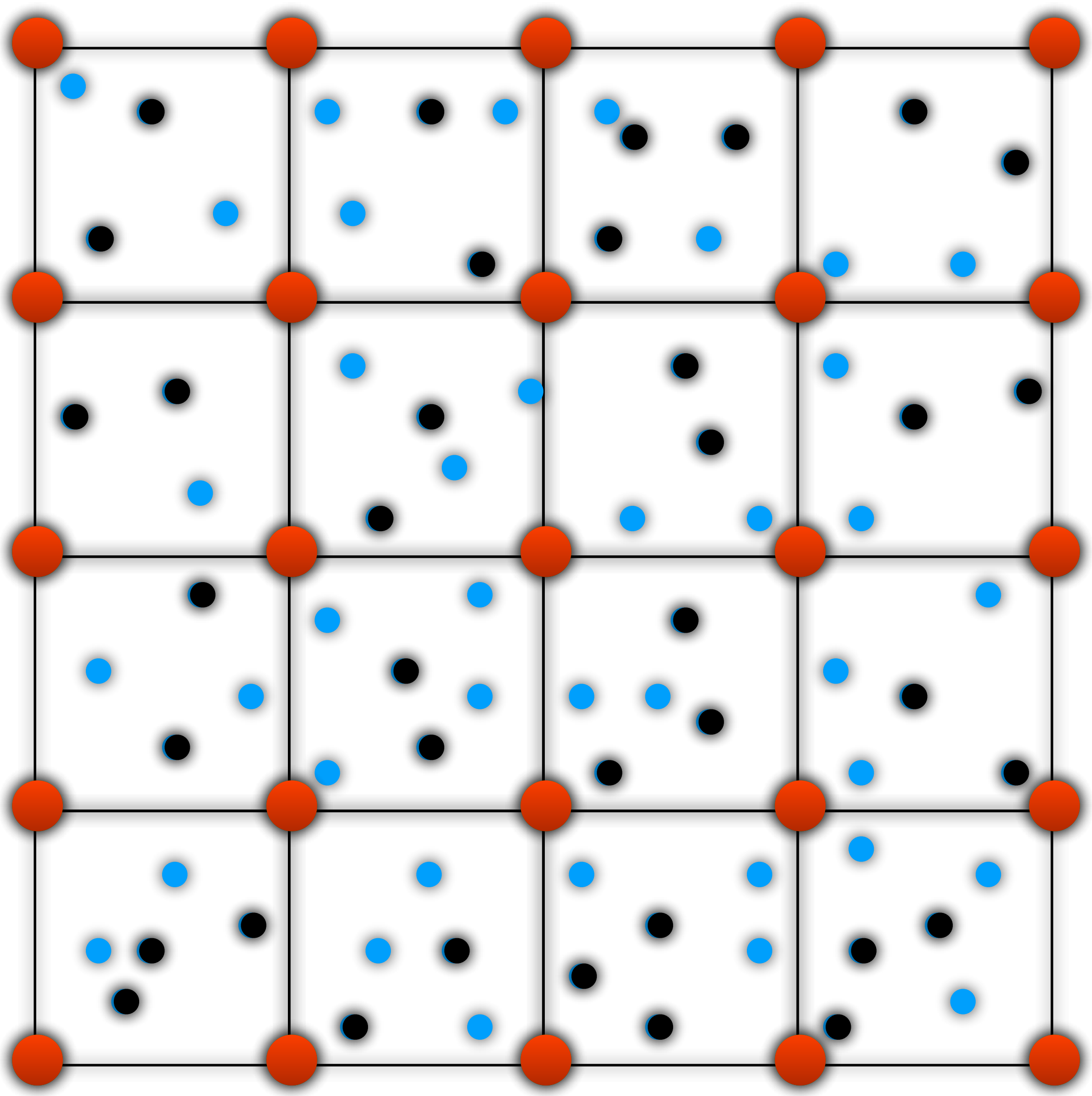

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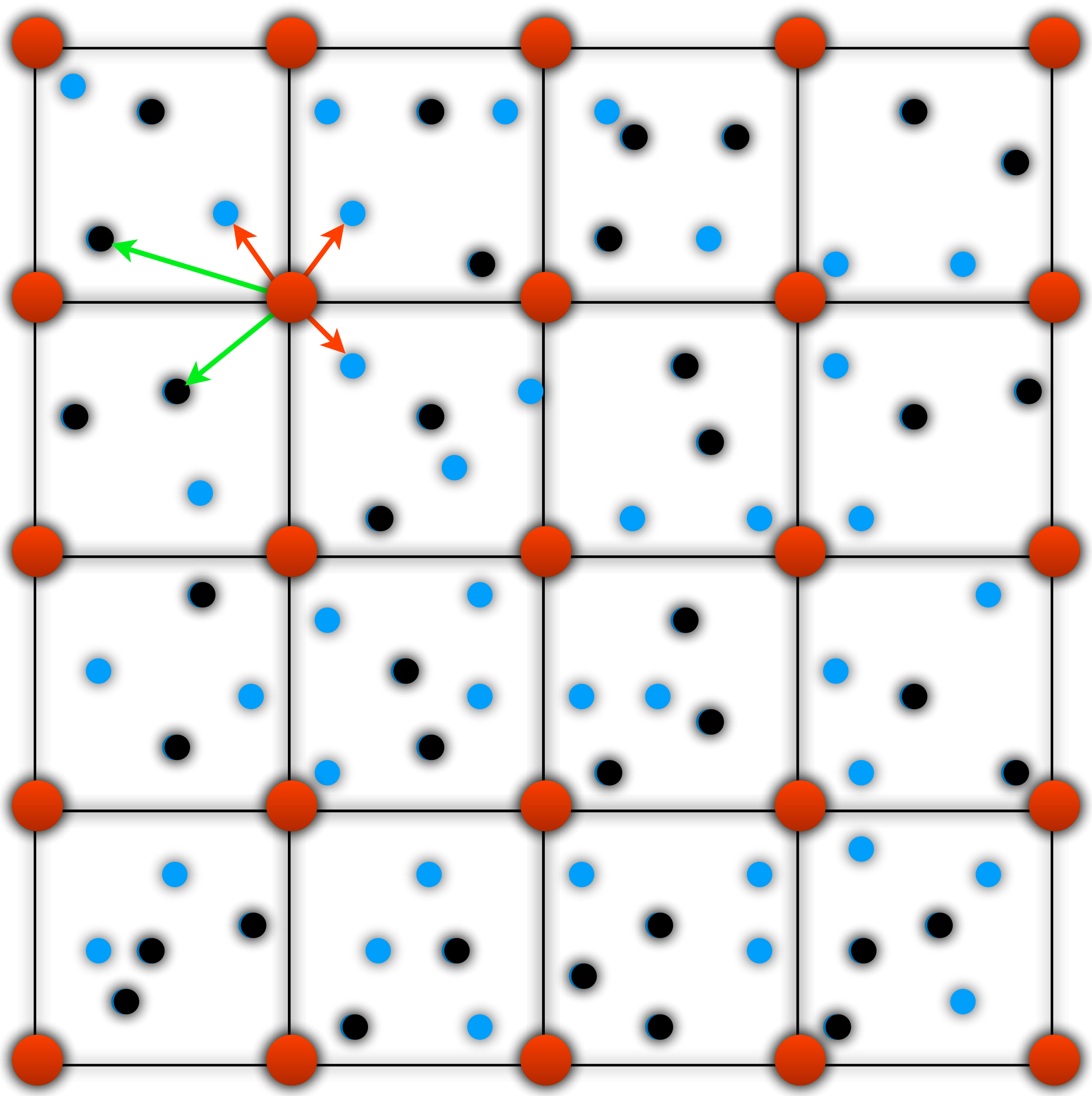

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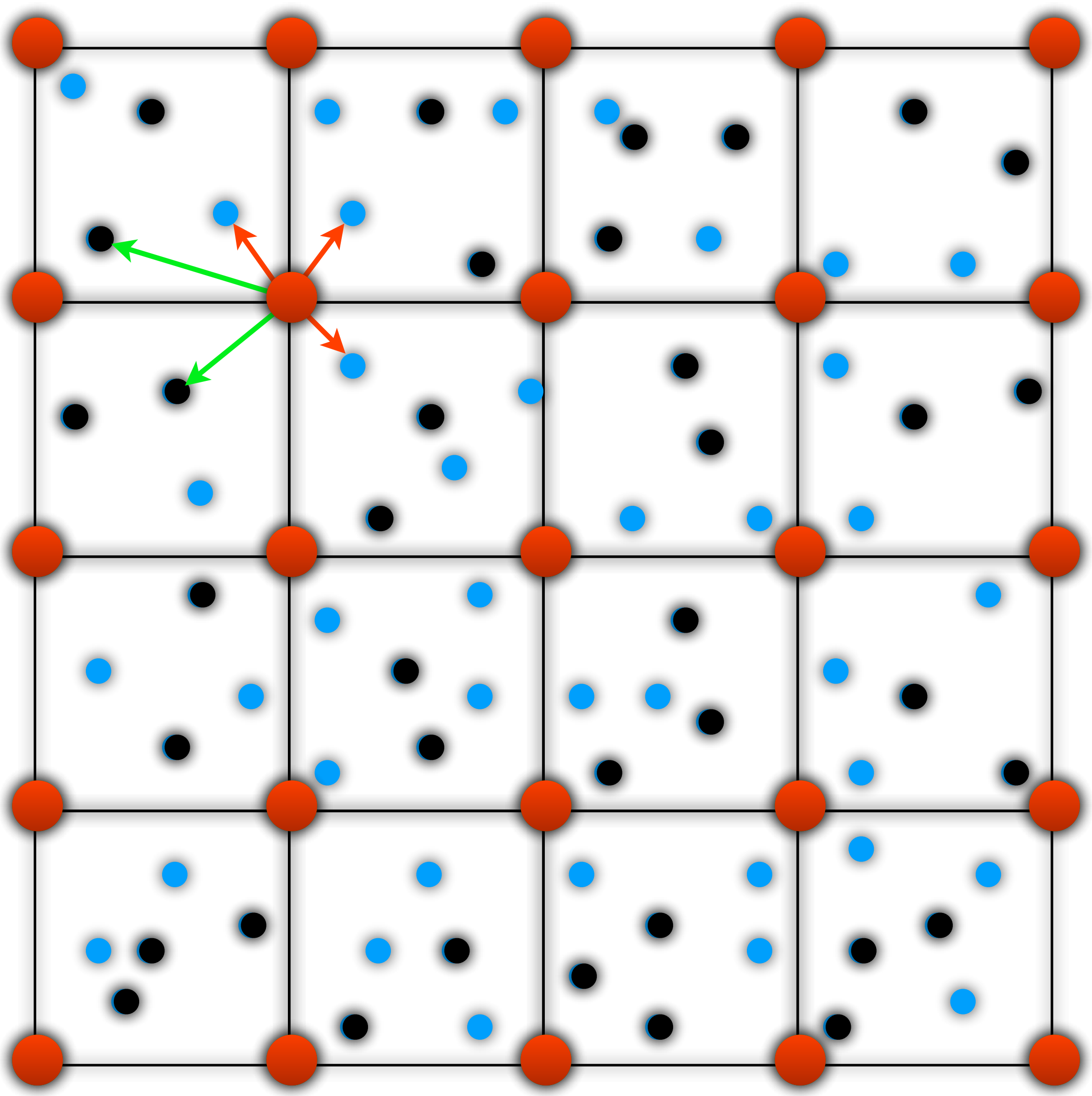

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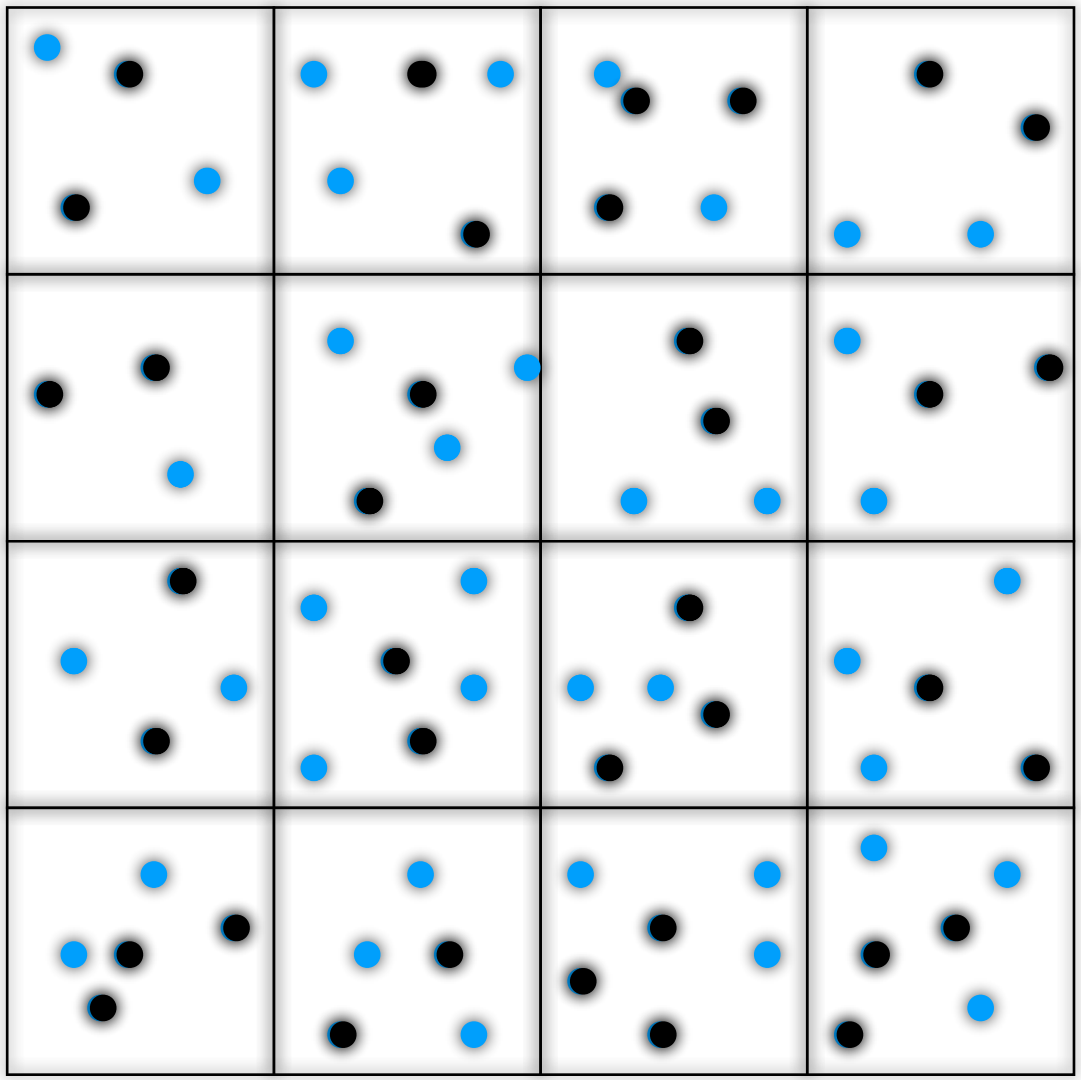

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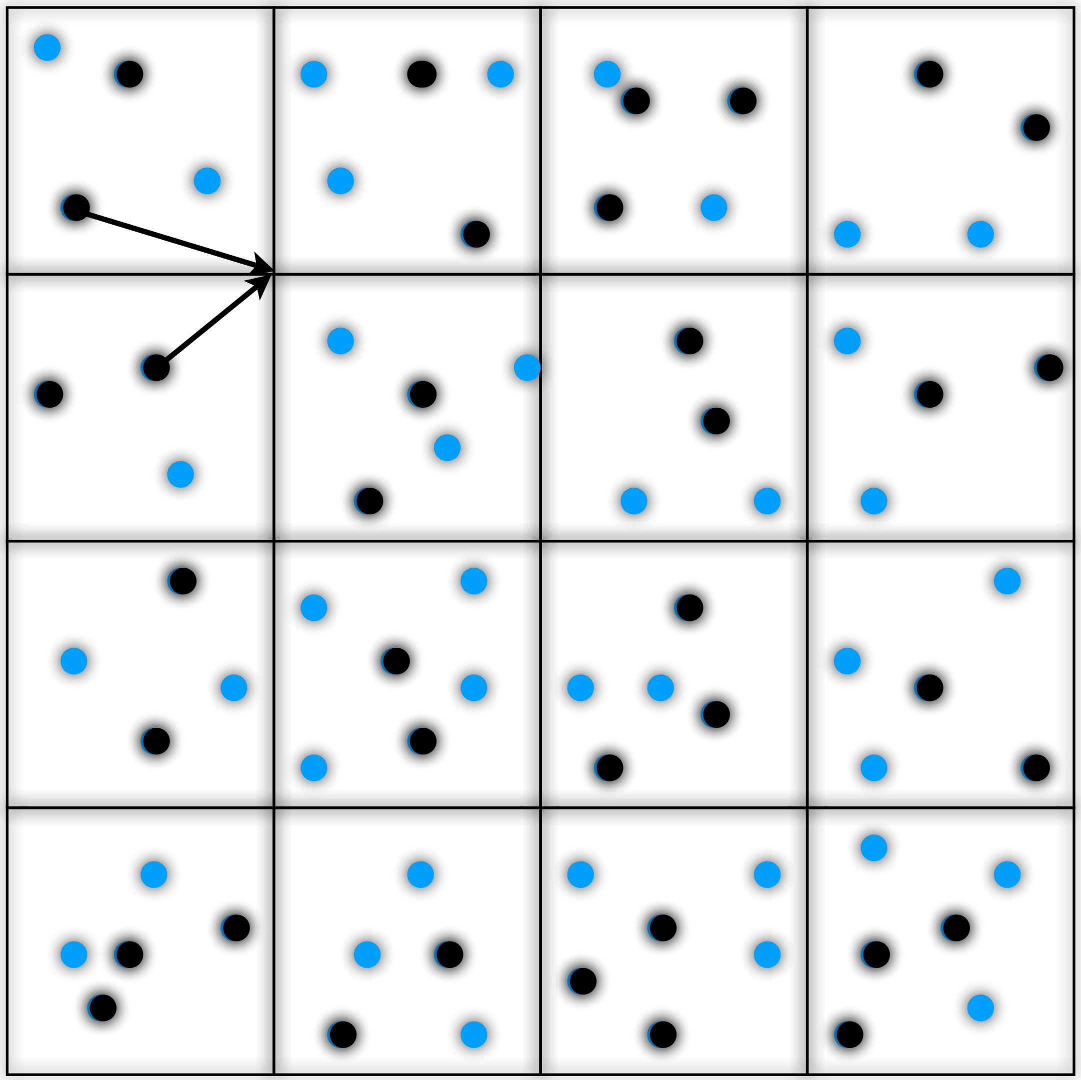

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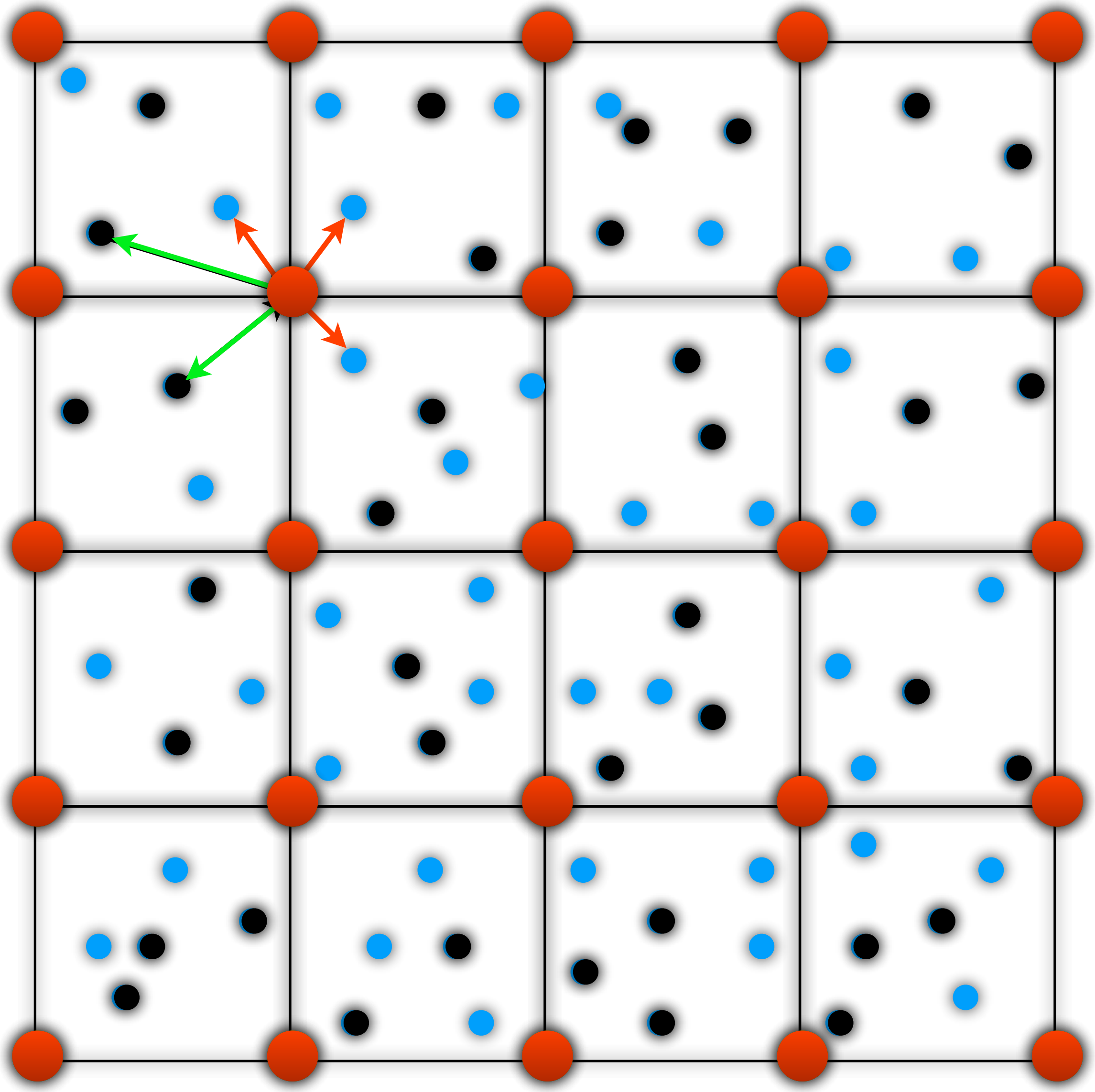
For instance, for a Yukawa potential:

$$V(R) \propto \frac{e^{-mr}}{r} \Rightarrow G(k) \propto \frac{1}{k^2 + m^2}$$

N-body algorithms for interacting Dark Energy

In interacting DE the coupling determines two different gravitational forces for dark matter and baryons:

Particle-Mesh (for interacting DE)



1) Assign mass to grid nodes for one species only, obtain density on the grid

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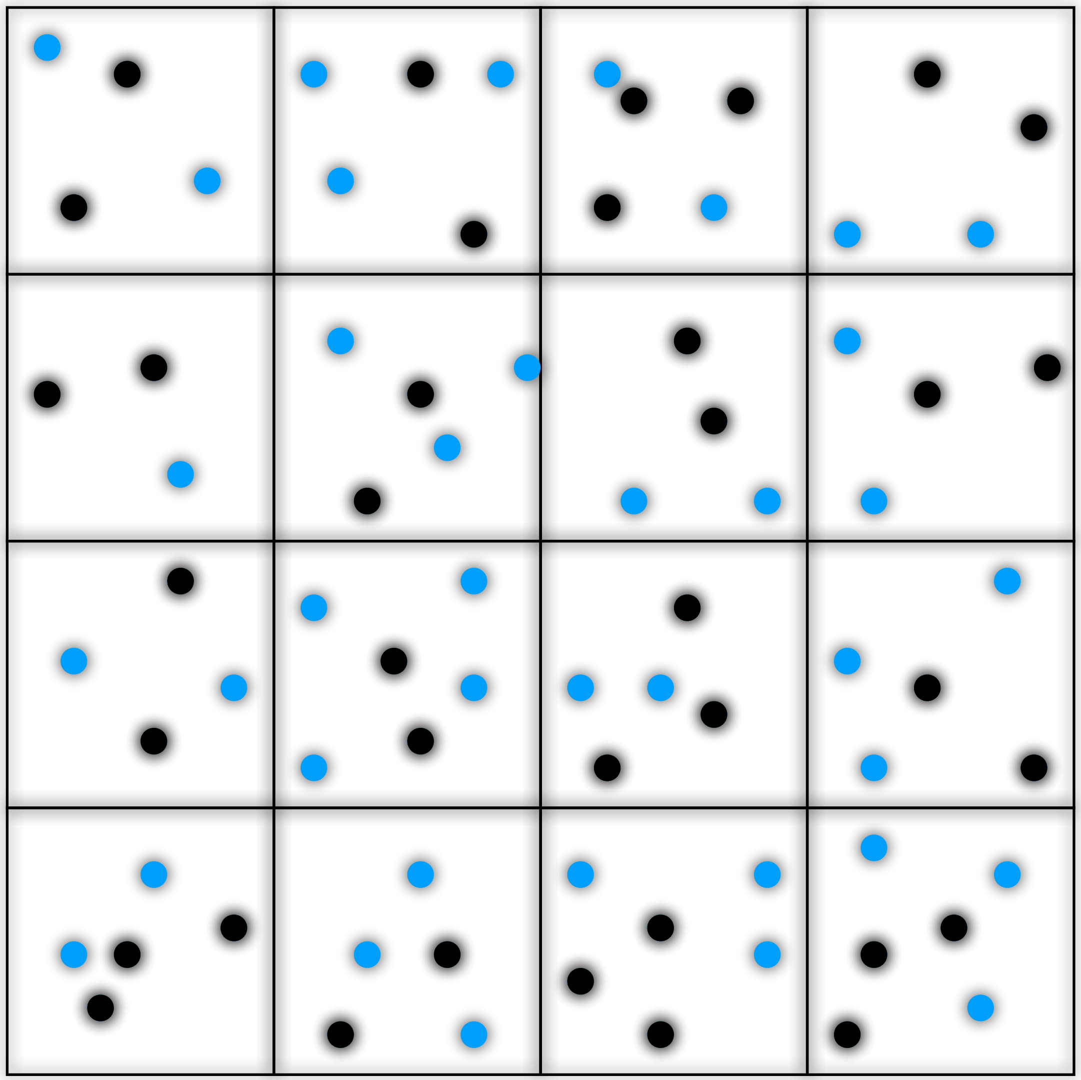

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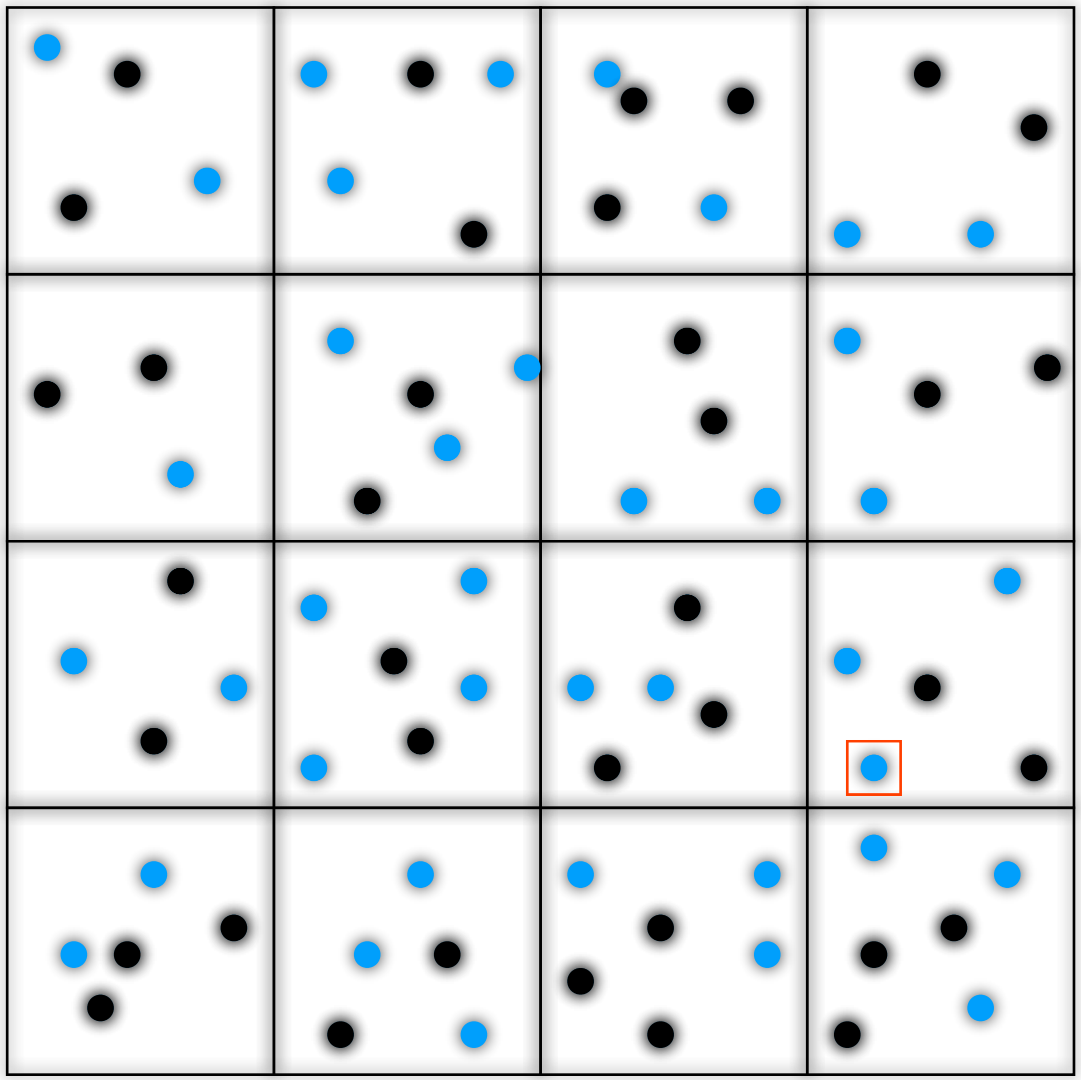

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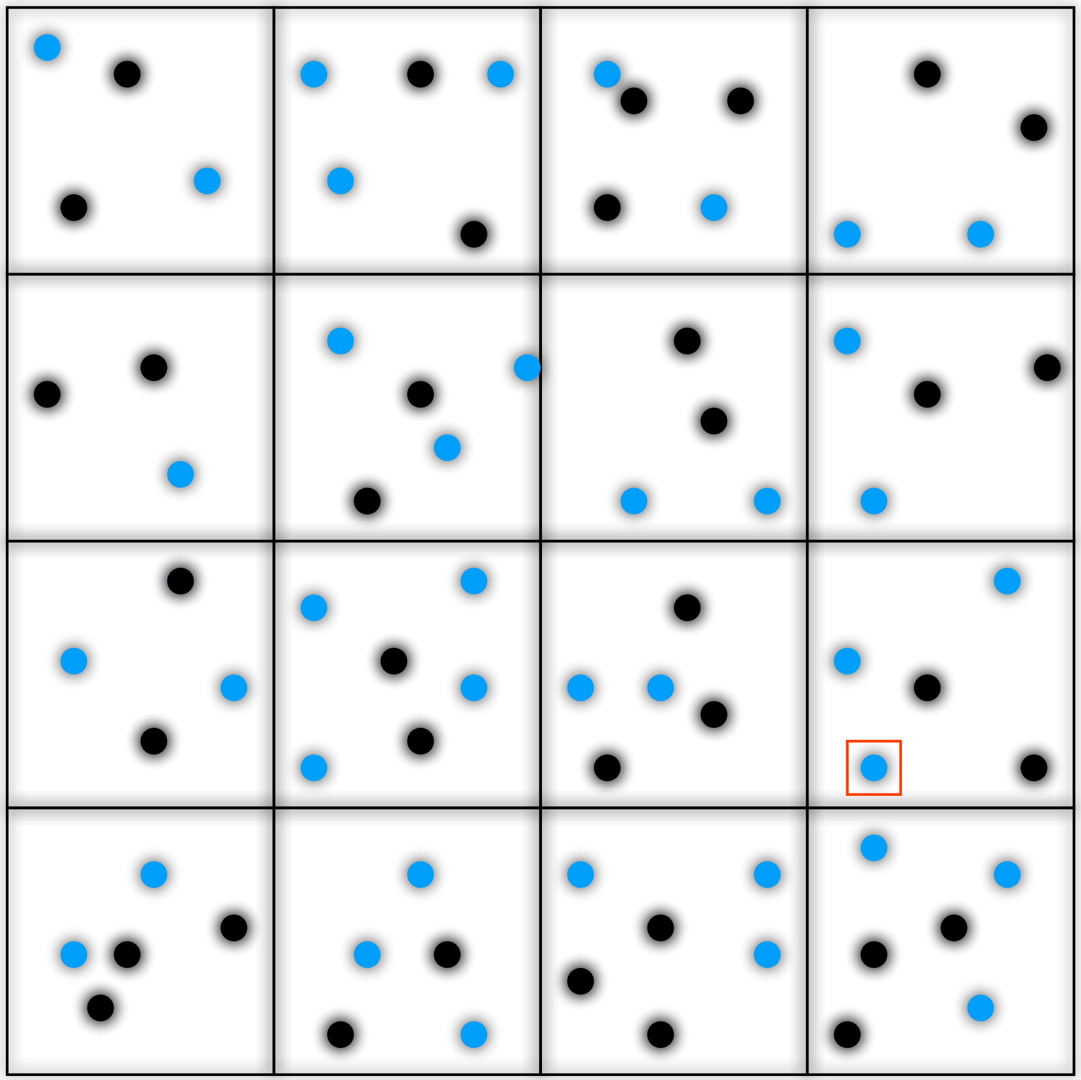

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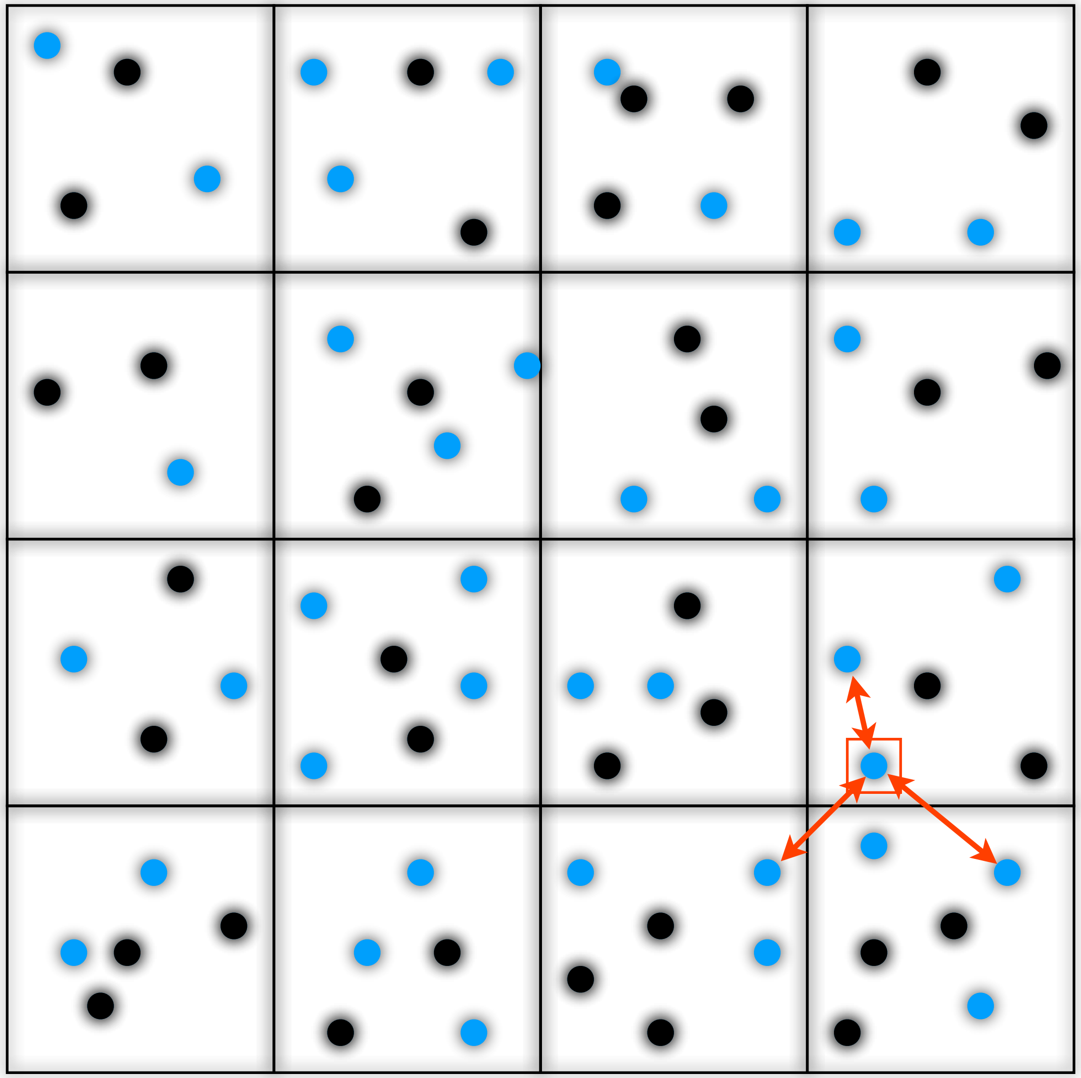

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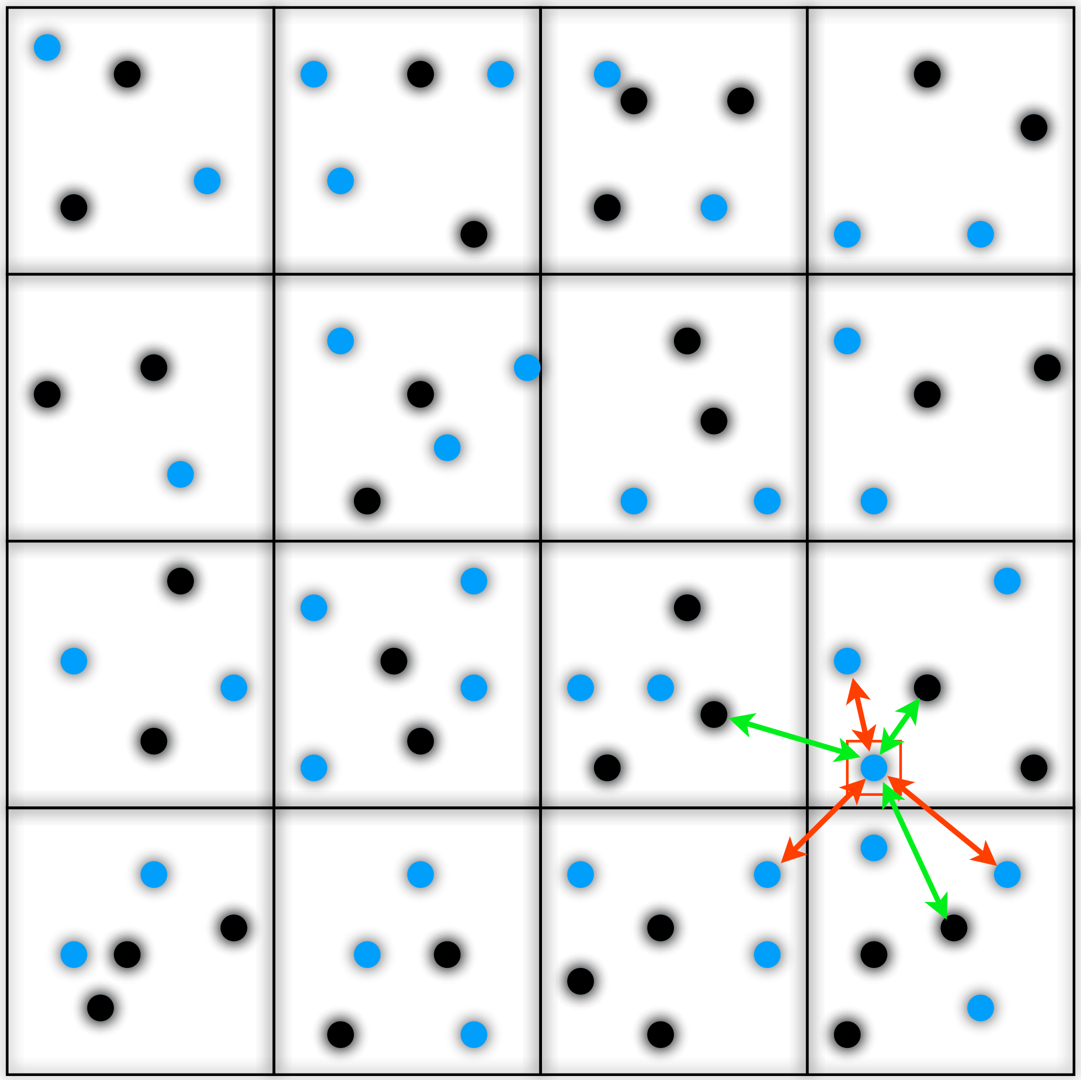

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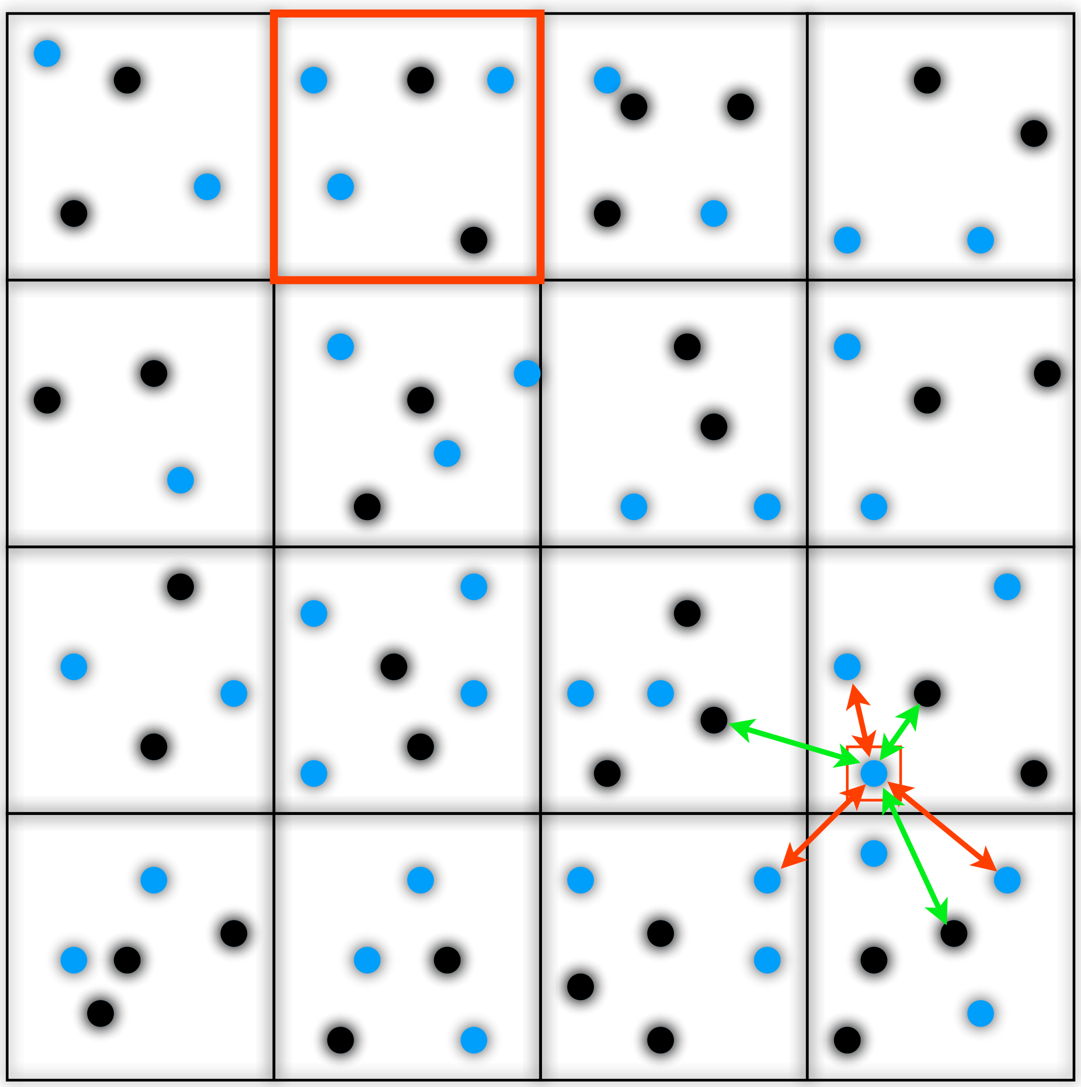

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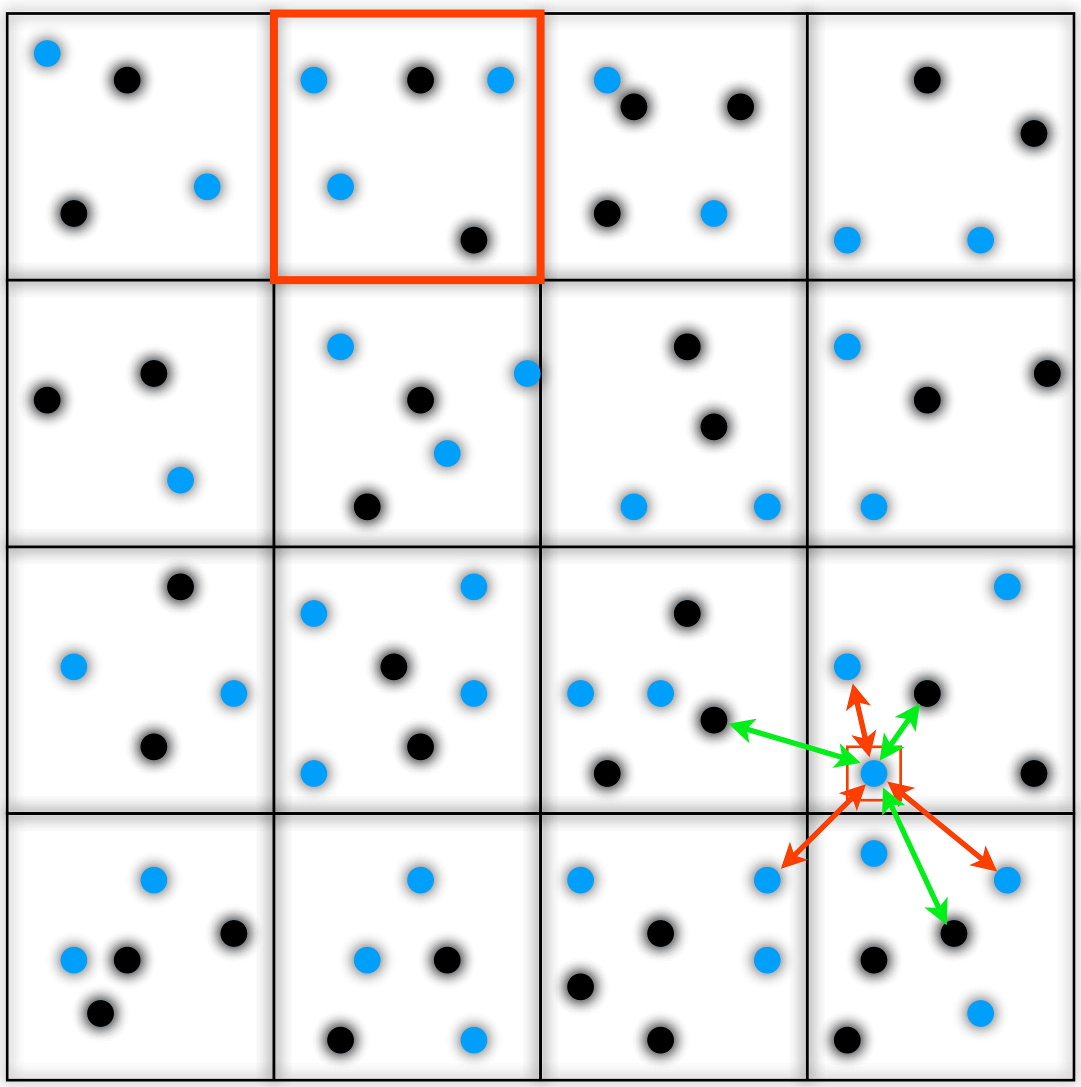

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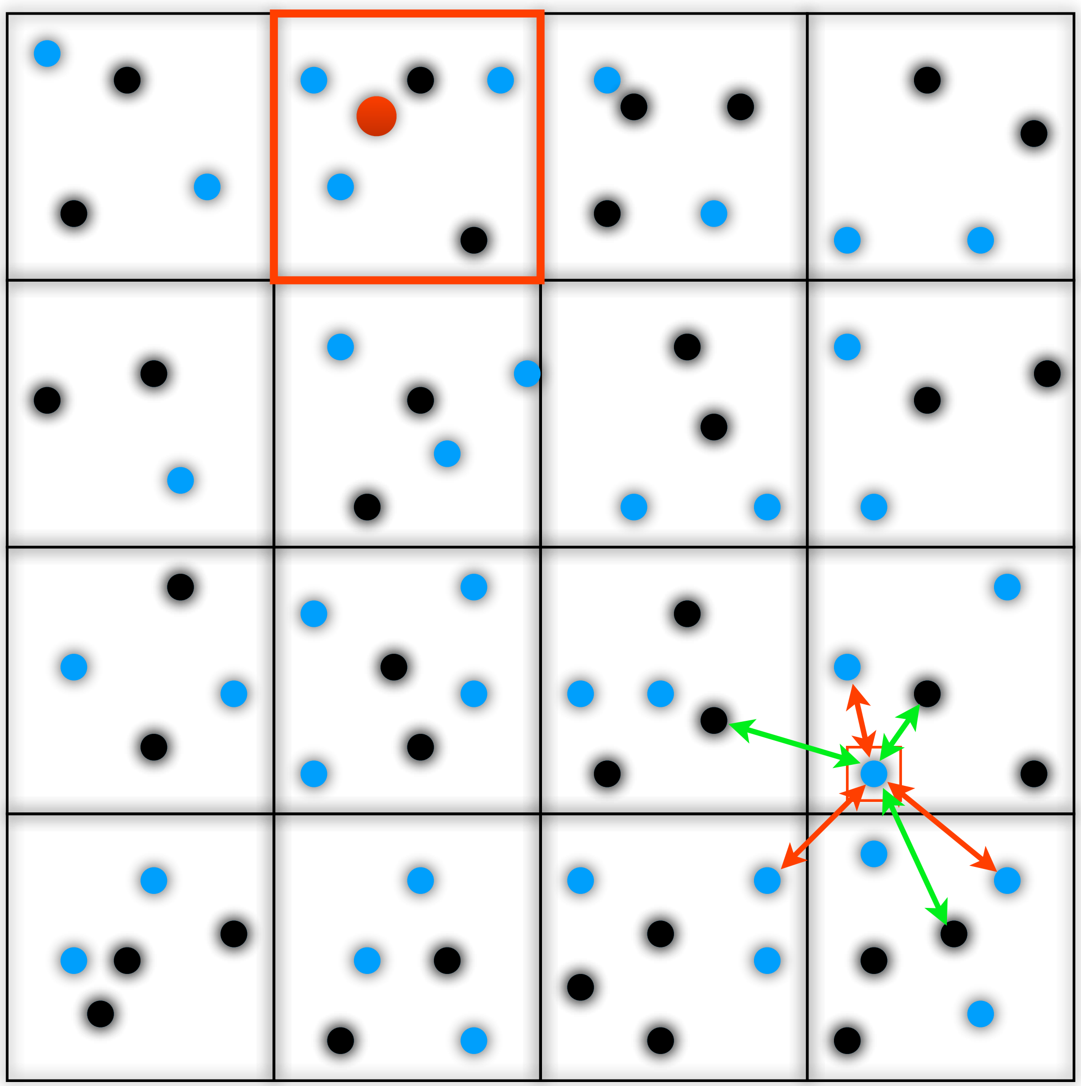

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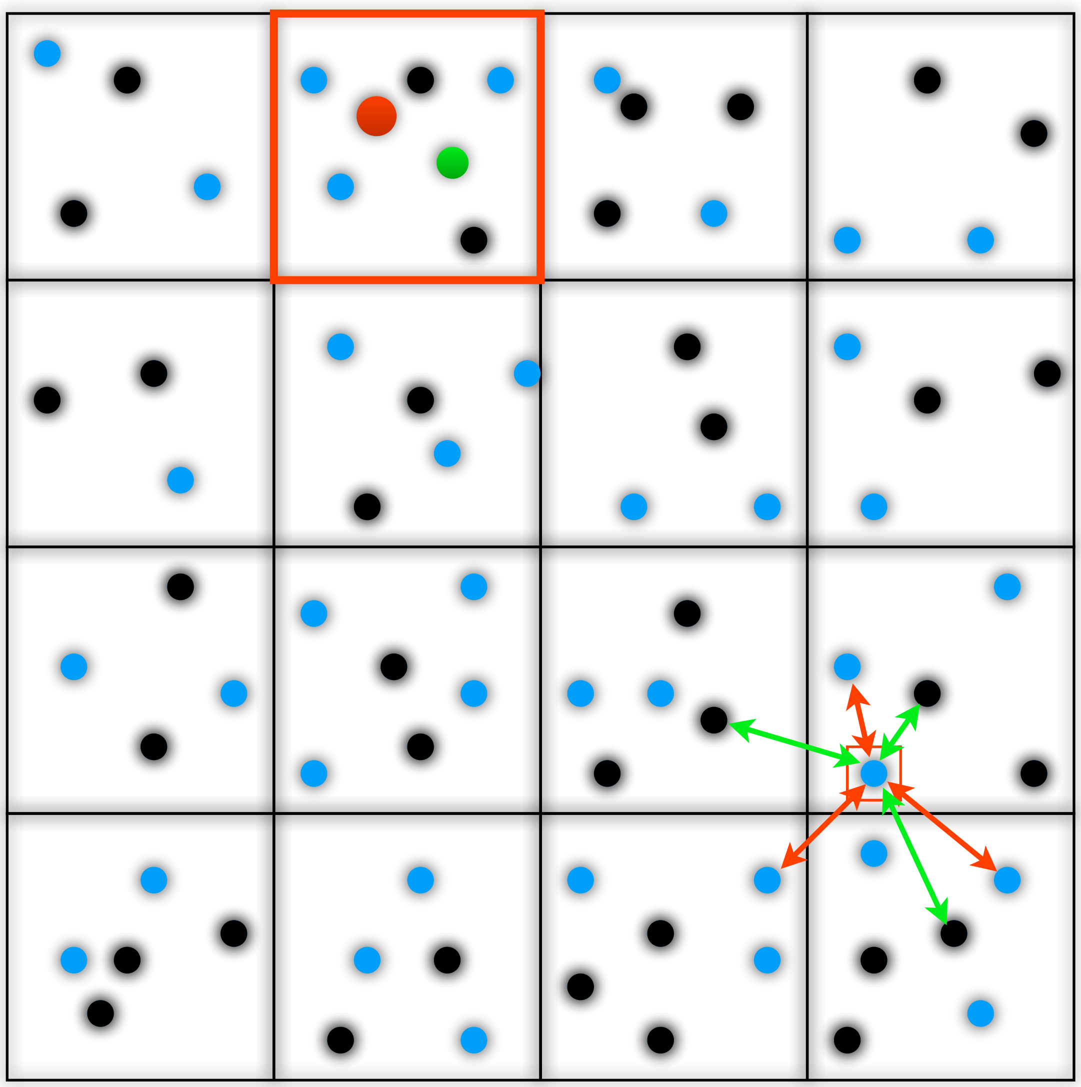

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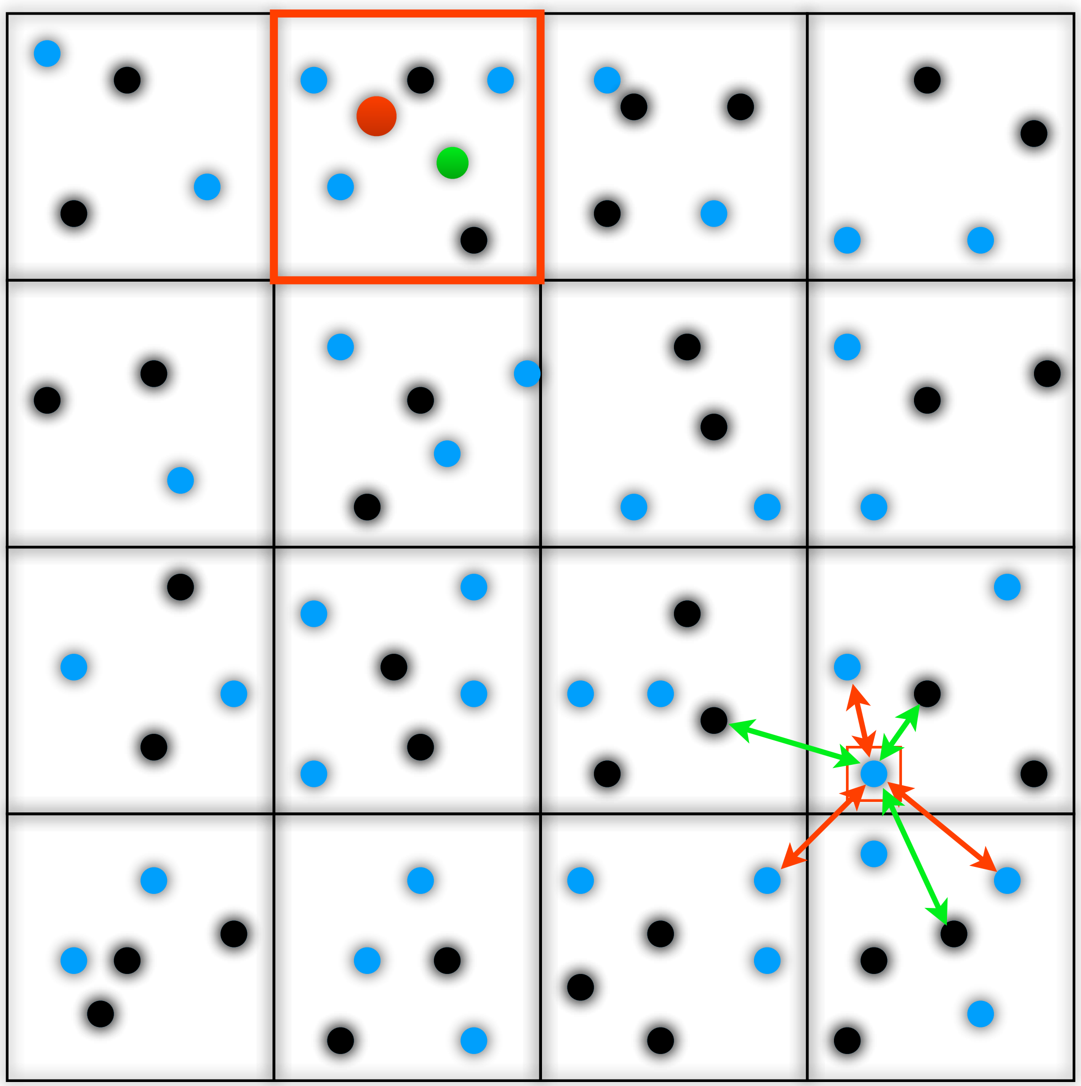

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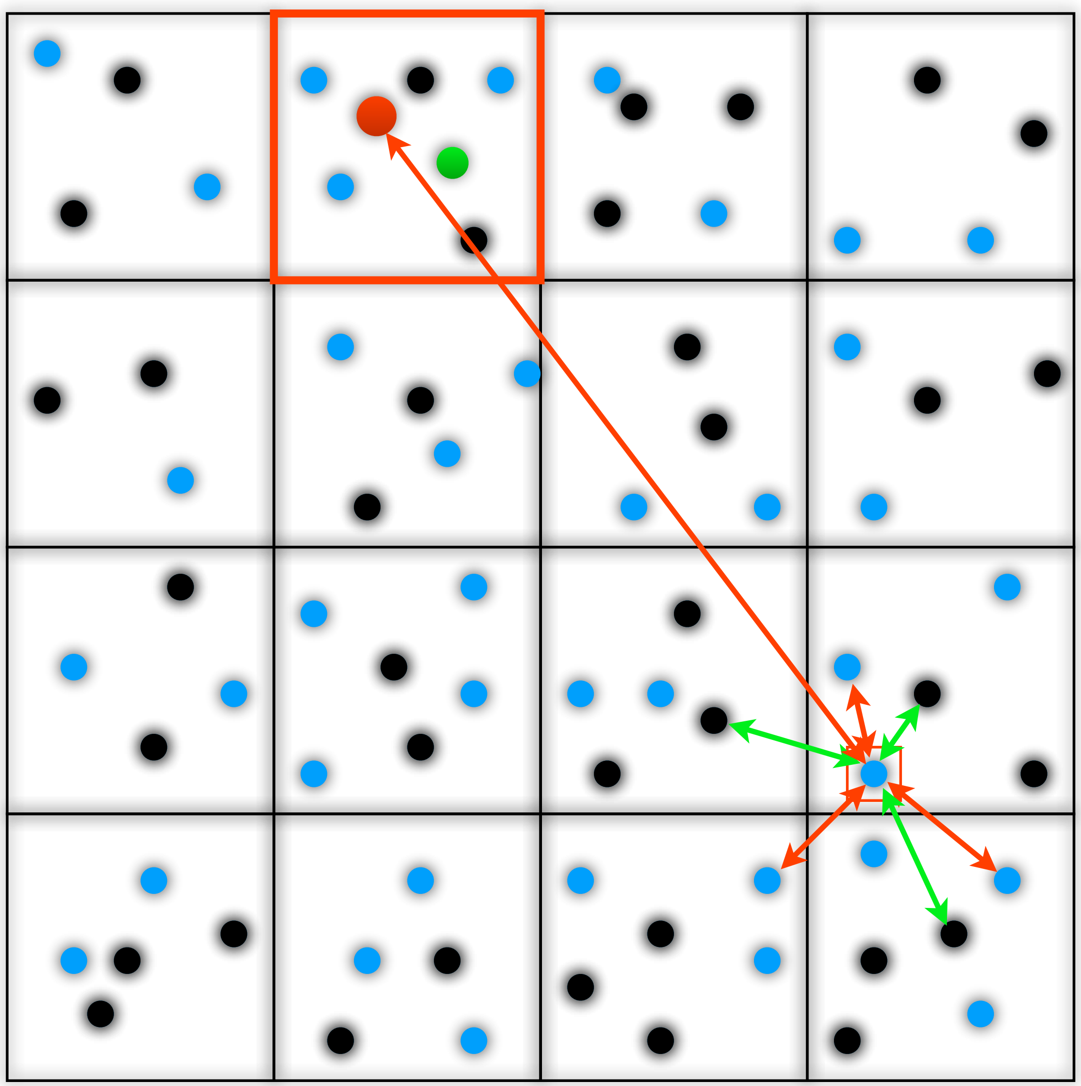

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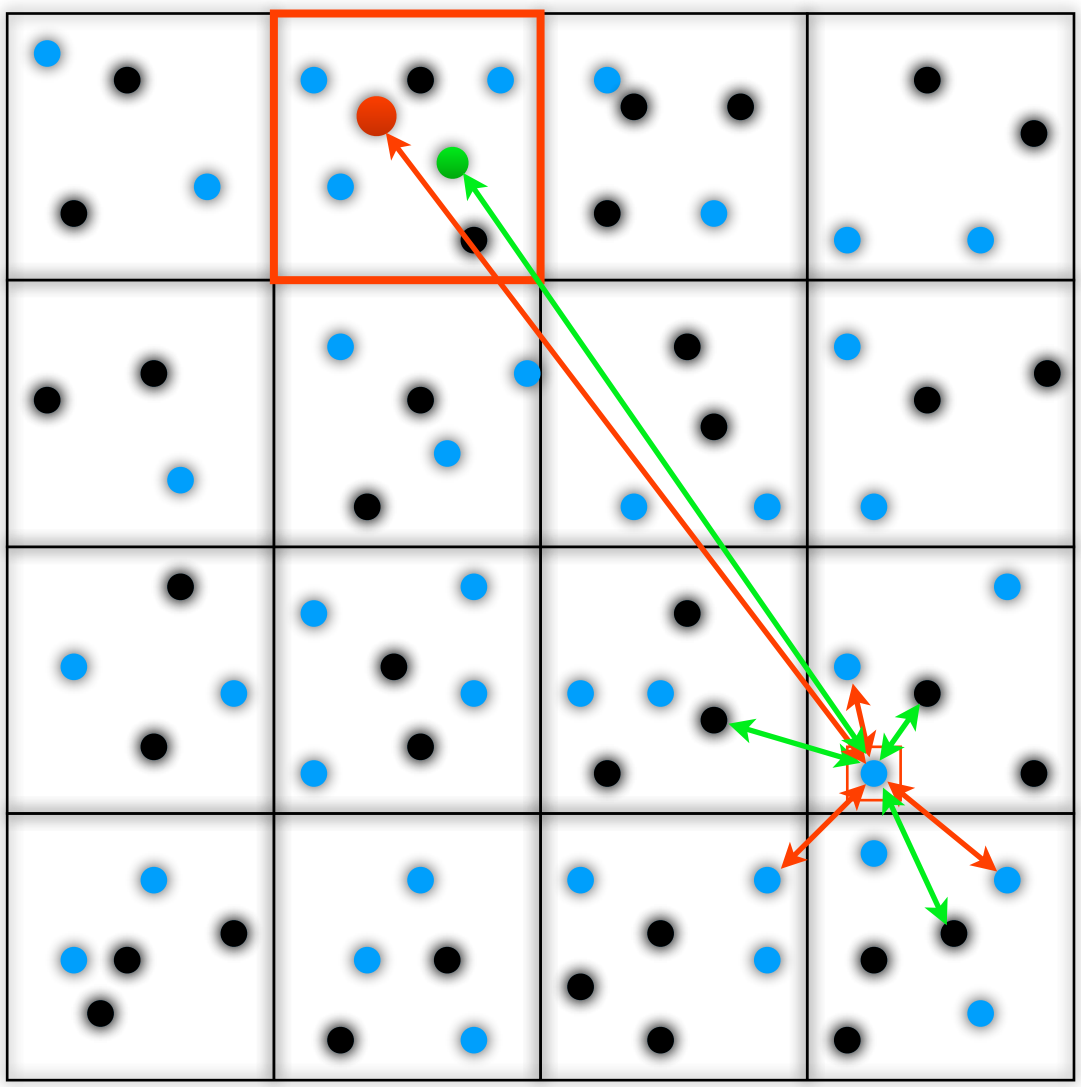

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Modified Gravity

N-body algorithms for Modified Gravity: *f(R)*

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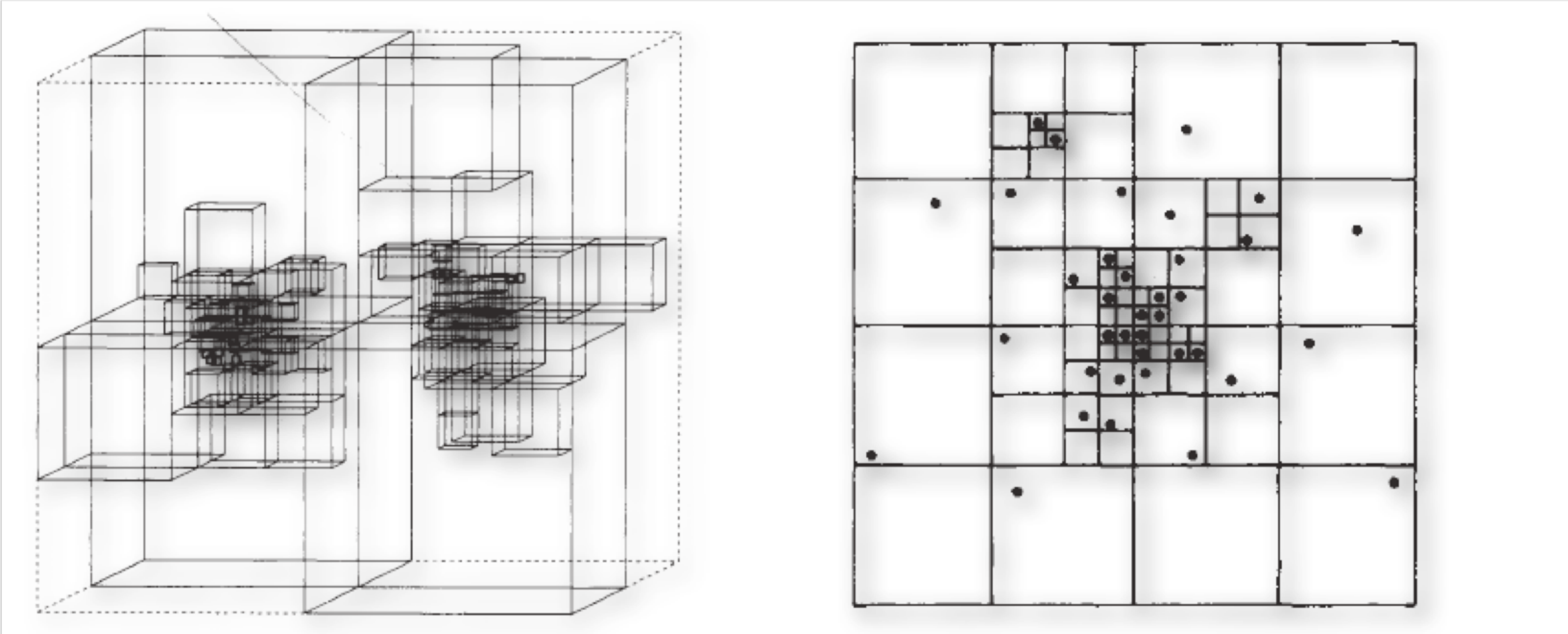

Barnes & Hut 1986

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- Employs multi-grid acceleration to achieve faster convergence

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• Once f_R is known, $\delta R(f_R)$ is also known, and the Poisson equation

$$\nabla^2\Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R$$

can be solved using the standard Gadget TreePM algorithm by:

i) associate an effective particle mass $m_{\delta R}$ to the density perturbations δR
ii) Apply the standard TreePM integration to the particles with mass $m + m_{\delta R}$

Massive Neutrinos

MASSIVE NEUTRINOS IN GADGET

Massive neutrinos have been included in N-body codes by different groups (see e.g. Brandbyge et al. 2008, Viel et al 2010, Wagner et al. 2012)

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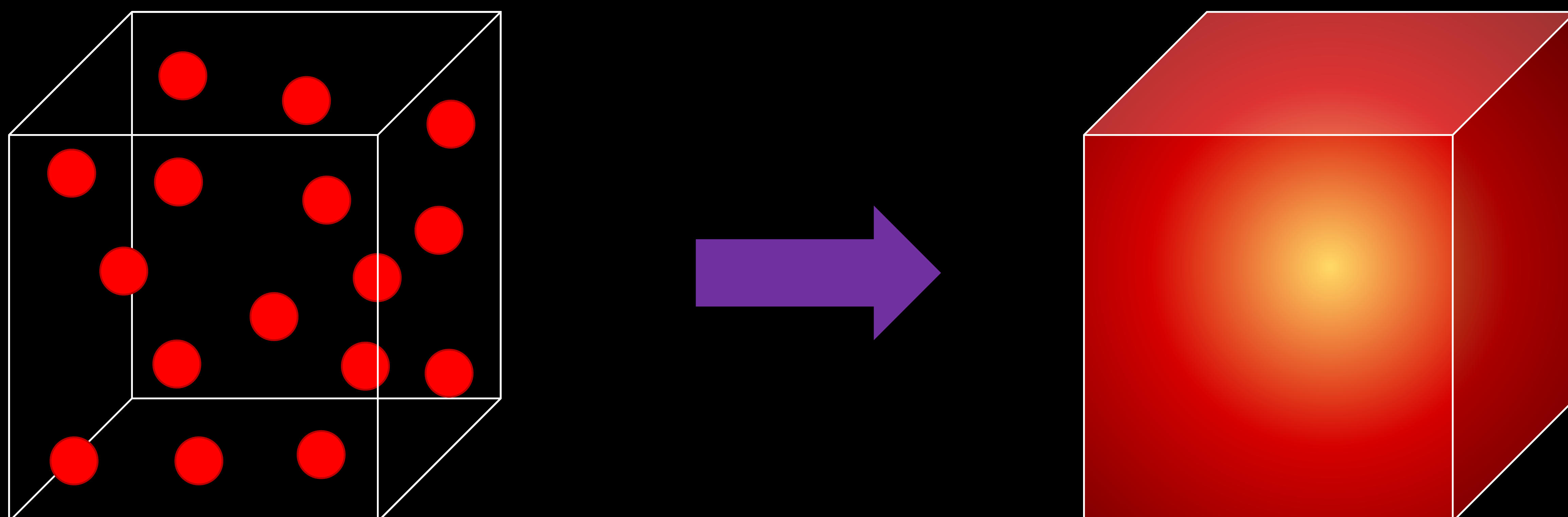
We adopt the latter method (more accurate in the non-linear regime, see e.g. Bird et al 2012)

Axions

SPH implementation

$\{\vec{r}_i, O_i\}$ $\quad$ $O(\vec{r})$



$$\vec{\nabla}\mathrm{Q} = \vec{\nabla}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

$$O_i = \sum_{j} m_j W_{ij} \frac{O_j}{\rho_j}$$

$$\vec{\nabla} O_i = \sum_{j} m_j \vec{\nabla} W_{ij} \frac{O_j}{\rho_j}$$

3° derivatives

=

$3 \cdot N \cdot N_{nn}$

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$$\vec{\nabla}\rho_i = \sum_{\mathrm{j}} m_j \vec{\nabla} W_{ij}$$

$$\nabla^2\rho_i = \sum_{\mathrm{j}} m_j \nabla^2 W_{ij}$$

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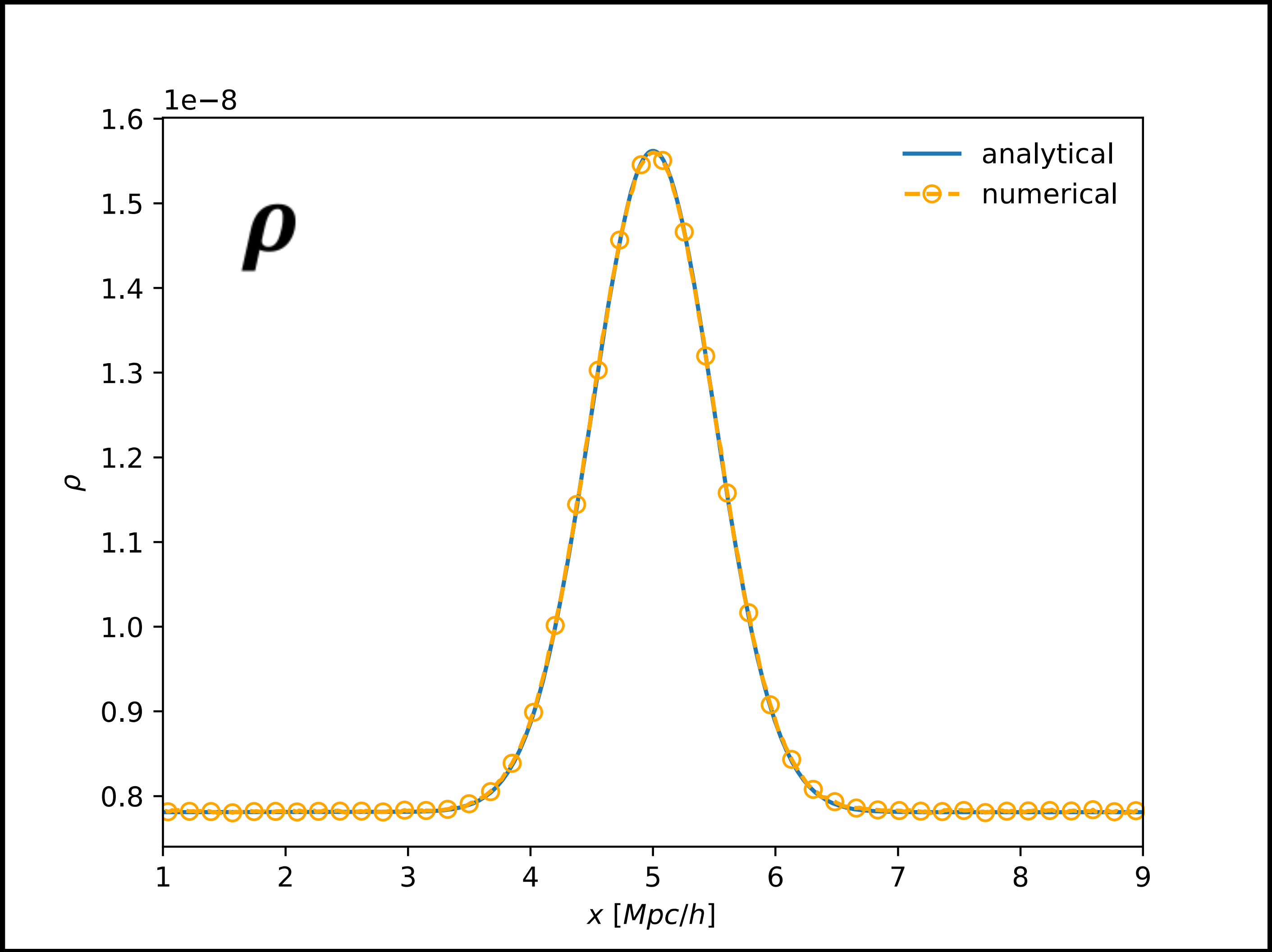

Gaussian overdensity $\rho \propto c + \exp(-x^2/2\sigma^2)$

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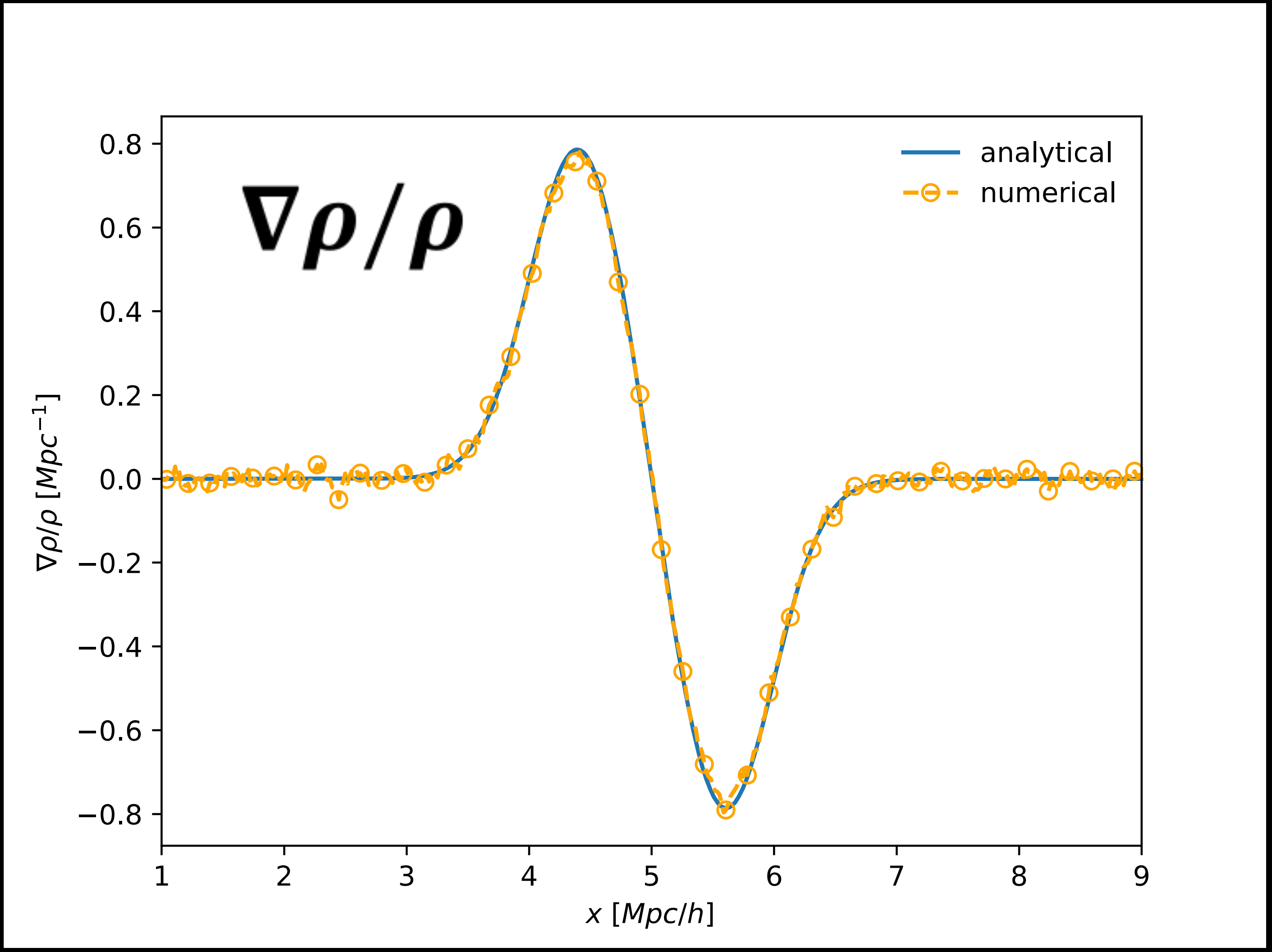

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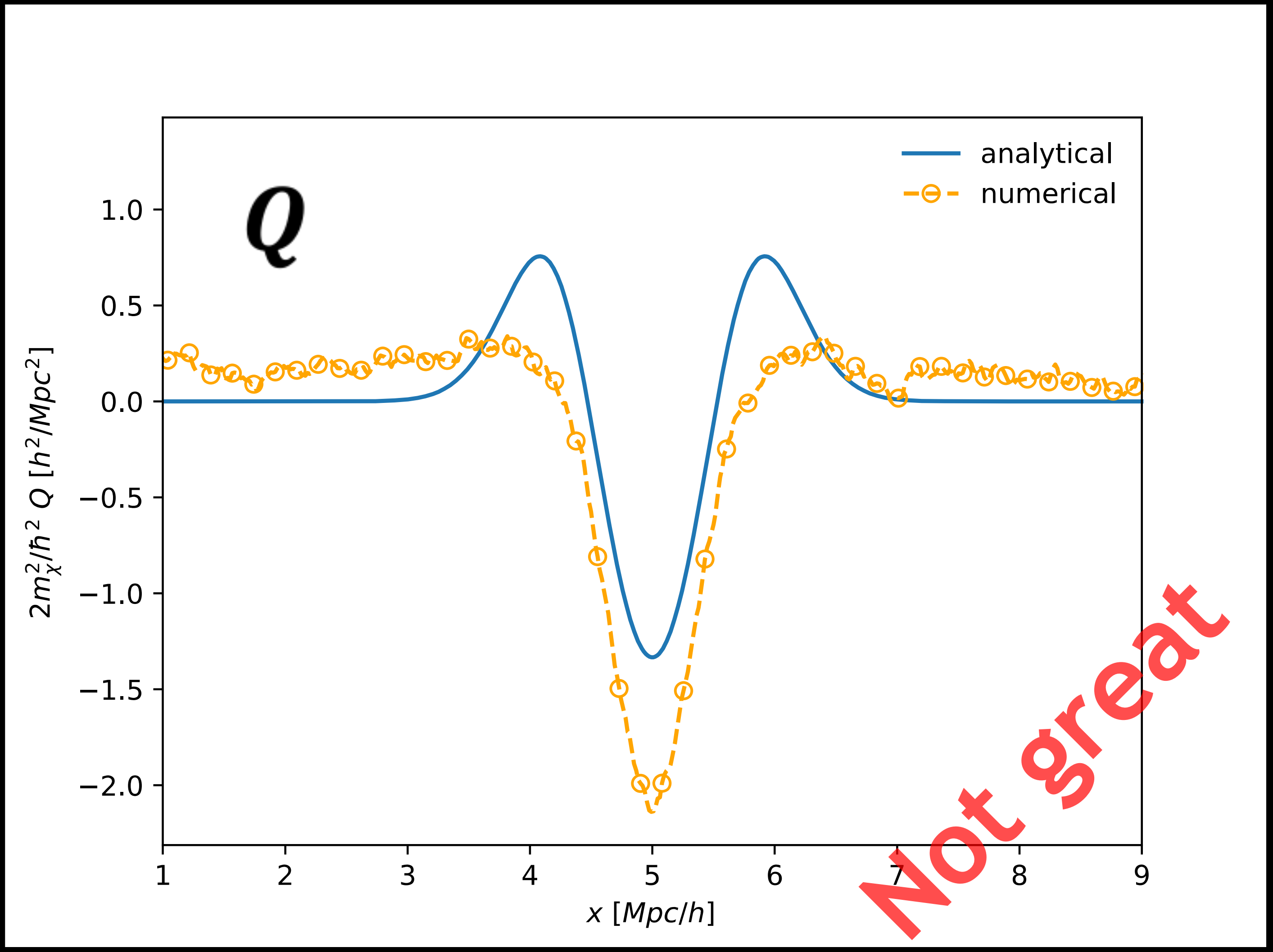

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Analytical equivalence

$\neq$

Numerical equivalence

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$$\vec{\nabla}\rho_i = \sum_{\mathrm{j}} m_j \vec{\nabla} W_{ij}$$

$$\vec{\nabla}\rho = \frac{1}{\phi}\left[\vec{\nabla}(\phi\rho) - \rho\vec{\nabla}\phi\right]$$

$$\nabla^2\rho_i = \sum_{\mathrm{j}} m_j \nabla^2 W_{ij}$$

$$\nabla^2\rho = \frac{1}{\phi}\left[\nabla^2(\phi\rho) - \rho\nabla^2\phi - 2\,\vec{\nabla}\rho\cdot\vec{\nabla}\phi\right]$$

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SPH implementation

$$\vec{\nabla}\rho_i = \sum_{\mathrm{j}} m_j \vec{\nabla} W_{ij} \frac{\rho_j - \rho_i}{\rho_j \phi_i/\phi_j}$$

In literature

$$\phi = \begin{cases} 1 \\ \sqrt{\rho} \\ \rho \end{cases}$$

$$\nabla^2\rho_i = \sum_{\mathrm{j}} m_j \nabla^2 W_{ij} \frac{\rho_j - \rho_i}{\rho_j \phi_i/\phi_j} - \frac{2}{\phi_i}\vec{\nabla}\rho_i \cdot \vec{\nabla}\phi_i$$

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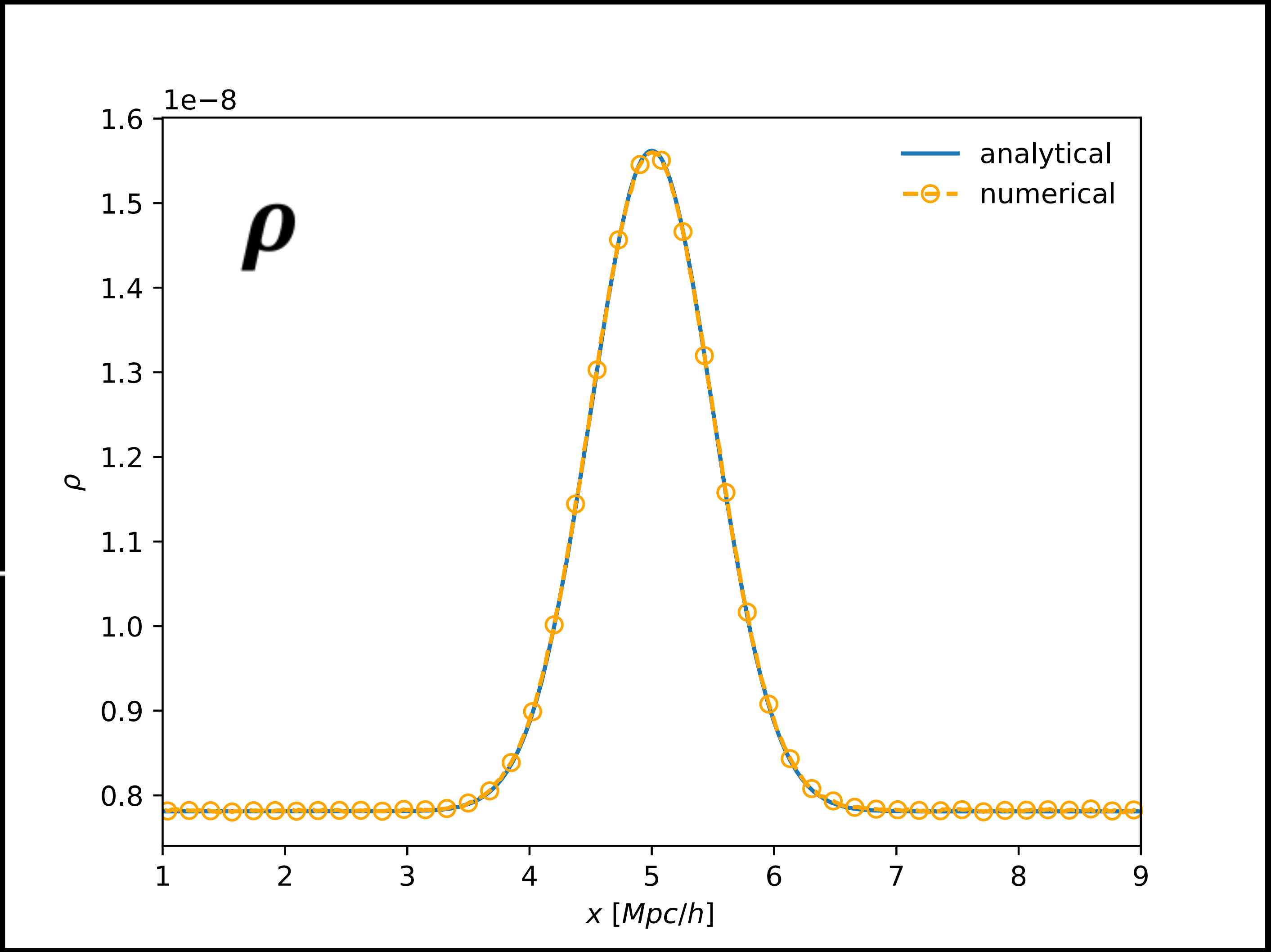

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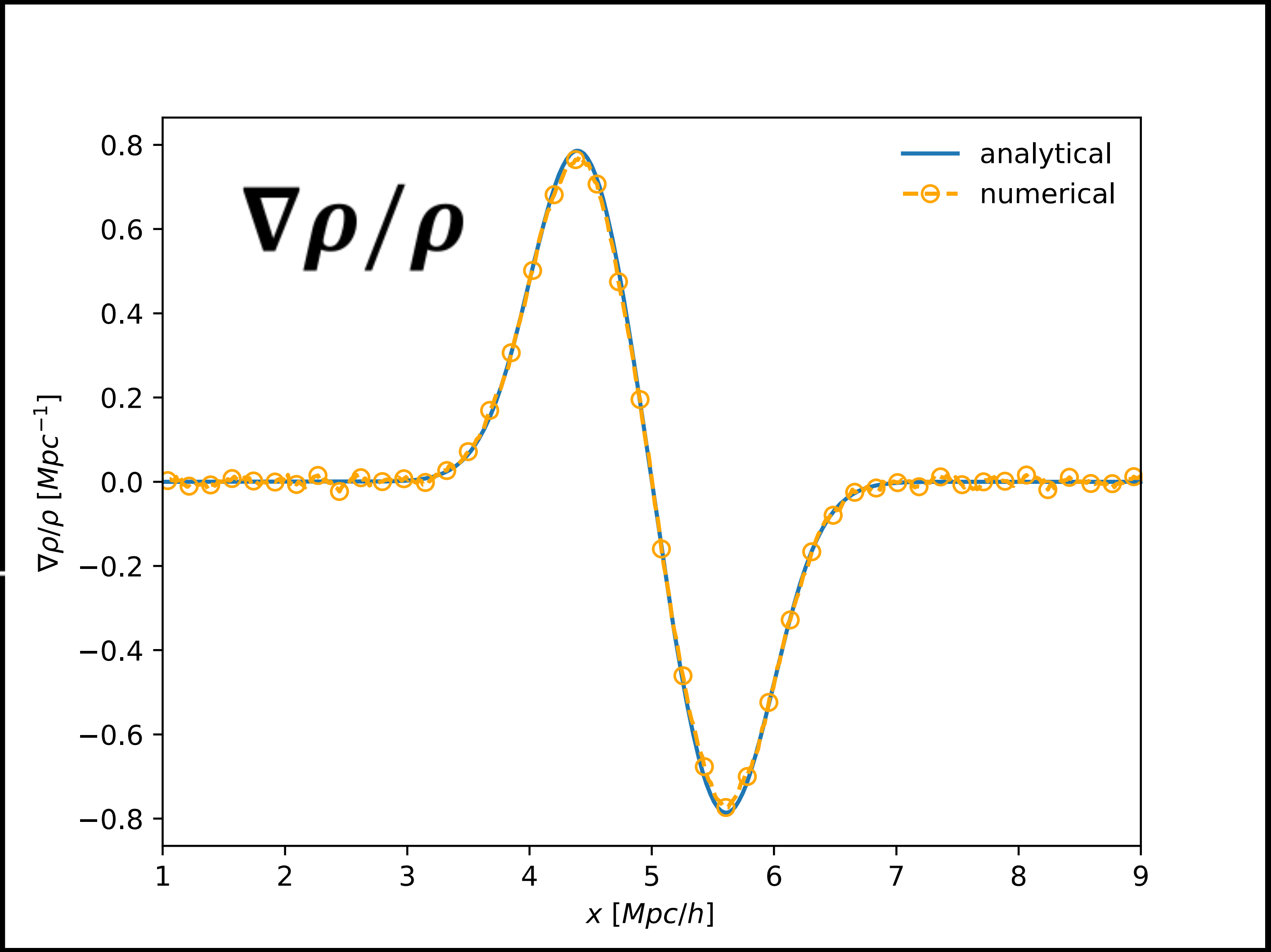

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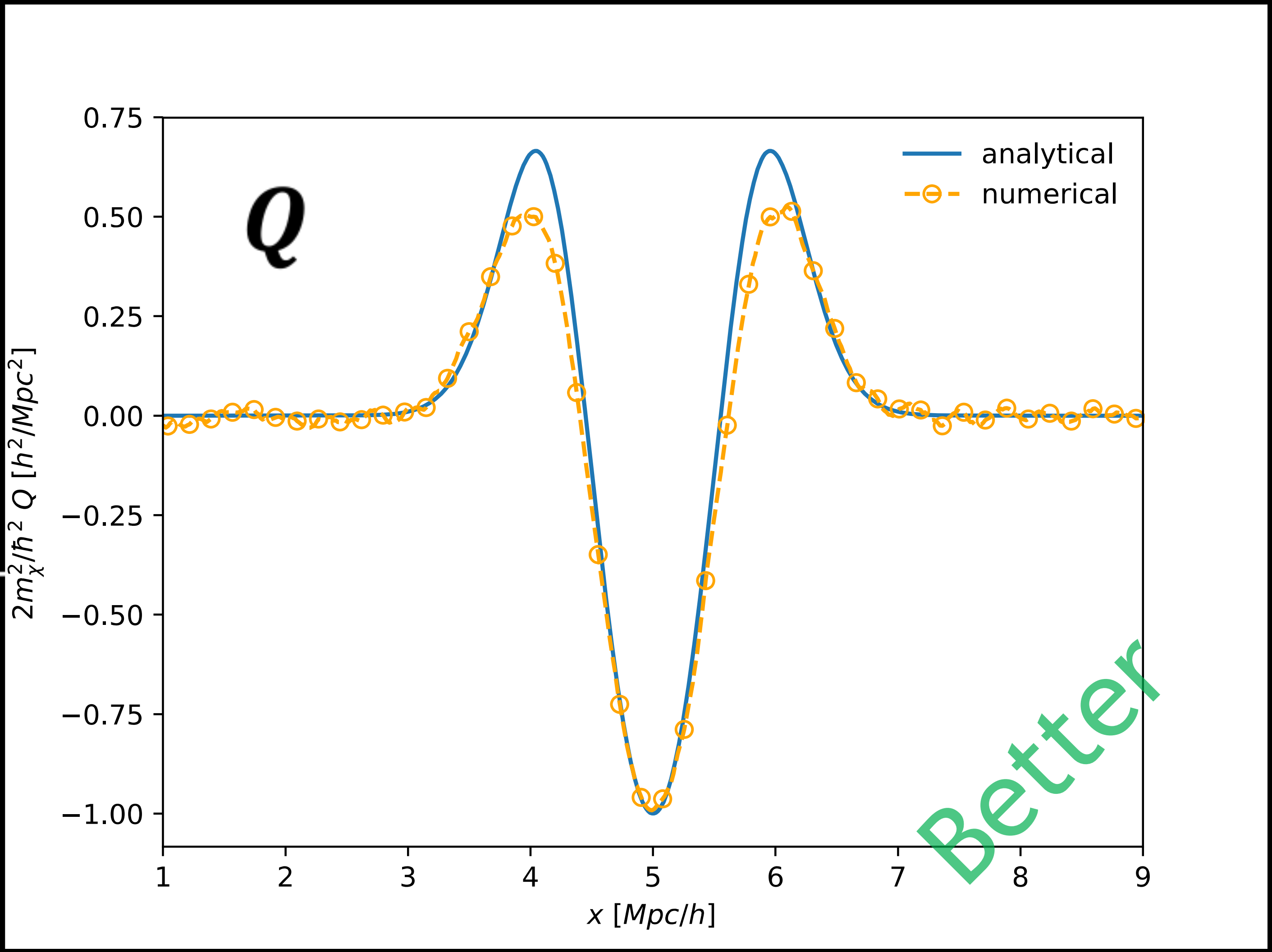

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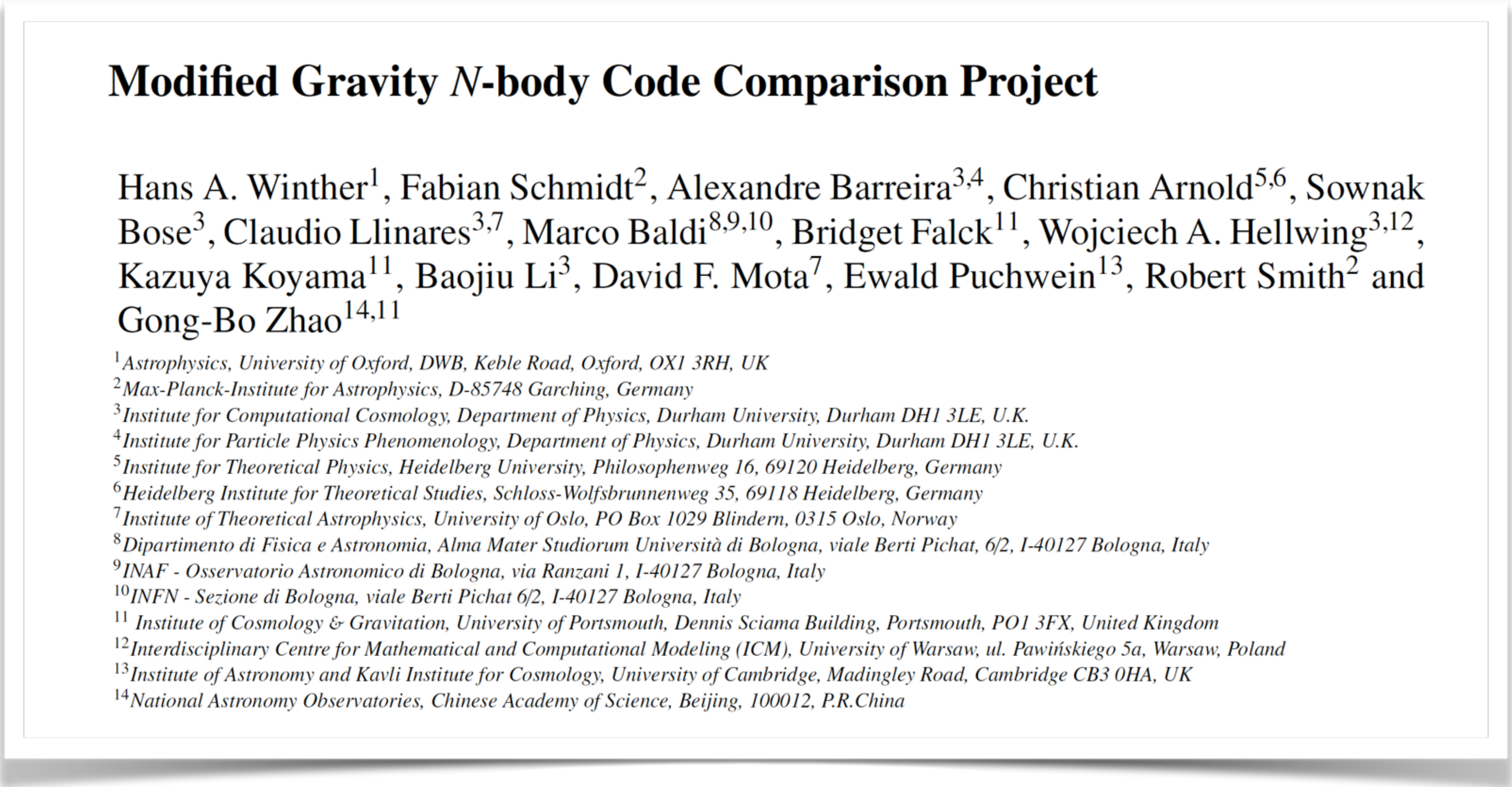
ARE NON-STANDARD CODES RELIABLE?

Comparing codes accuracy

N-body codes for non-standard models seem to be mature for accurate simulations. One example: comparison of MG codes (Winther et al. arXiv:1506.06384):



Modified Gravity *N*-body Code Comparison Project

Hans A. Winther[1], Fabian Schmidt[2], Alexandre Barreira[3,4], Christian Arnold[5,6], Sownak Bose[3], Claudio Llinares[3,7], Marco Baldi[8,9,10], Bridget Falck[11], Wojciech A. Hellwing[3,12], Kazuya Koyama[11], Baojiu Li[3], David F. Mota[7], Ewald Puchwein[13], Robert Smith[2] and Gong-Bo Zhao[14,11]

[1] *Astrophysics, University of Oxford, DWB, Keble Road, Oxford, OX1 3RH, UK*
[2] *Max-Planck-Institute for Astrophysics, D-85748 Garching, Germany*
[3] *Institute for Computational Cosmology, Department of Physics, Durham University, Durham DH1 3LE, U.K.*
[4] *Institute for Particle Physics Phenomenology, Department of Physics, Durham University, Durham DH1 3LE, U.K.*
[5] *Institute for Theoretical Physics, Heidelberg University, Philosophenweg 16, 69120 Heidelberg, Germany*
[6] *Heidelberg Institute for Theoretical Studies, Schloss-Wolfsbrunnenweg 35, 69118 Heidelberg, Germany*
[7] *Institute of Theoretical Astrophysics, University of Oslo, PO Box 1029 Blindern, 0315 Oslo, Norway*
[8] *Dipartimento di Fisica e Astronomia, Alma Mater Studiorum Università di Bologna, viale Berti Pichat, 6/2, I-40127 Bologna, Italy*
[9] *INAF - Osservatorio Astronomico di Bologna, via Ranzani 1, I-40127 Bologna, Italy*
[10] *INFN - Sezione di Bologna, viale Berti Pichat 6/2, I-40127 Bologna, Italy*
[11] *Institute of Cosmology & Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX, United Kingdom*
[12] *Interdisciplinary Centre for Mathematical and Computational Modeling (ICM), University of Warsaw, ul. Pawińskiego 5a, Warsaw, Poland*
[13] *Institute of Astronomy and Kavli Institute for Cosmology, University of Cambridge, Madingley Road, Cambridge CB3 0HA, UK*
[14] *National Astronomy Observatories, Chinese Academy of Science, Beijing, 100012, P.R.China*

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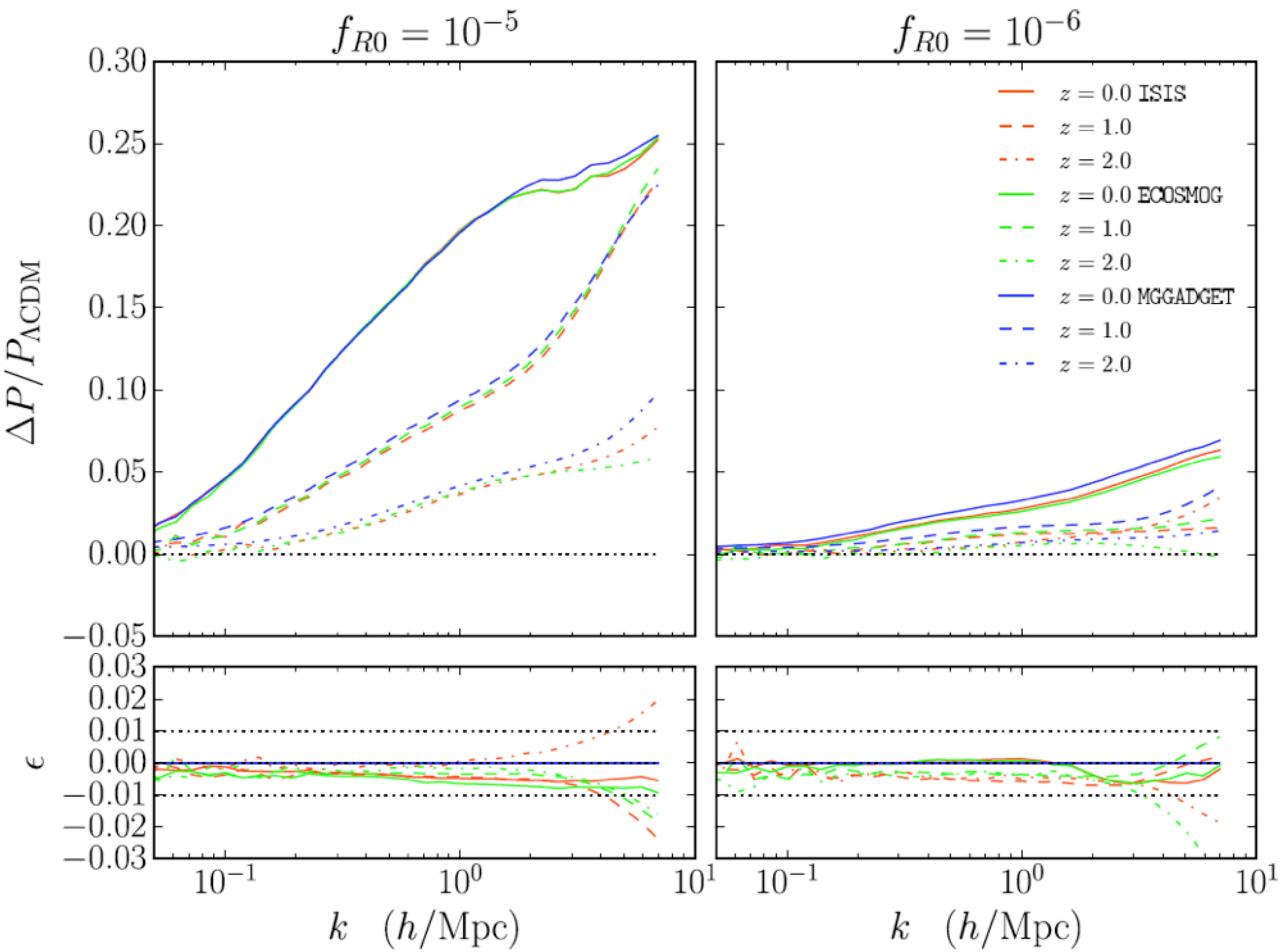

Found a relative accuracy in the power spectrum deviation better than 1% up to k~5 h/Mpc (at z=0)

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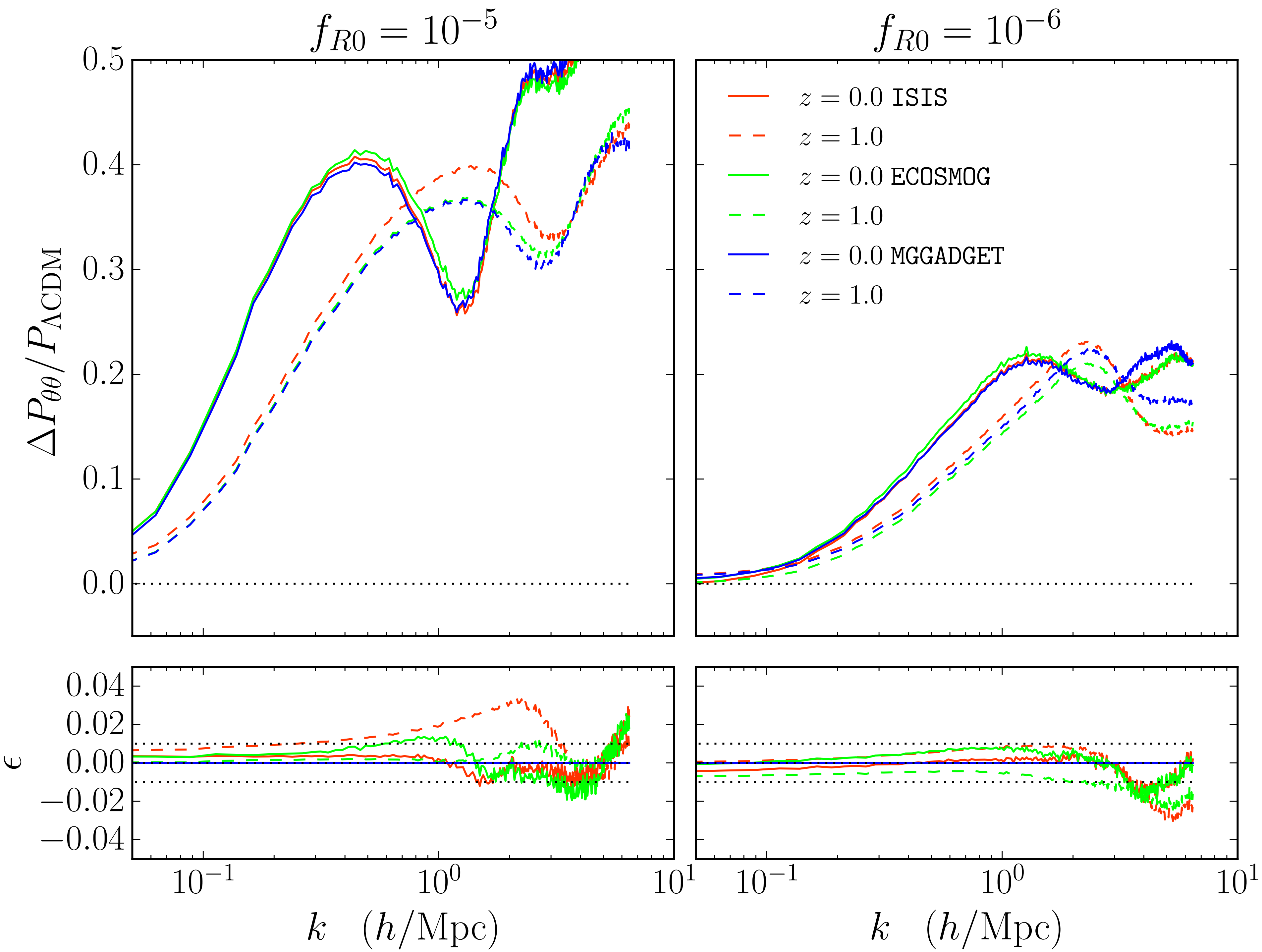
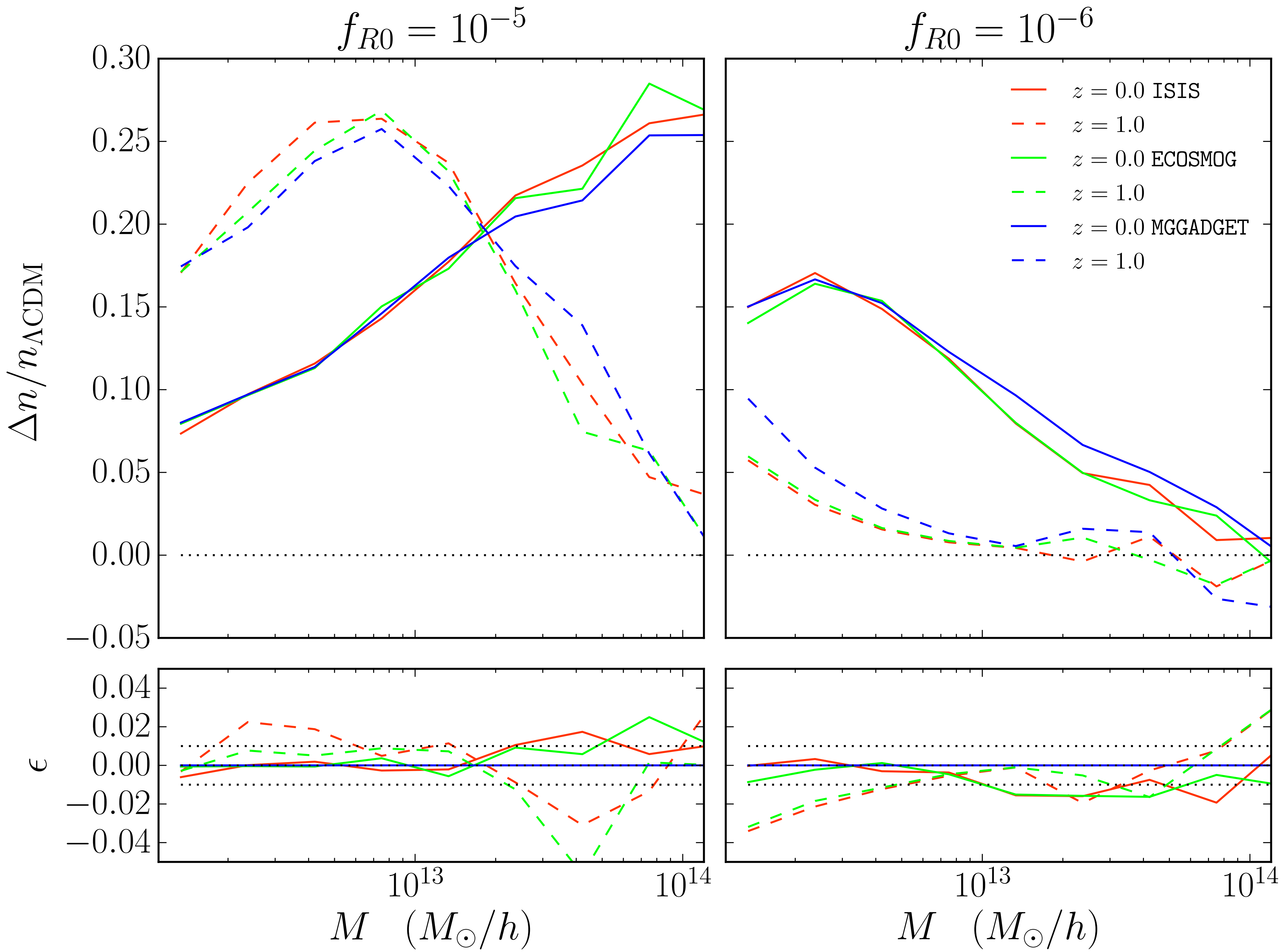


Found a good relative accuracy also in the velocity divergence power spectra

Comparing codes accuracy

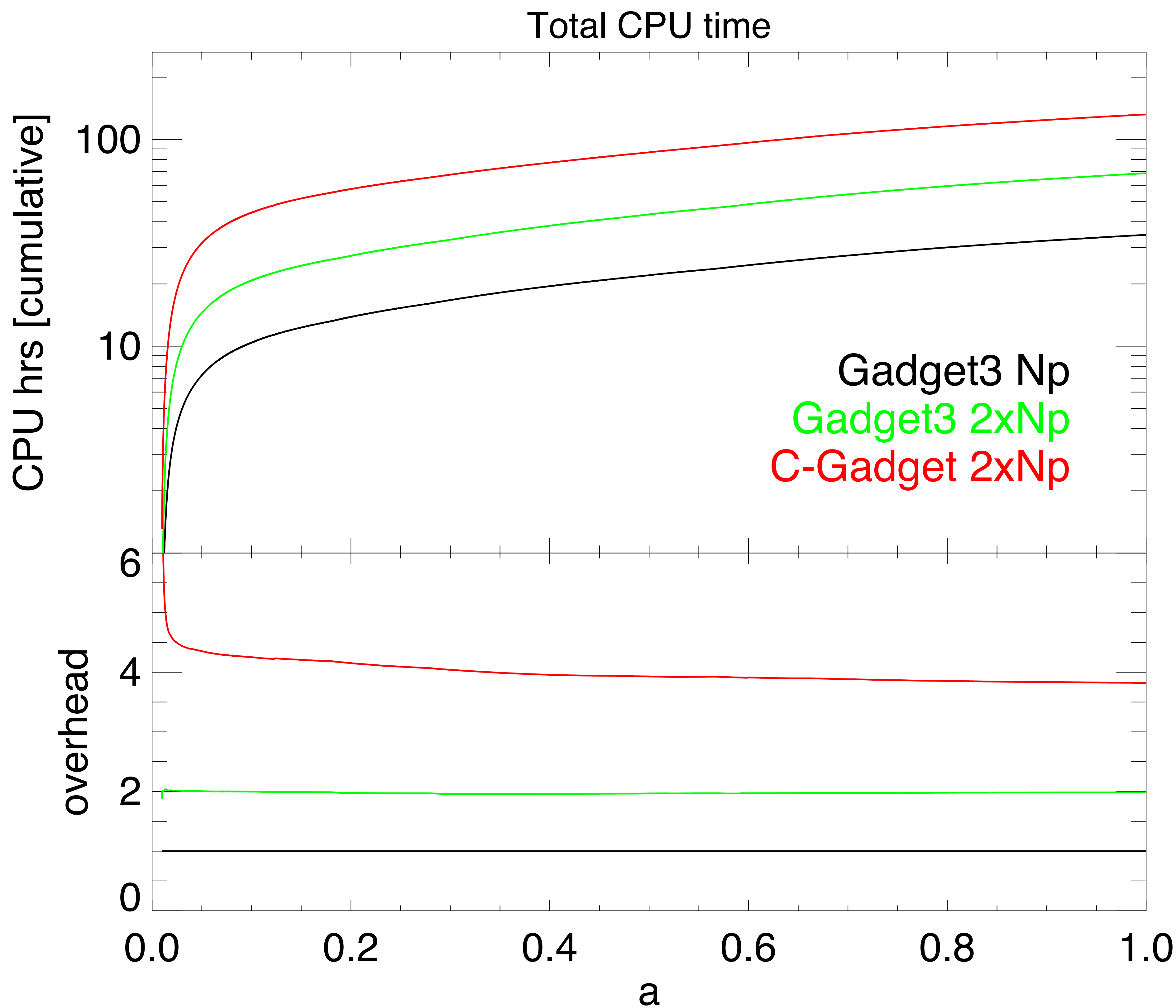
N-body codes for non-standard models seem to be mature for accurate simulations. One example: comparison of MG codes (Winther et al. arXiv:1506.06384):



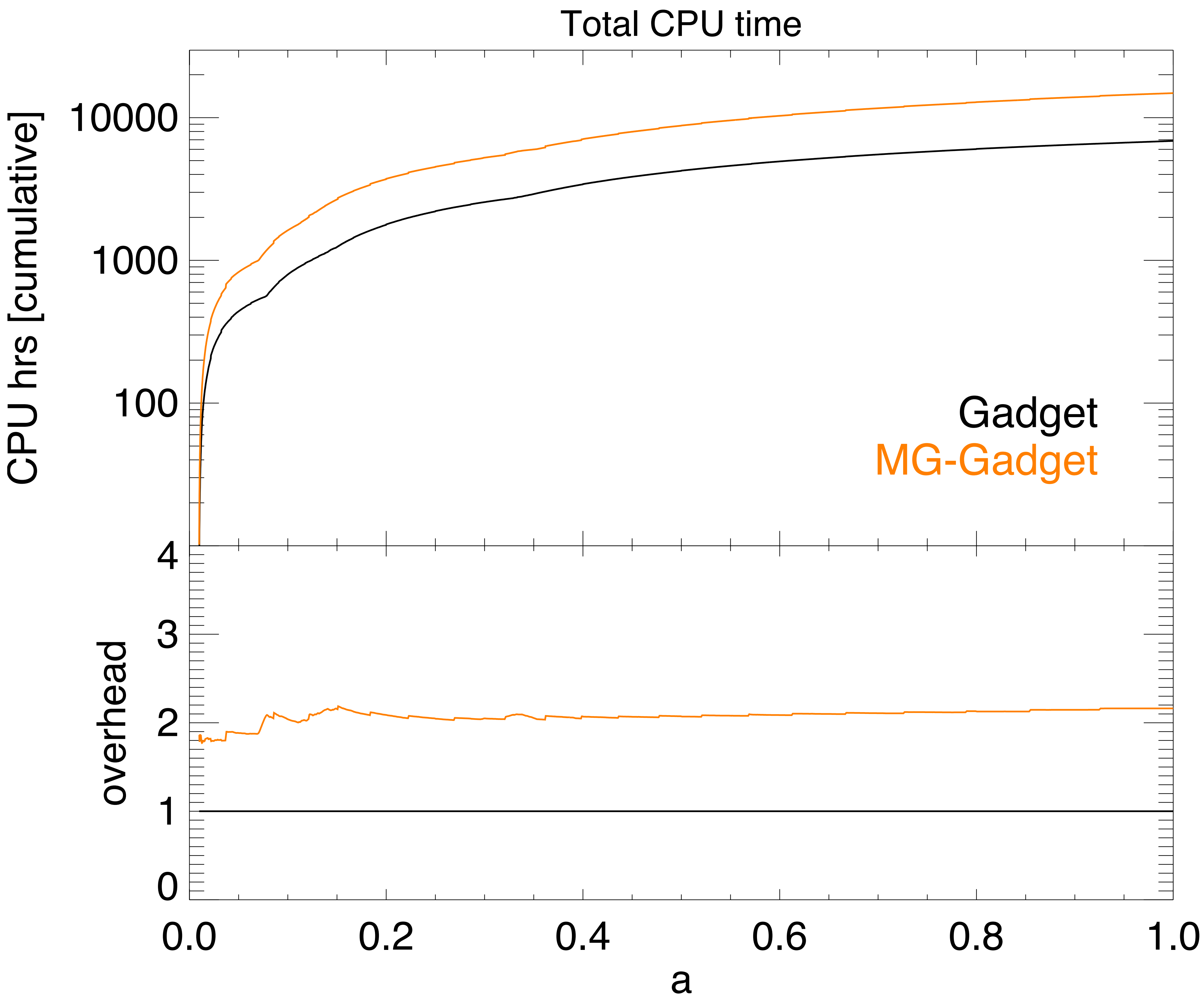
Found a good relative accuracy also in the halo abundance (though with a slightly larger scatter)

Focusing on code performances

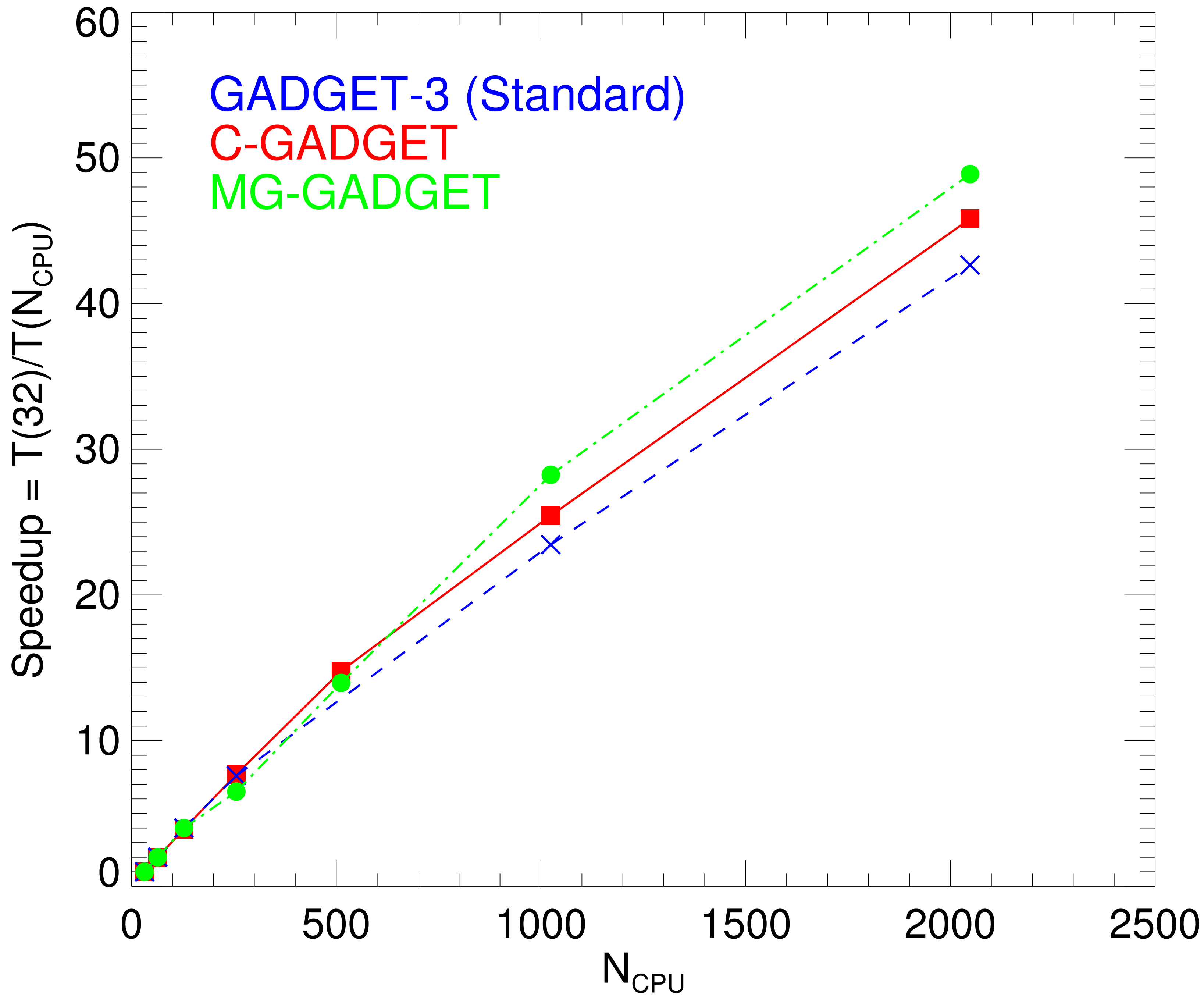
C-G PERFORMANCE: ALGORITHM OVERHEAD



MG-G PERFORMANCE: ALGORITHM OVERHEAD



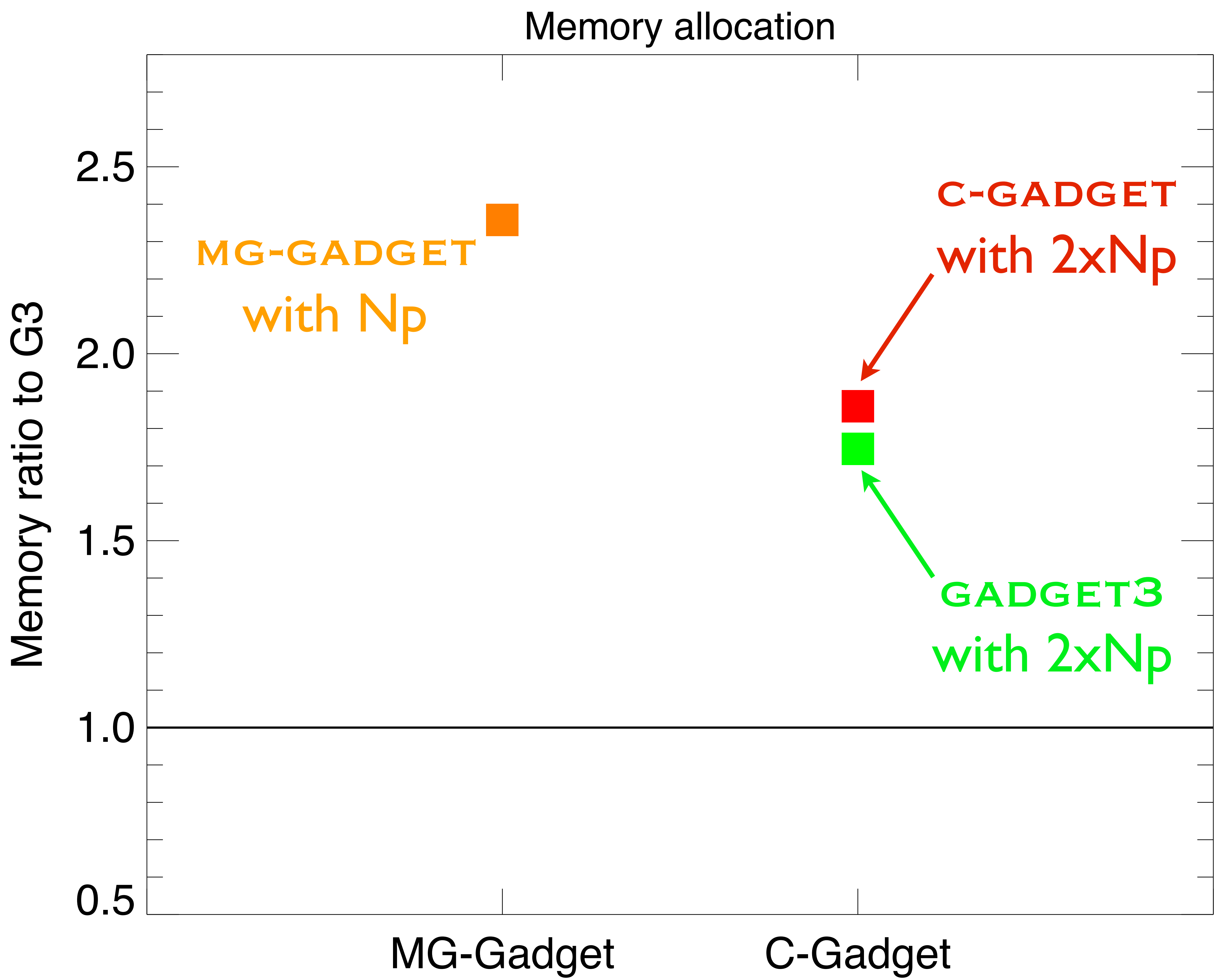
C-GADGET & MG-GADGET SCALING



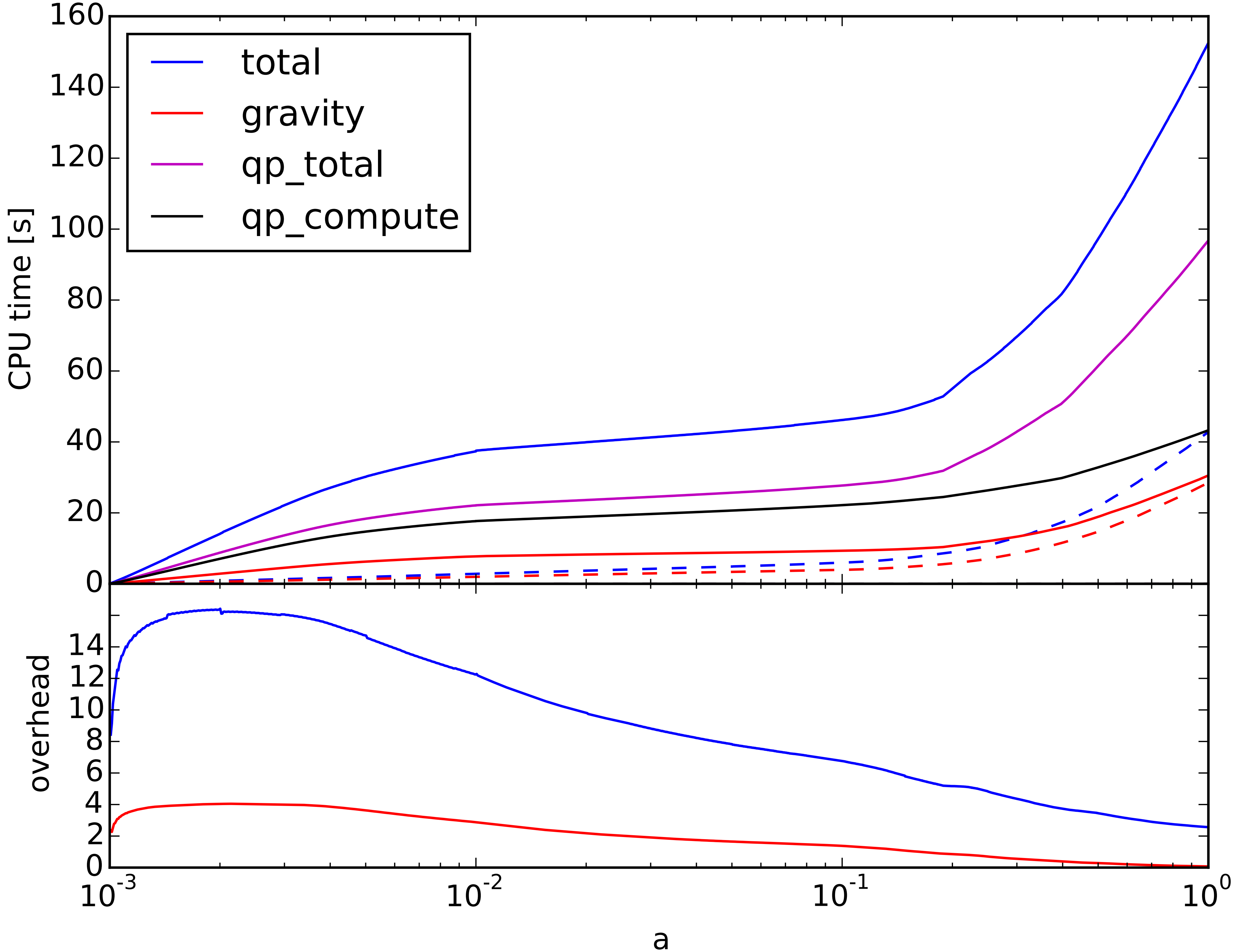
C-GADGET tested on 6656 cores @ MareNostrum (Barcelona)

MG-GADGET tested on 8192 cores @ SuperMuc (LRZ)

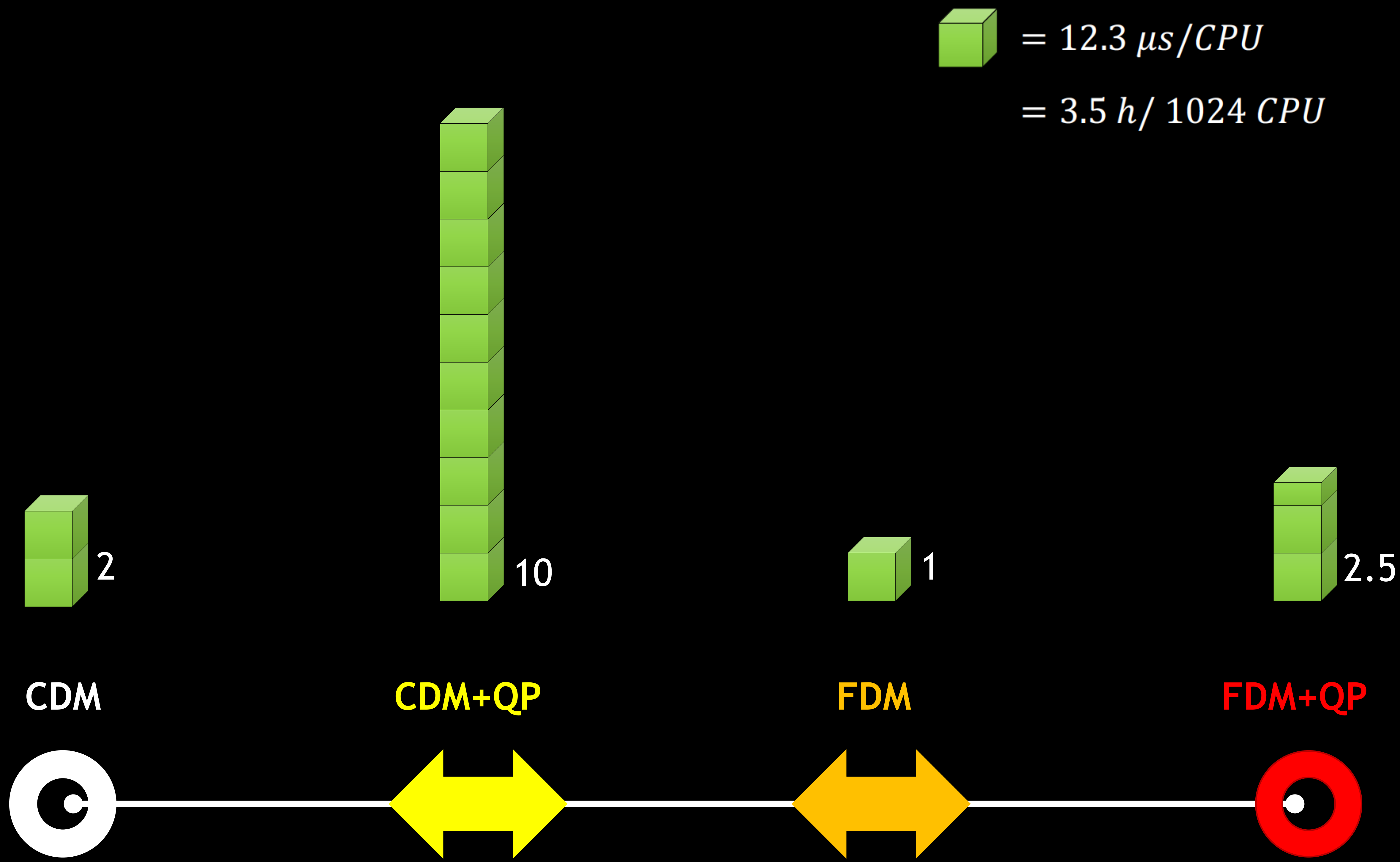
MEMORY REQS: C-GADGET VS MG-GADGET



AX-G PERFORMANCE: ALGORITHM OVERHEAD

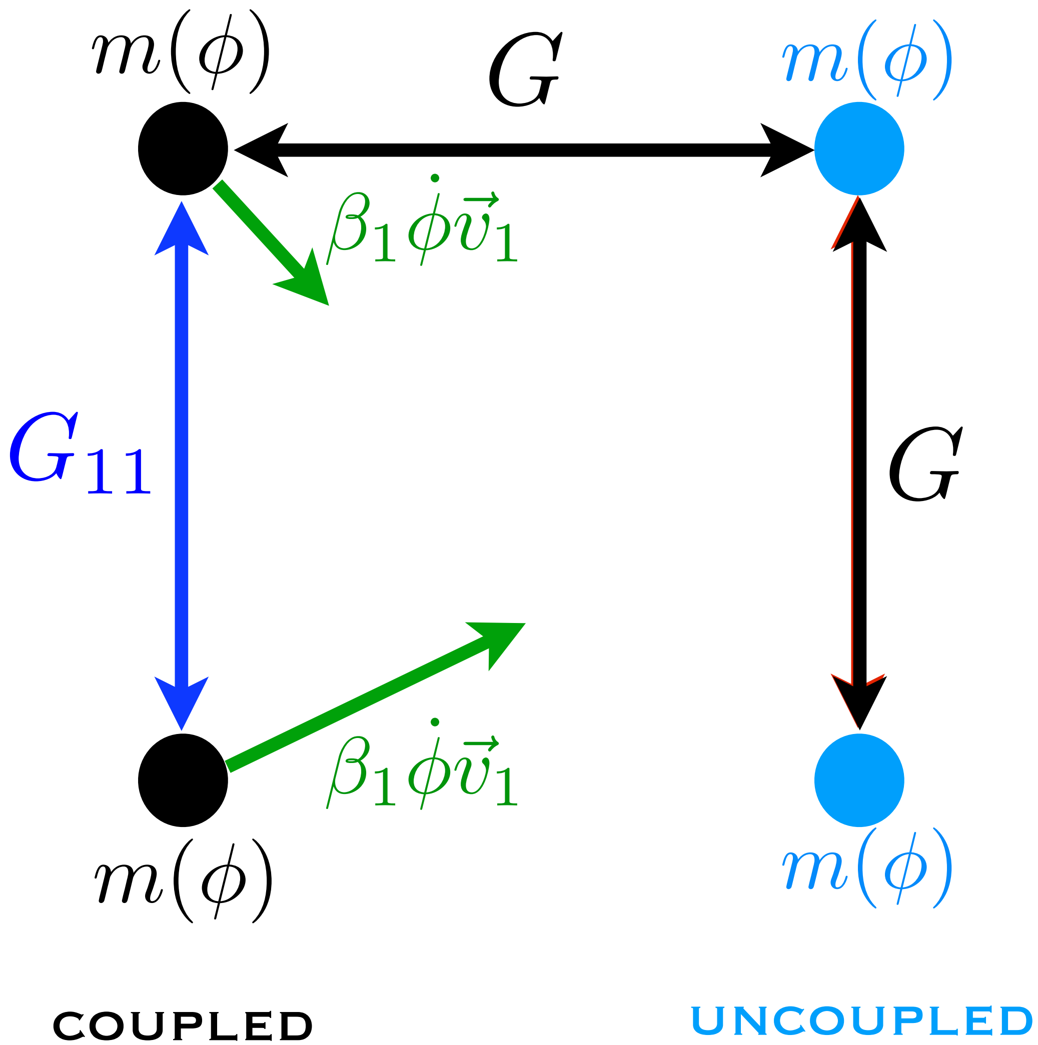


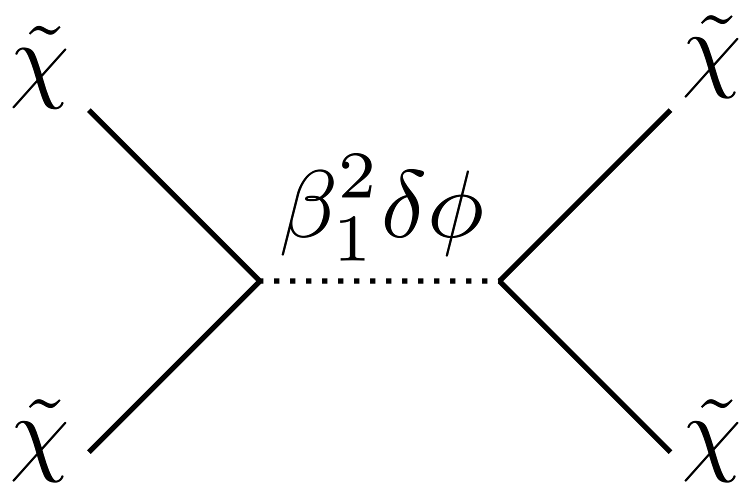
Performance

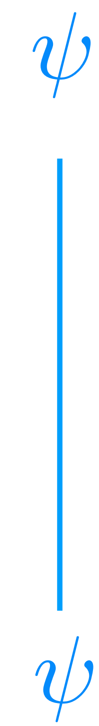


Results of non-standard cosmological simulations

Interacting Dark Energy







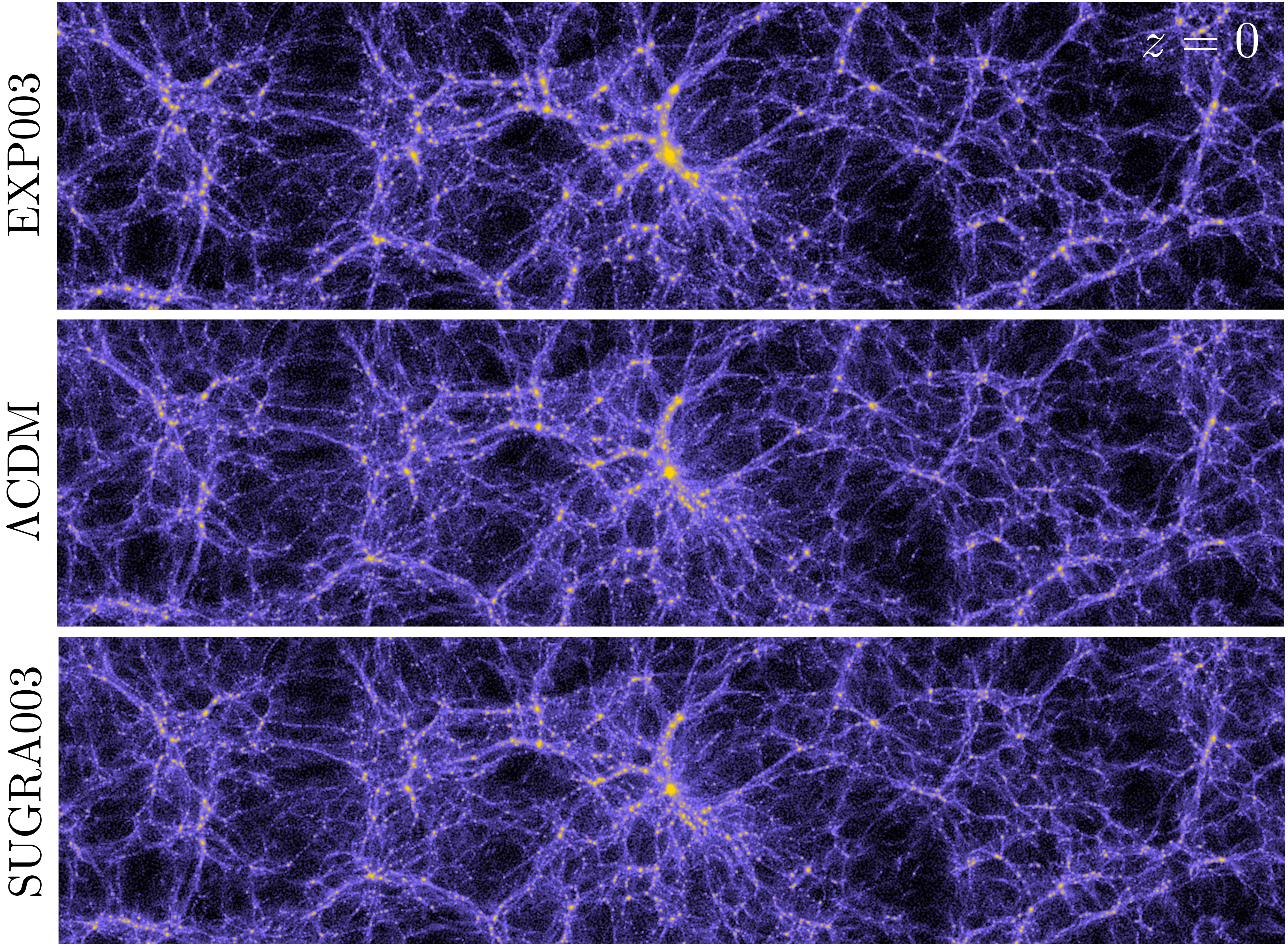
THE CoDECS PROJECT

A publicly available suite of cosmological N-body simulations for interacting Dark Energy models

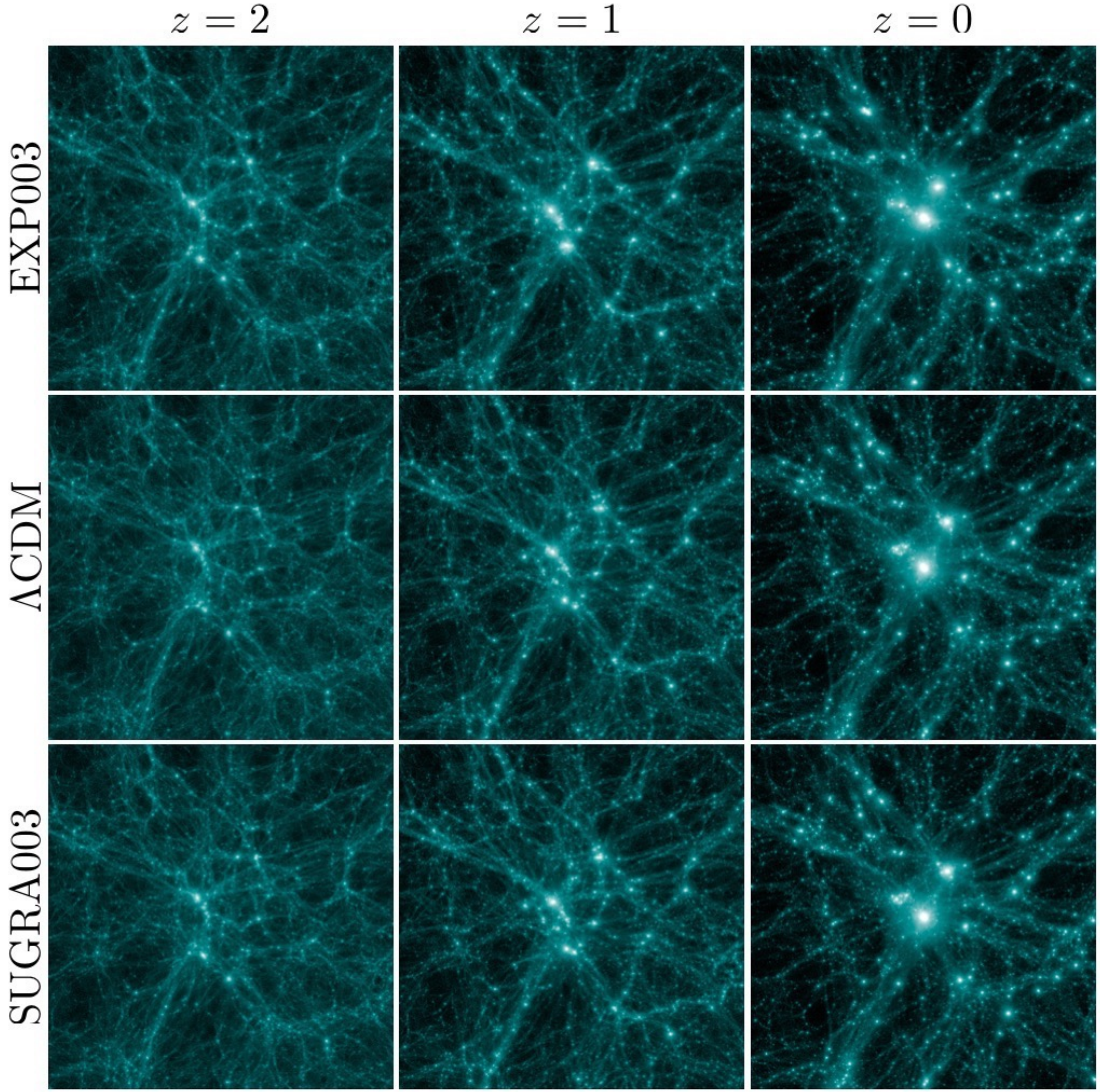
www.marcobaldi.it/CoDECS

MB, MNRAS 422 (2012), arXiv:1109.5695

Comparing LSS and halo properties in CoDECS



COMPARING LSS AND HALO PROPERTIES IN CoDECS

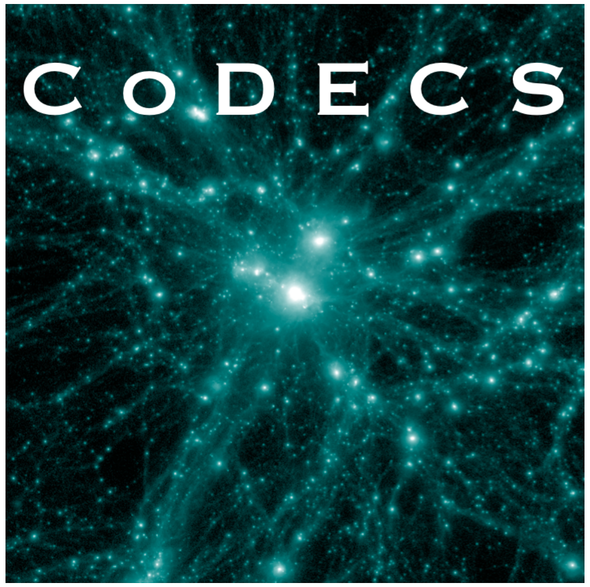


CoDECS results

CoDECS

Coupled Dark Energy Cosmological Simulations

(data publicly available at www.marcobaldi.it/CoDECS)



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Abundance of high z massive clusters
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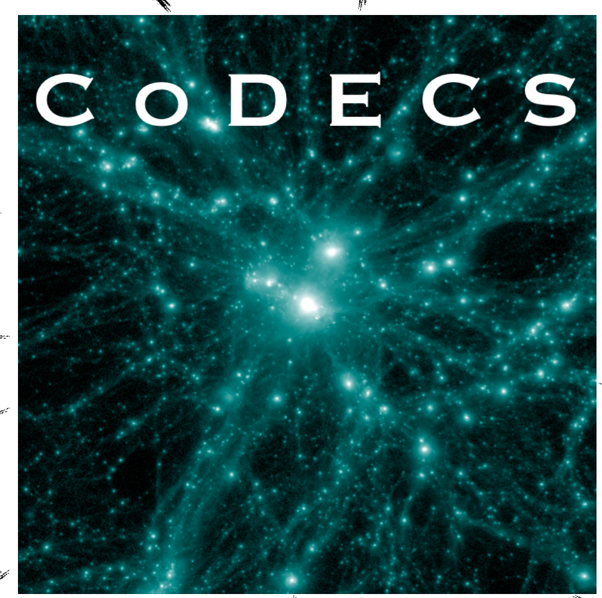

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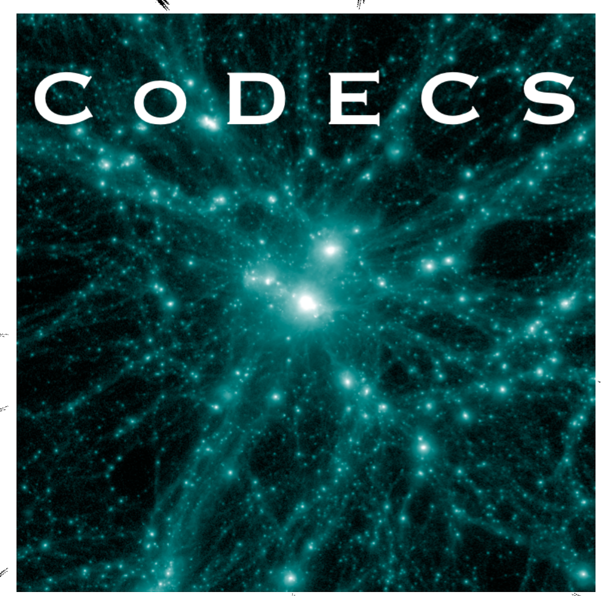

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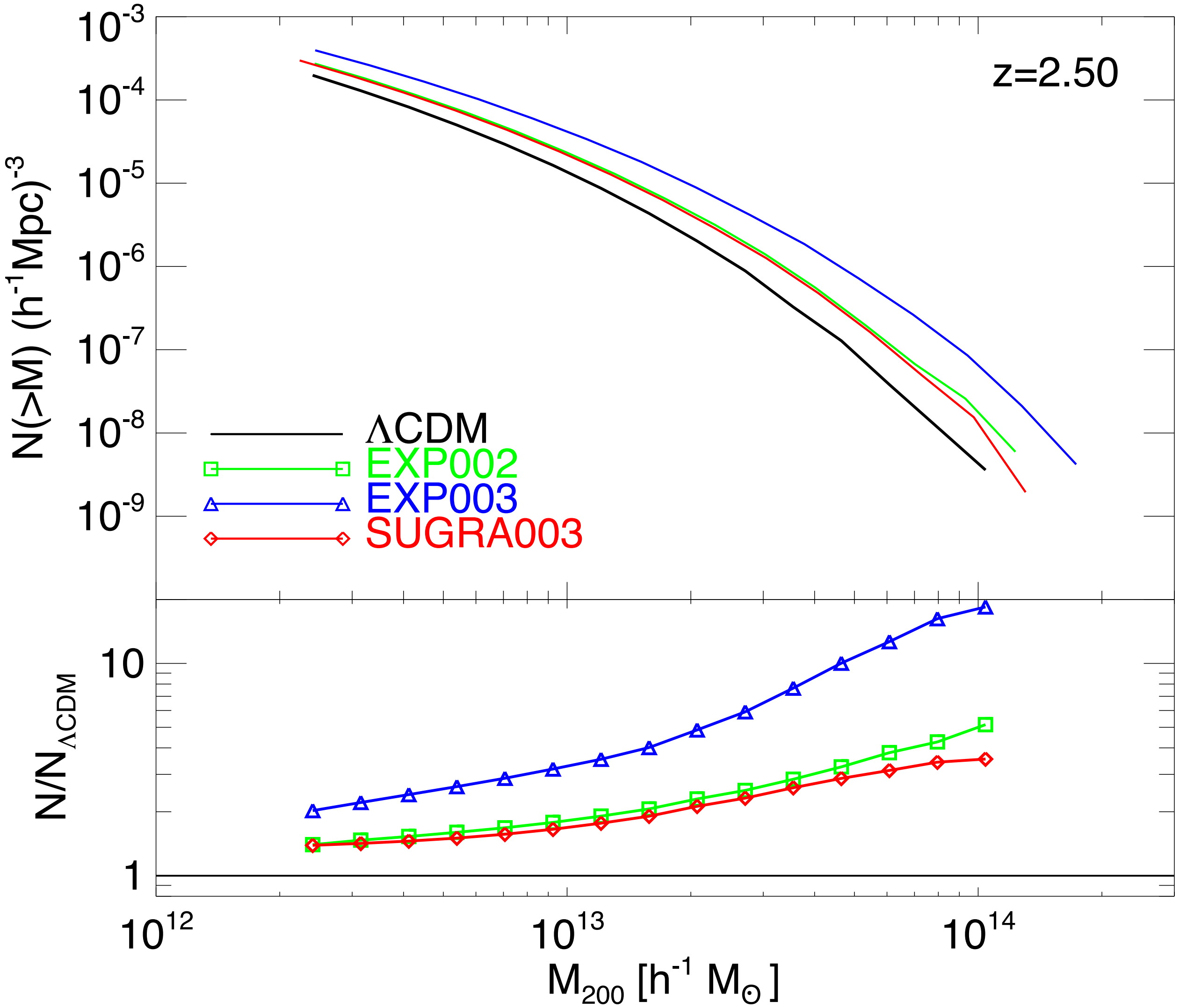

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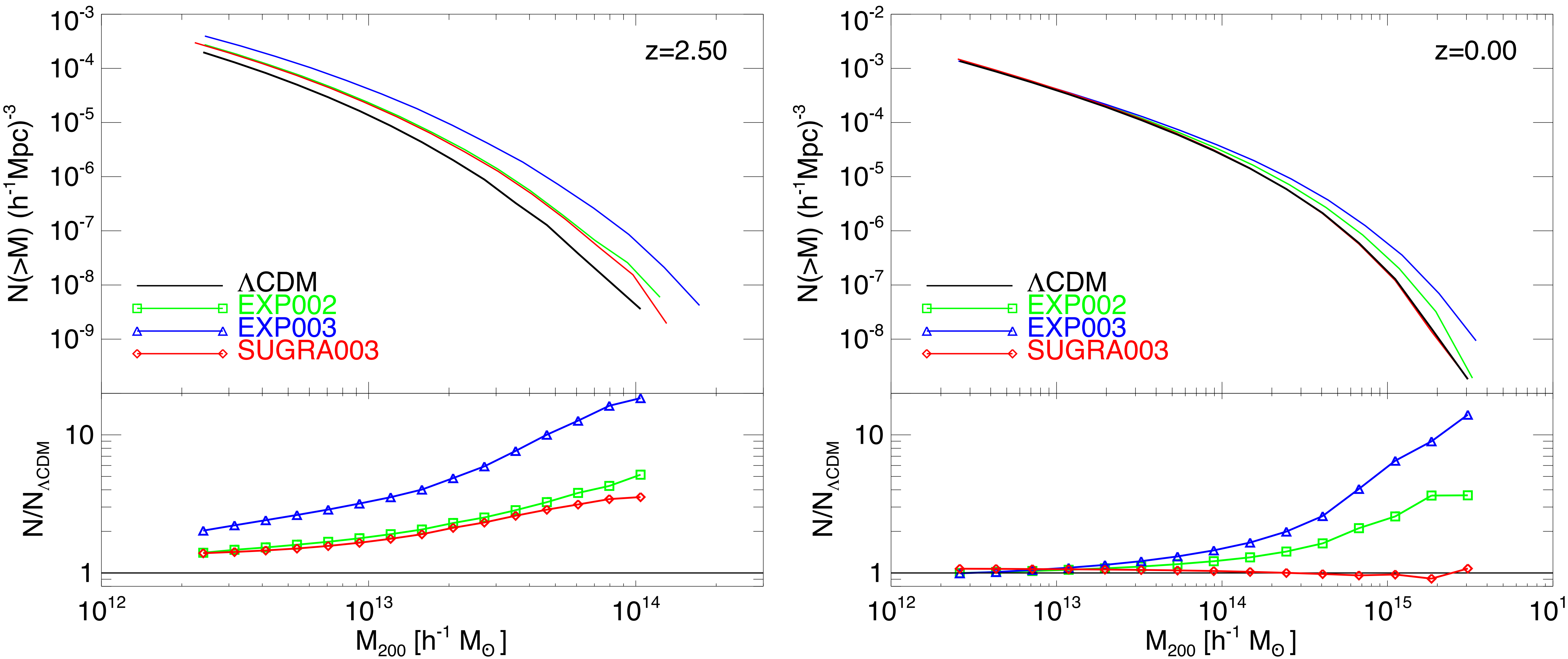

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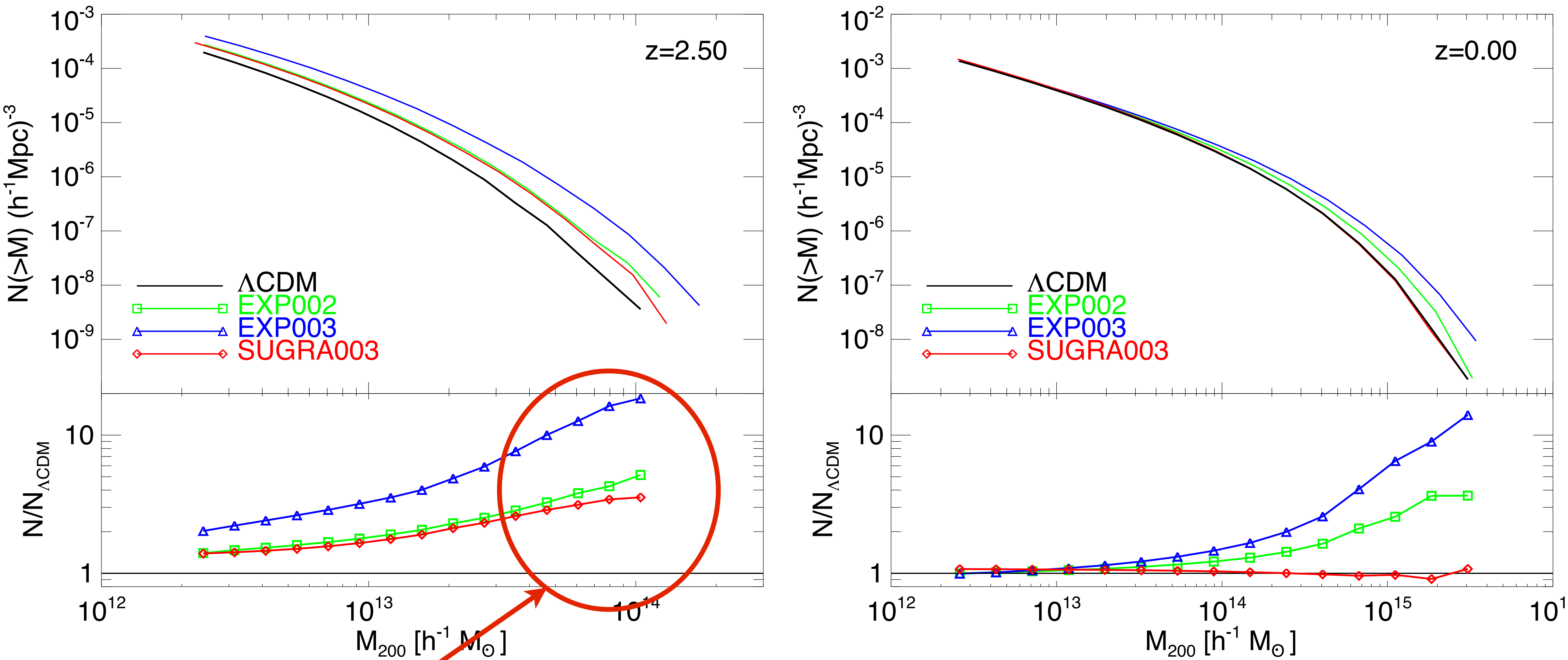

More massive clusters at high z

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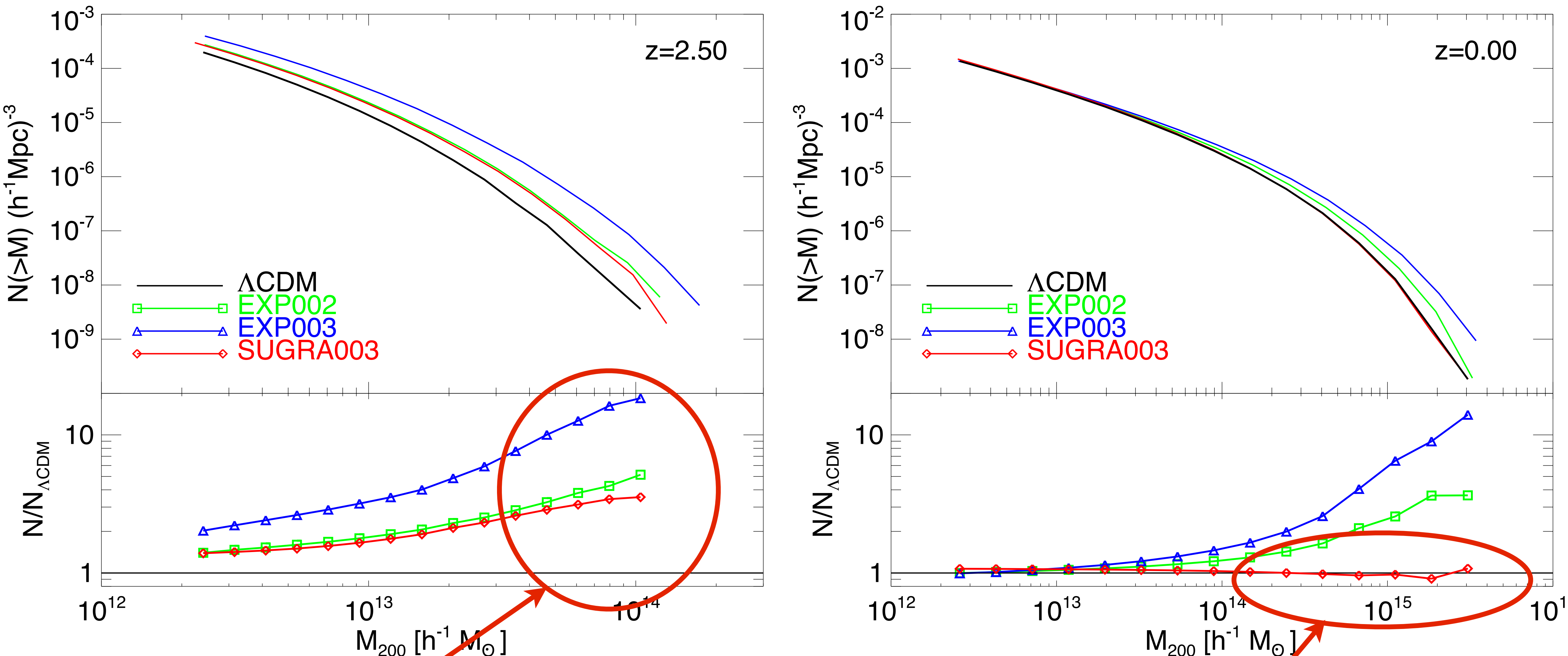

More massive clusters at high z

Same number of clusters as ΛCDM at z=0

THE CoDECS PROJECT: RESULTS SO FAR...

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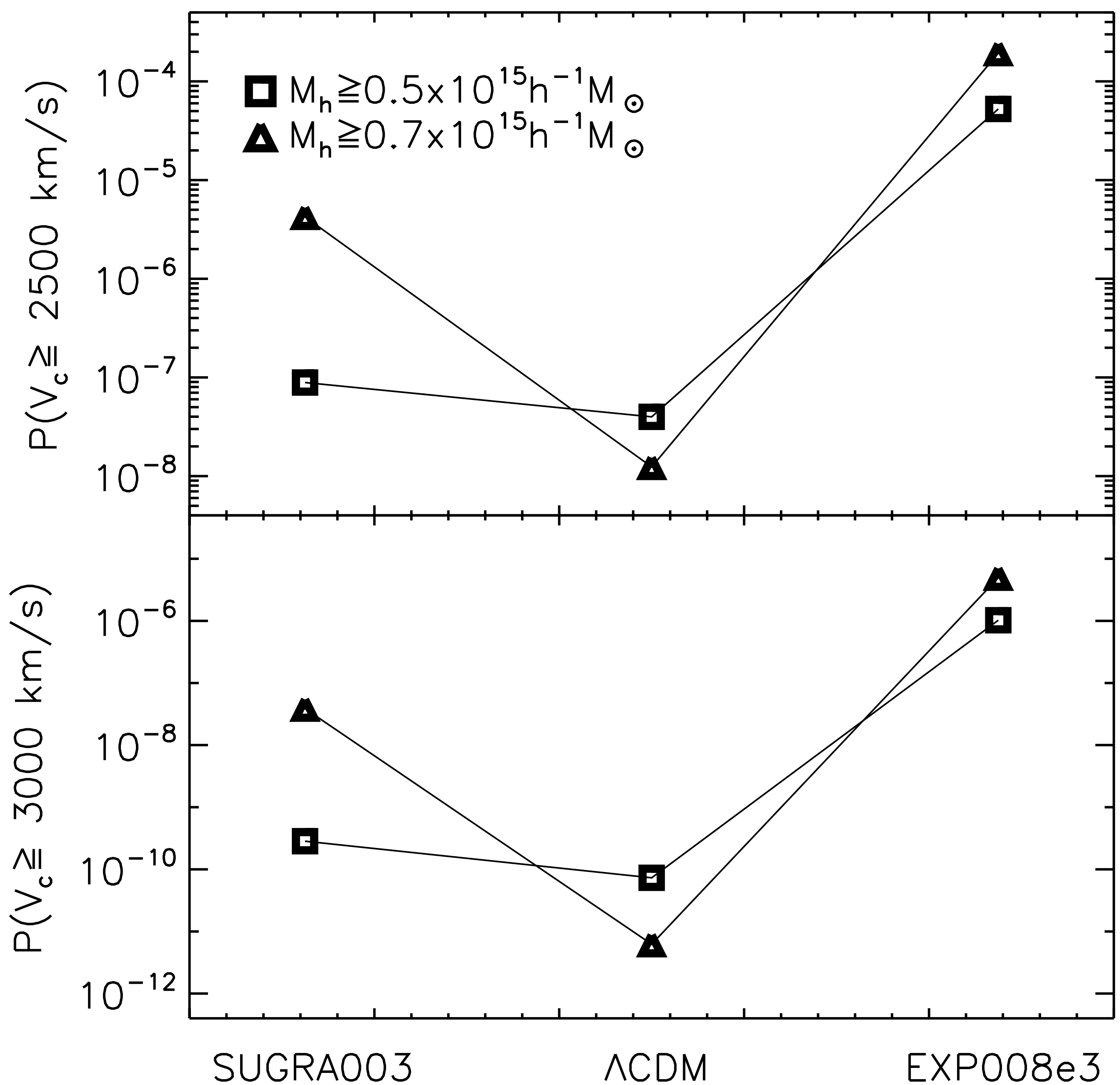
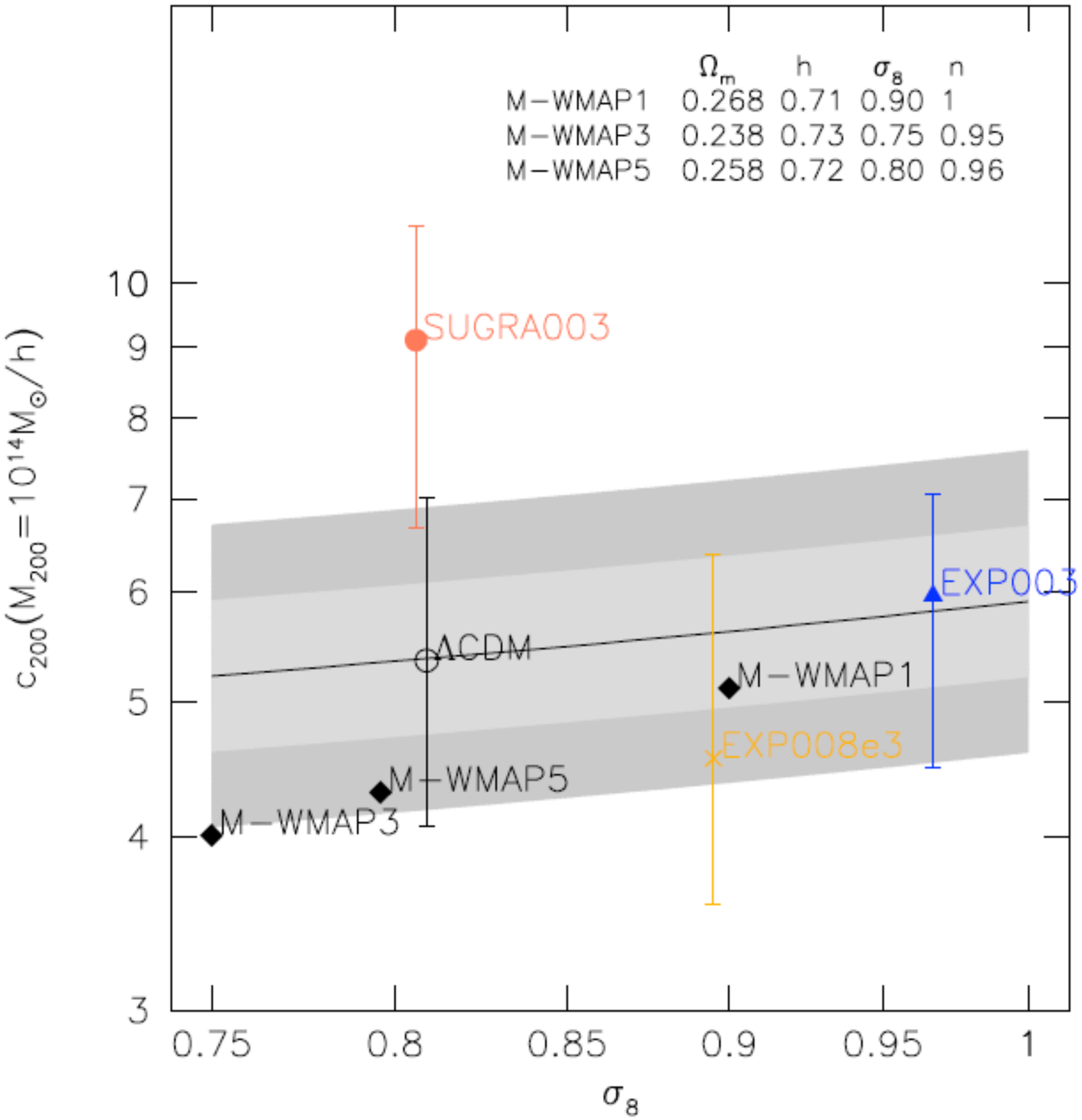


HALO STRUCTURAL PROPERTIES IN cDE

Breaking the c-σ_8 degeneracy for some specific cDE realizations



GIOCOLI, MB, ET AL. 2013

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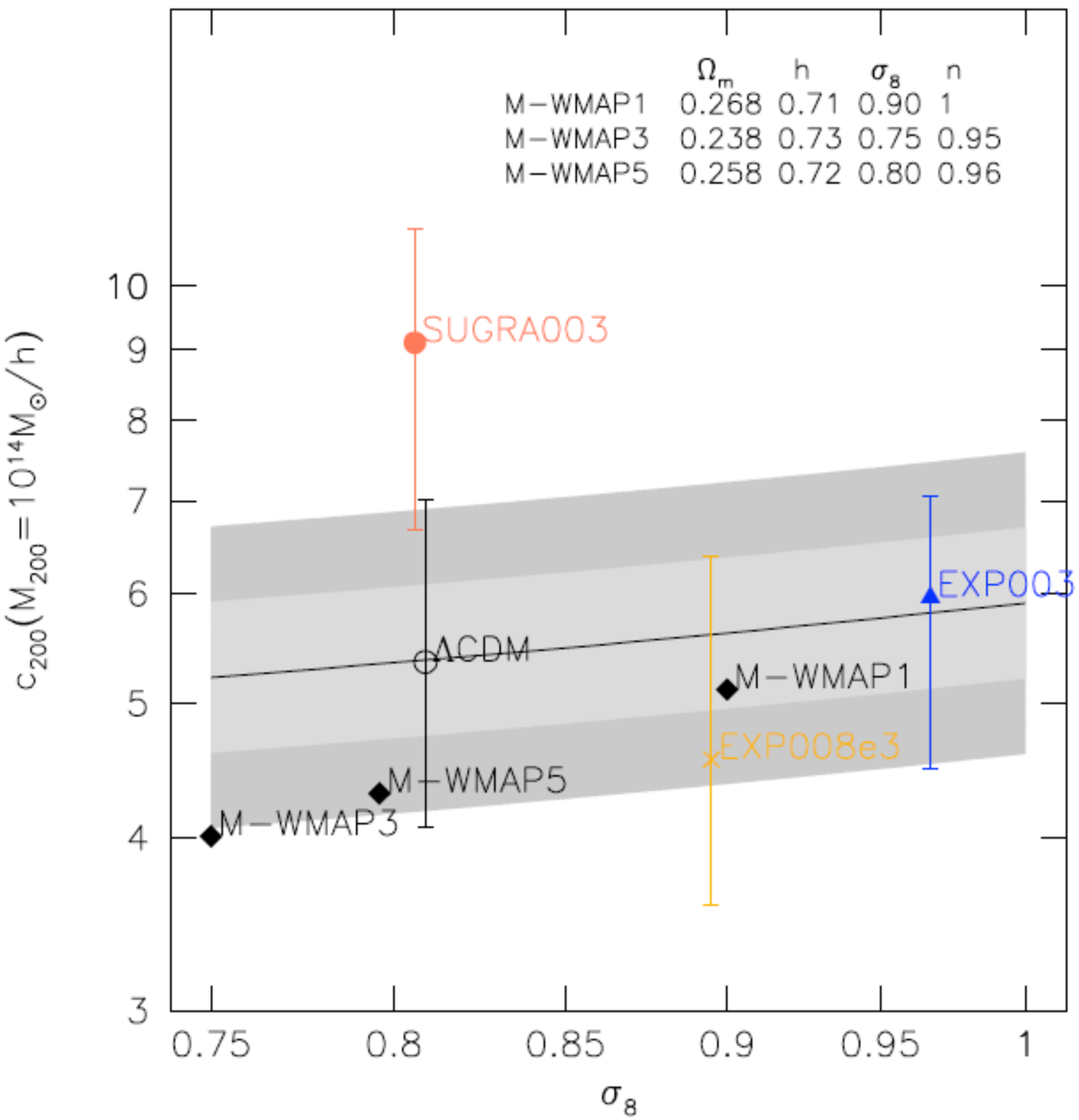
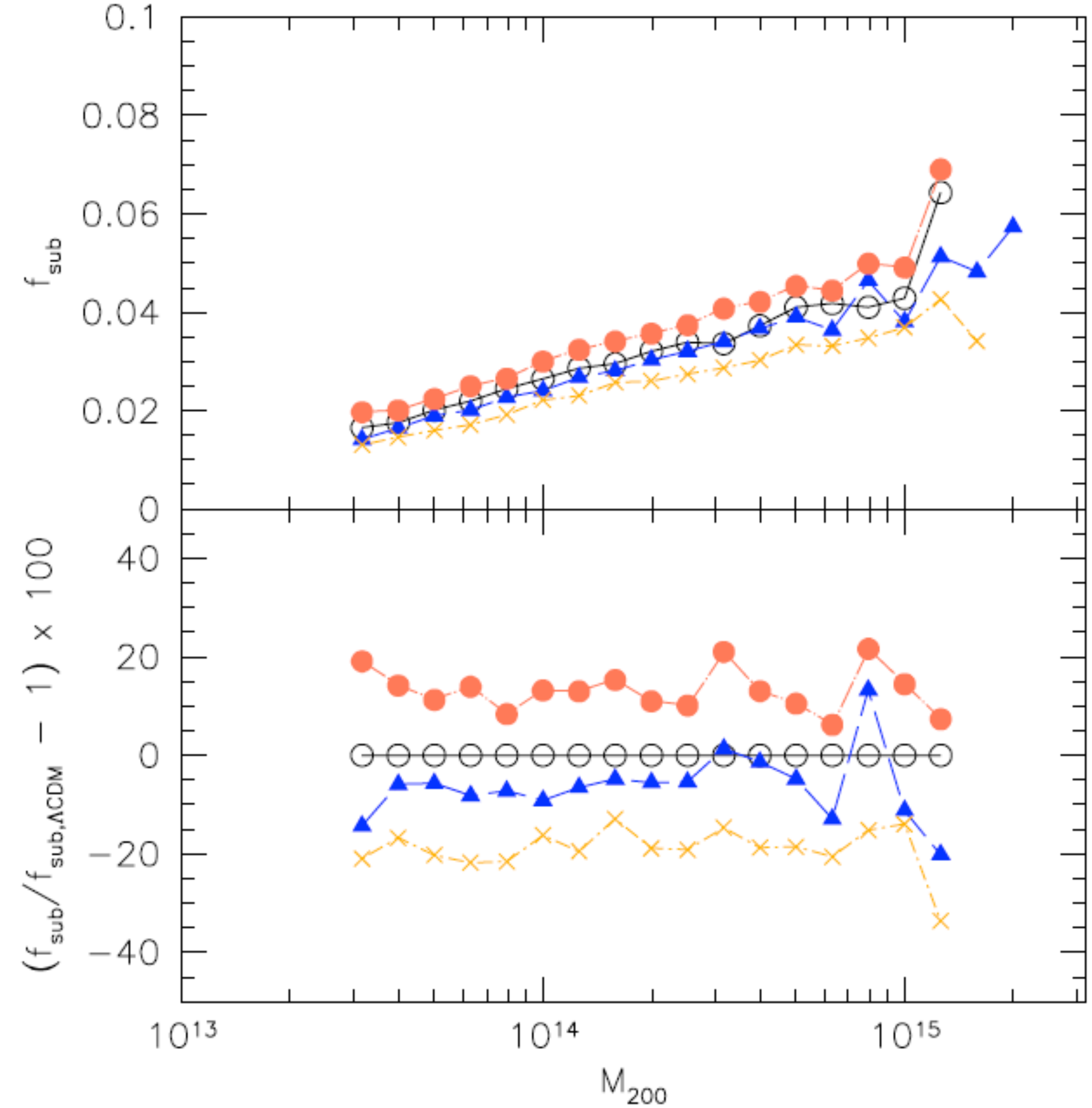
Breaking the c-σ_8 degeneracy for some specific cDE realizations

Substructure abundance discriminates among different cDE models





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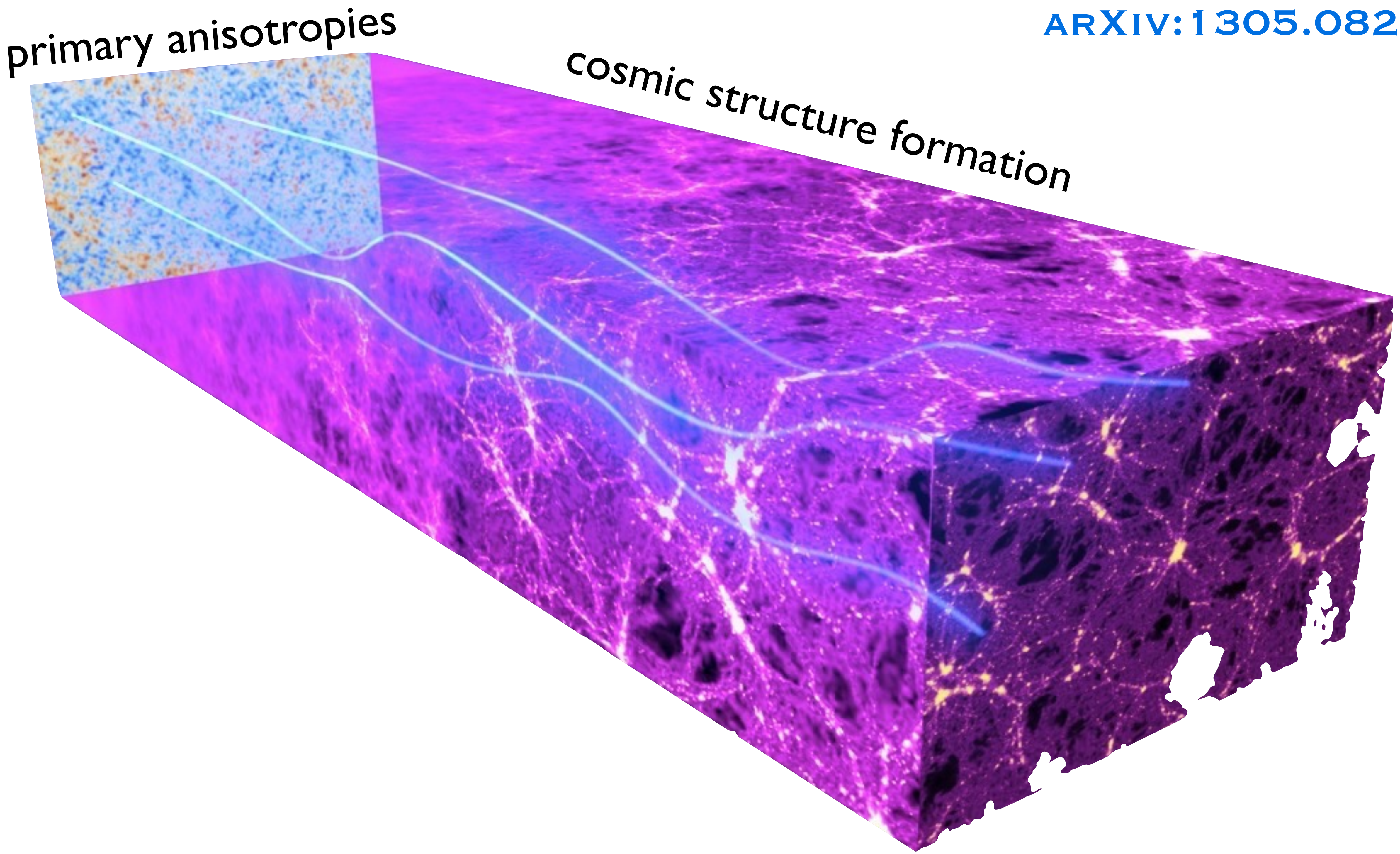
CMB lensing in interacting dark energy

Carbone et al.,2013, arXiv:1305.0829



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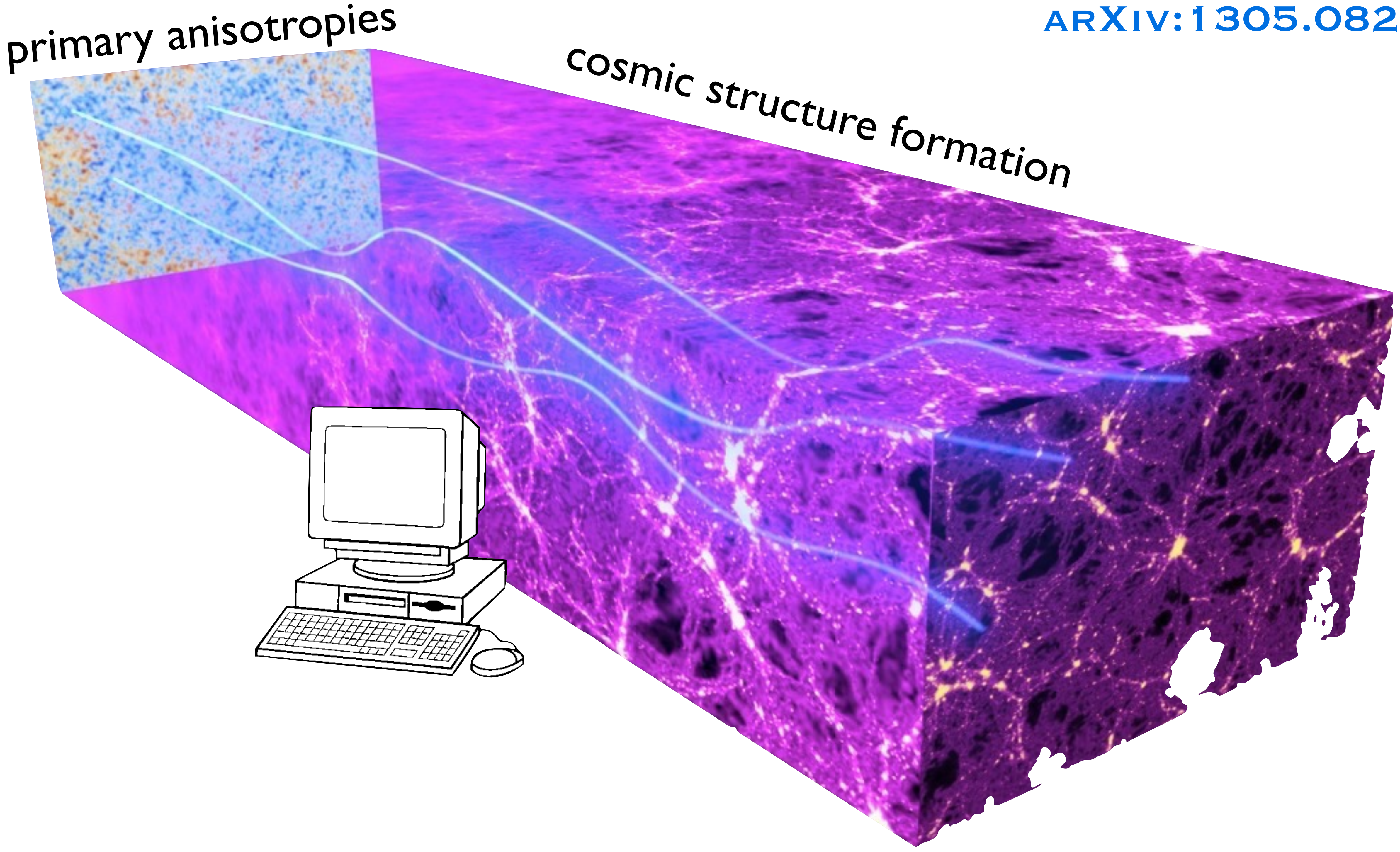

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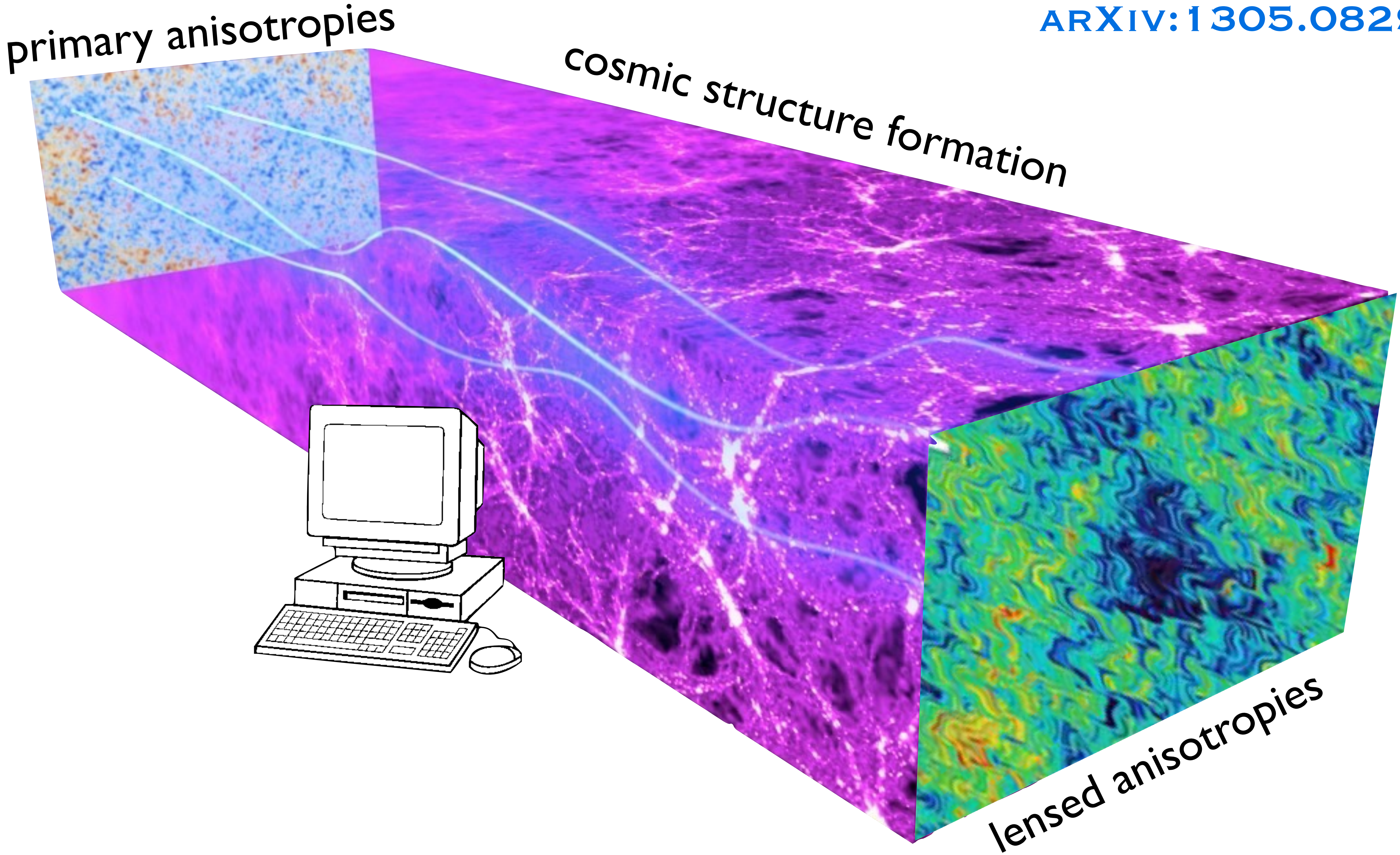

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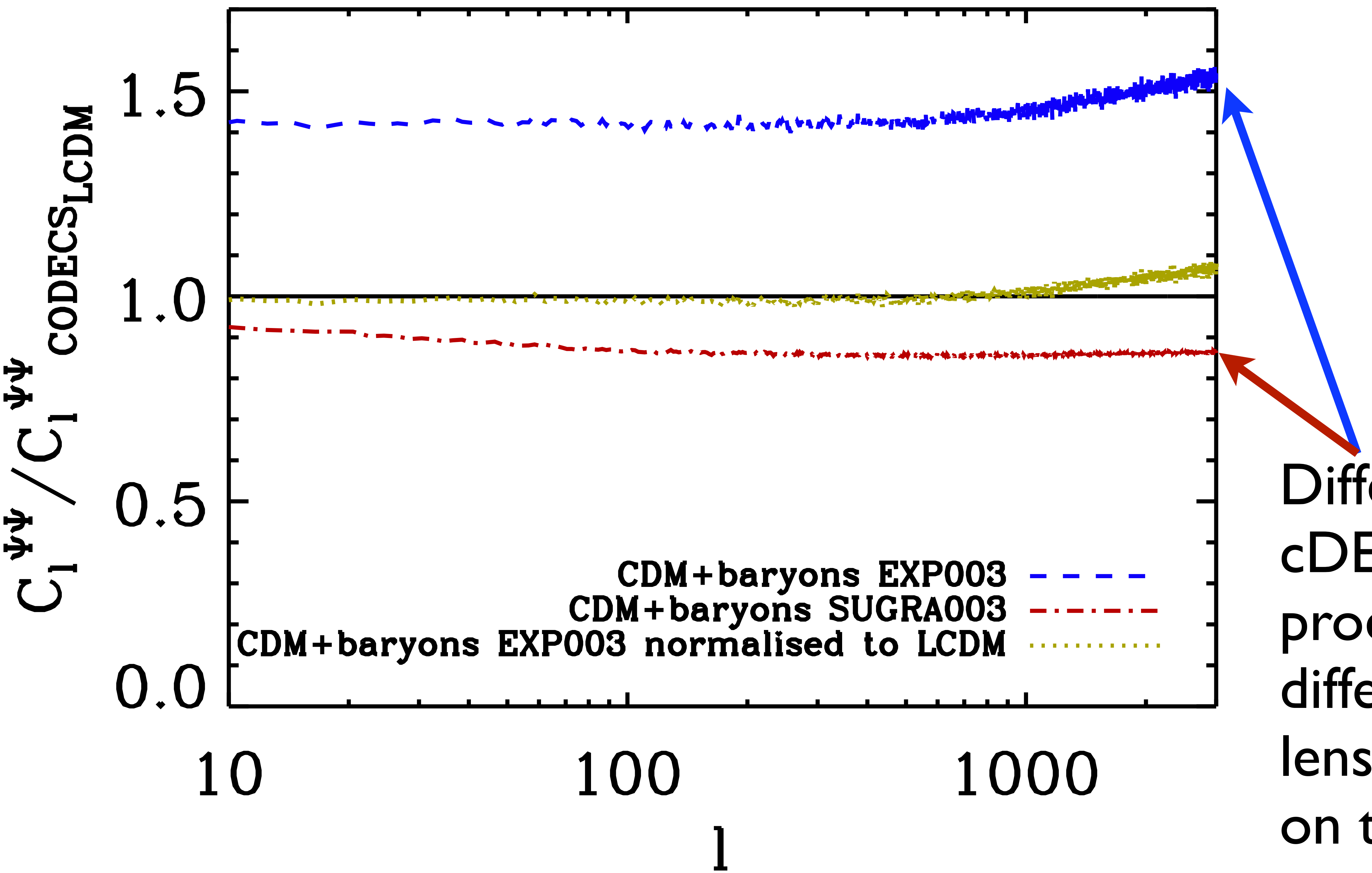
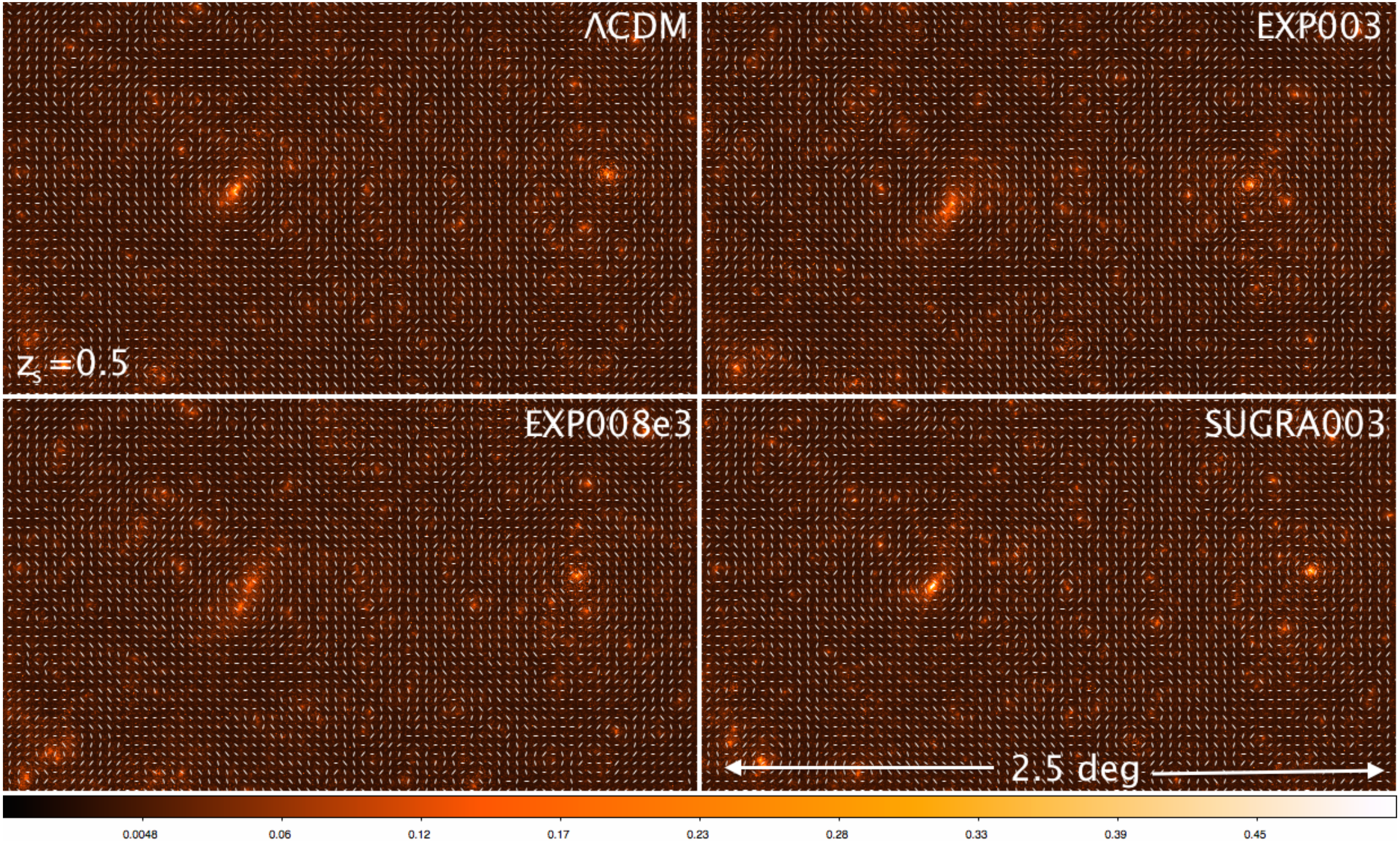


Different cDE models produce a different lensing signal on the CMB

Carbone, MB, et al., 2013

WEAK LENSING TOMOGRAPHY IN cDE

Ray-tracing with different source redshifts (Giocoli, MB, et al. 2015)



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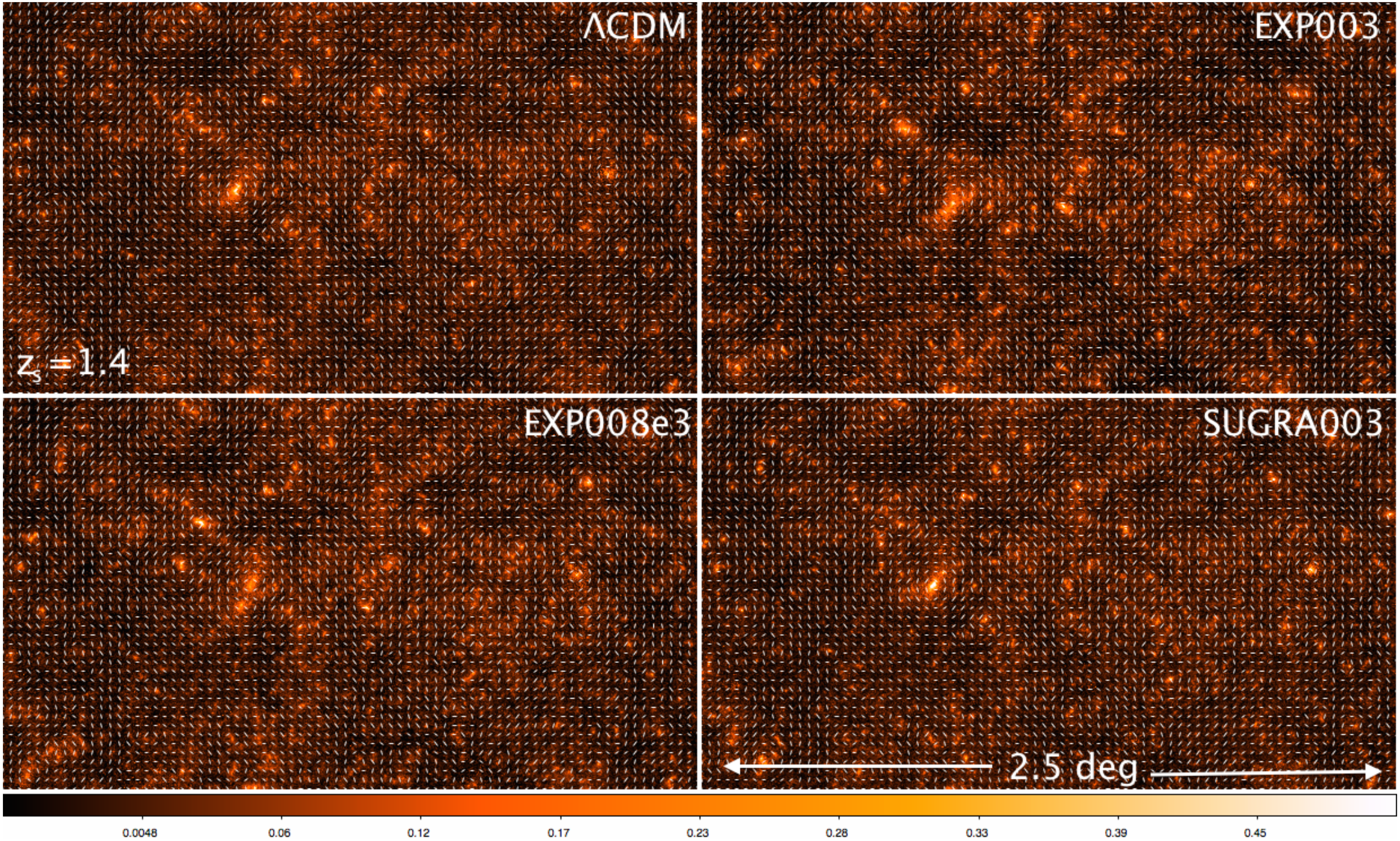

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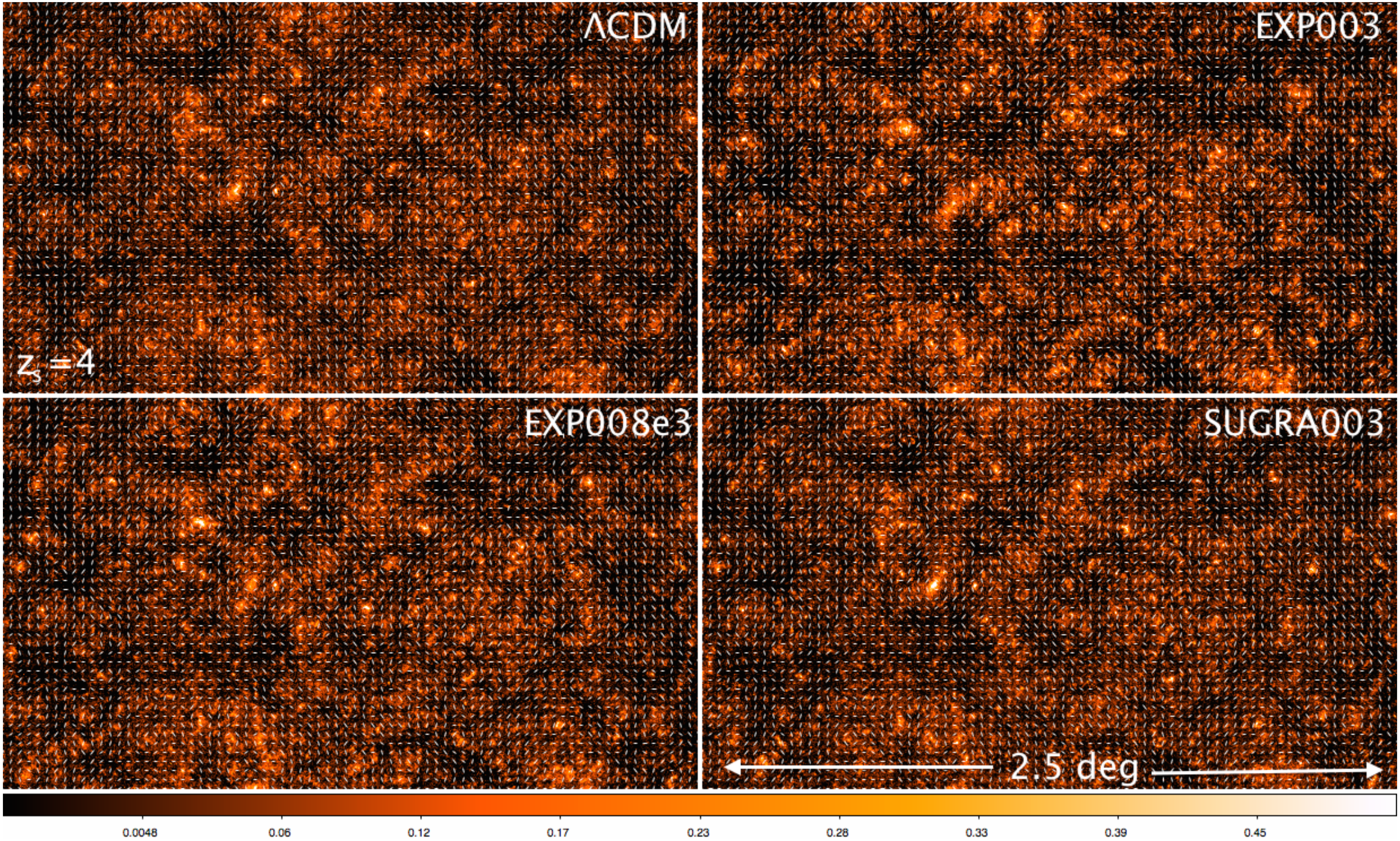

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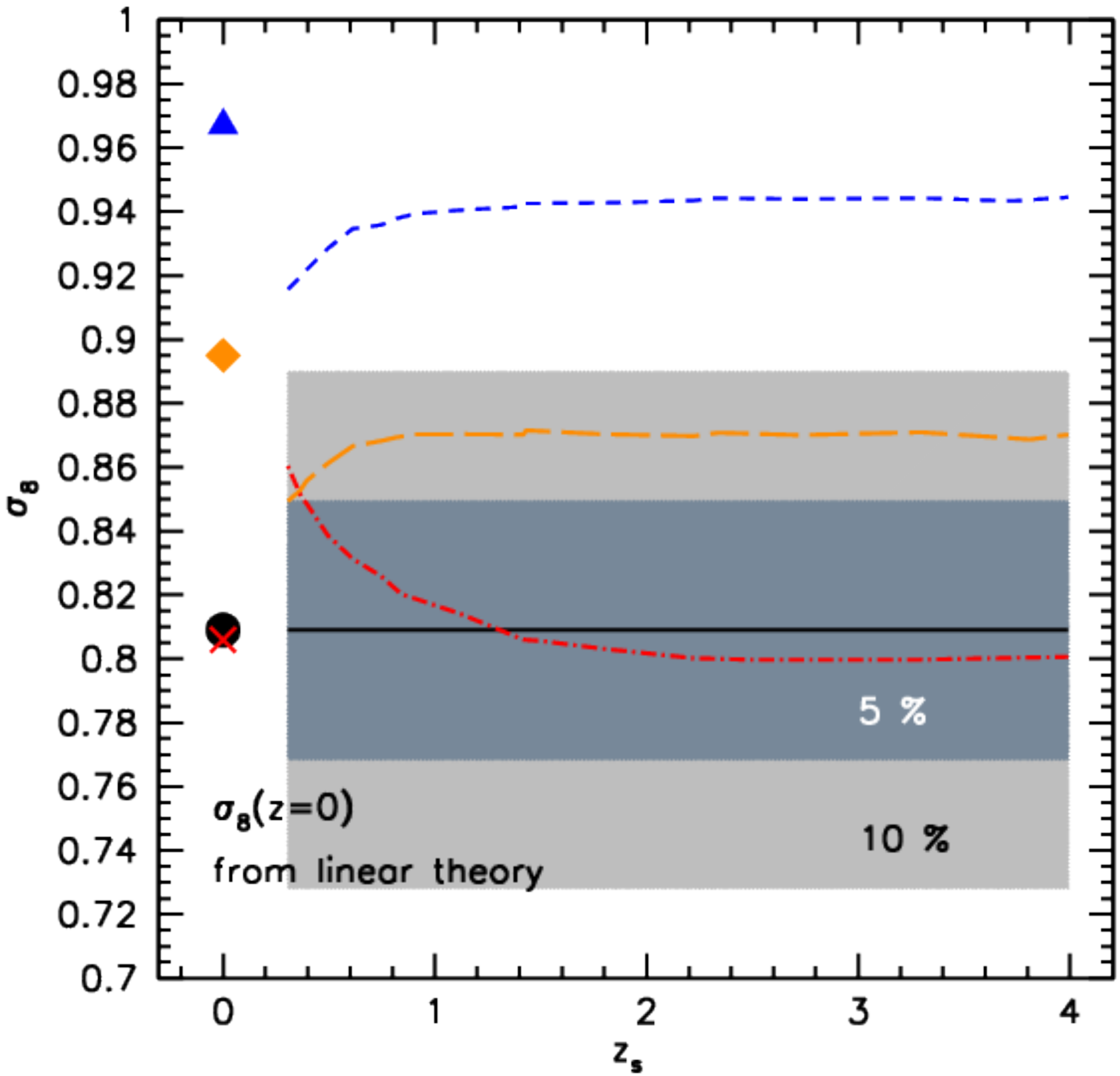
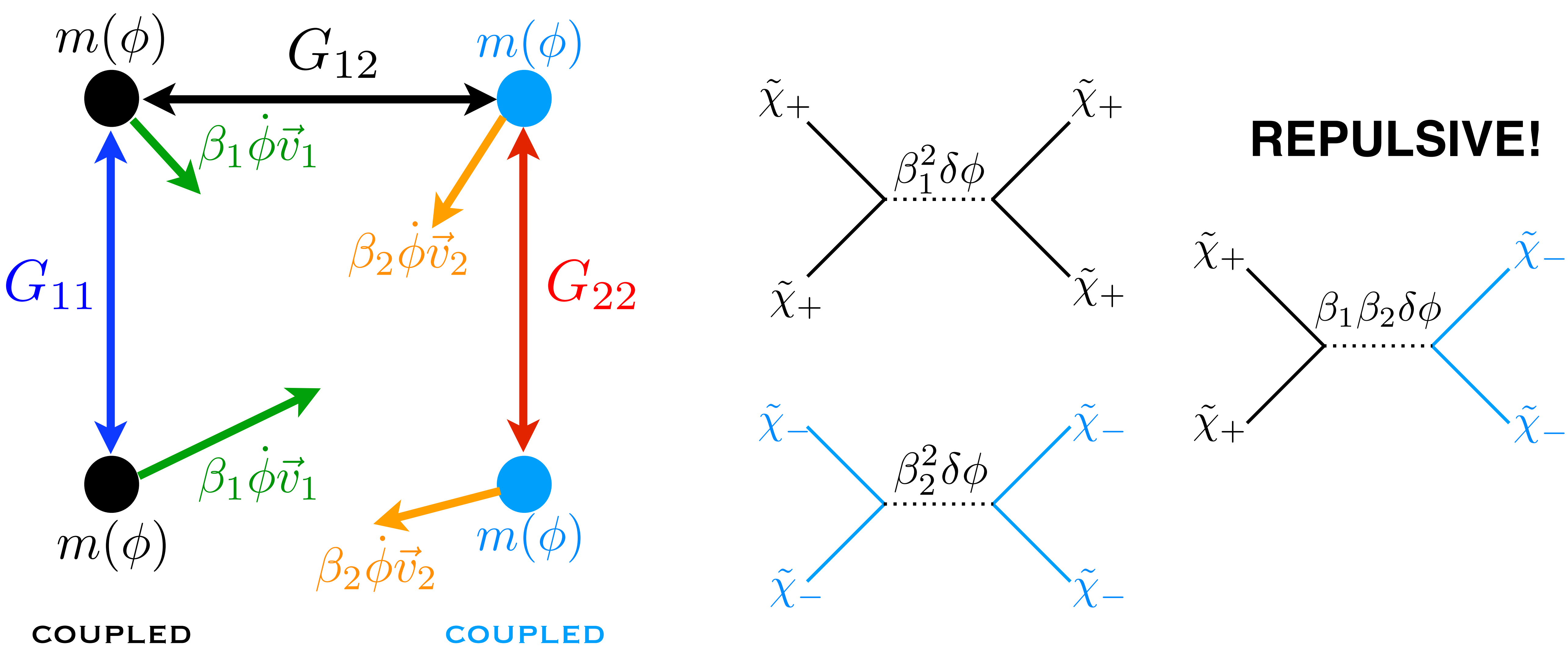


Use WL to constrain σ_8 assuming a ΛCDM cosmology:

if the best-fit value depends on the source redshift, then something is wrong in the assumed cosmology

Multi-coupled Dark Energy



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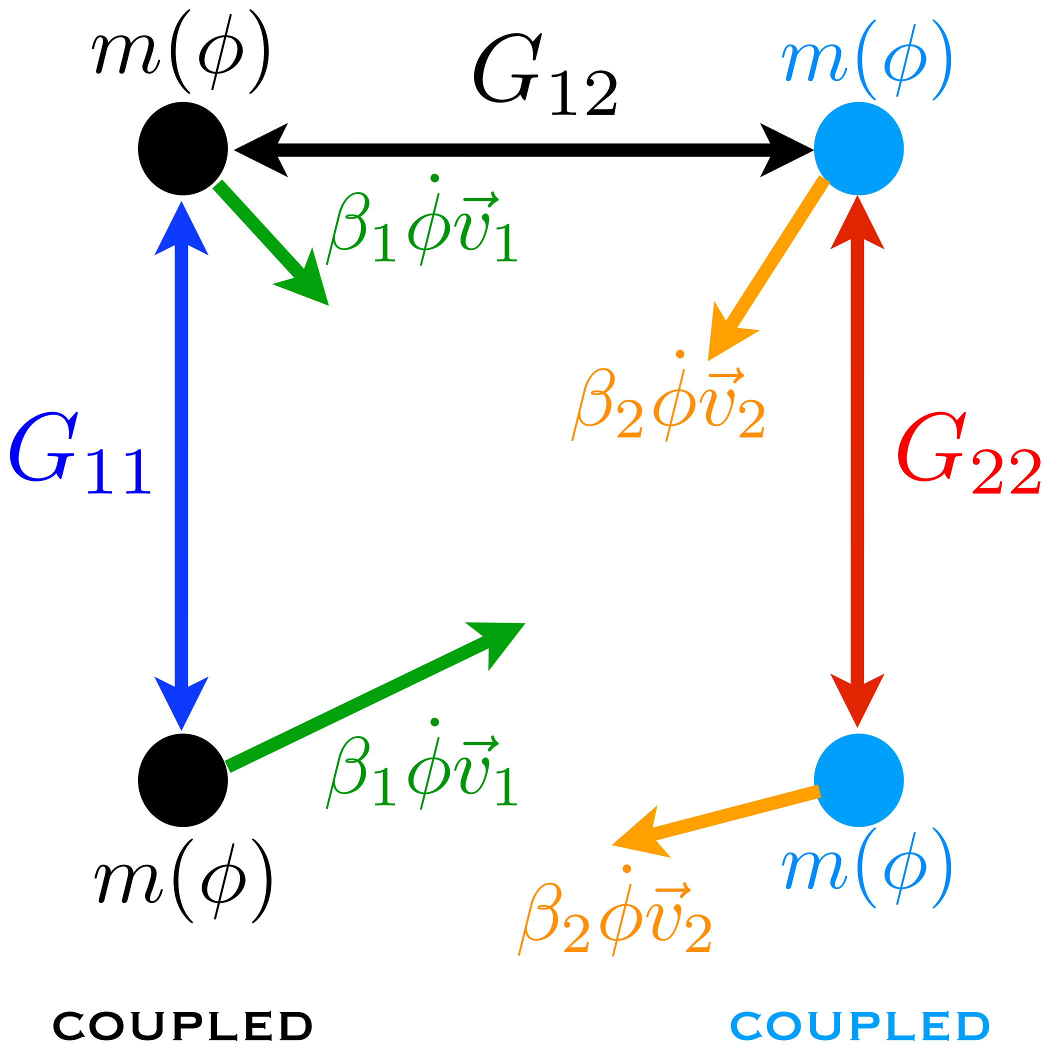
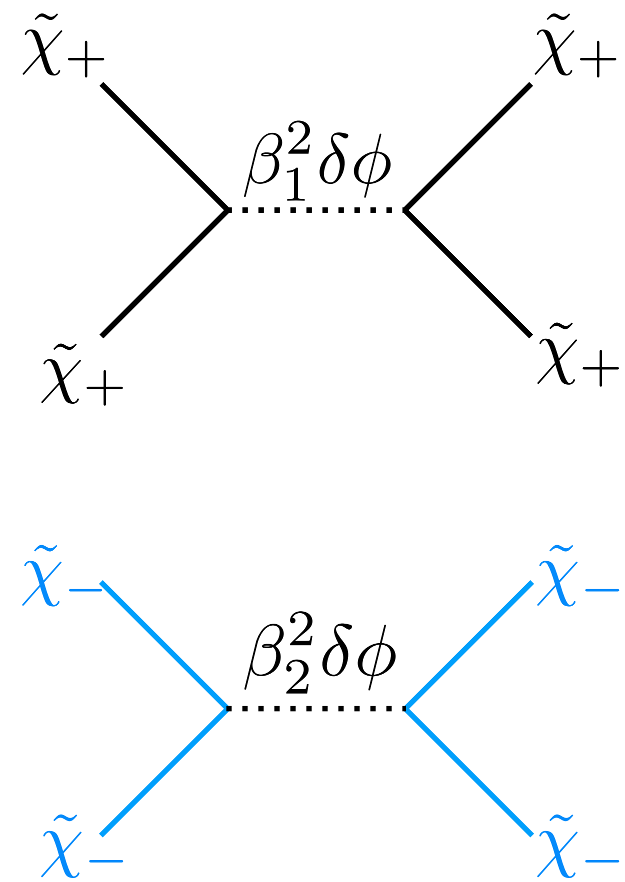
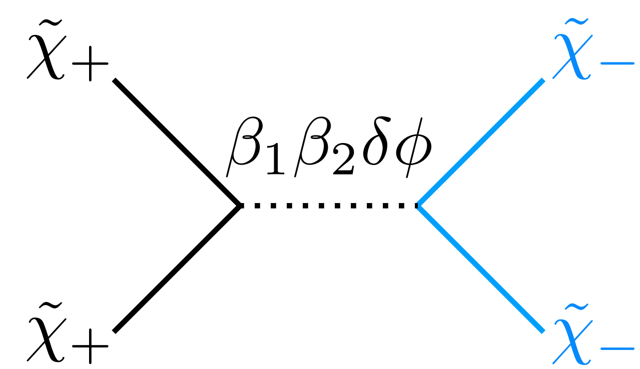




REPULSIVE!



STRUCTURE FORMATION IN MULTI-COUPLED DE

First low-res. simulations of multi-coupled Dark Energy [MB, MNRAS 428 2013]

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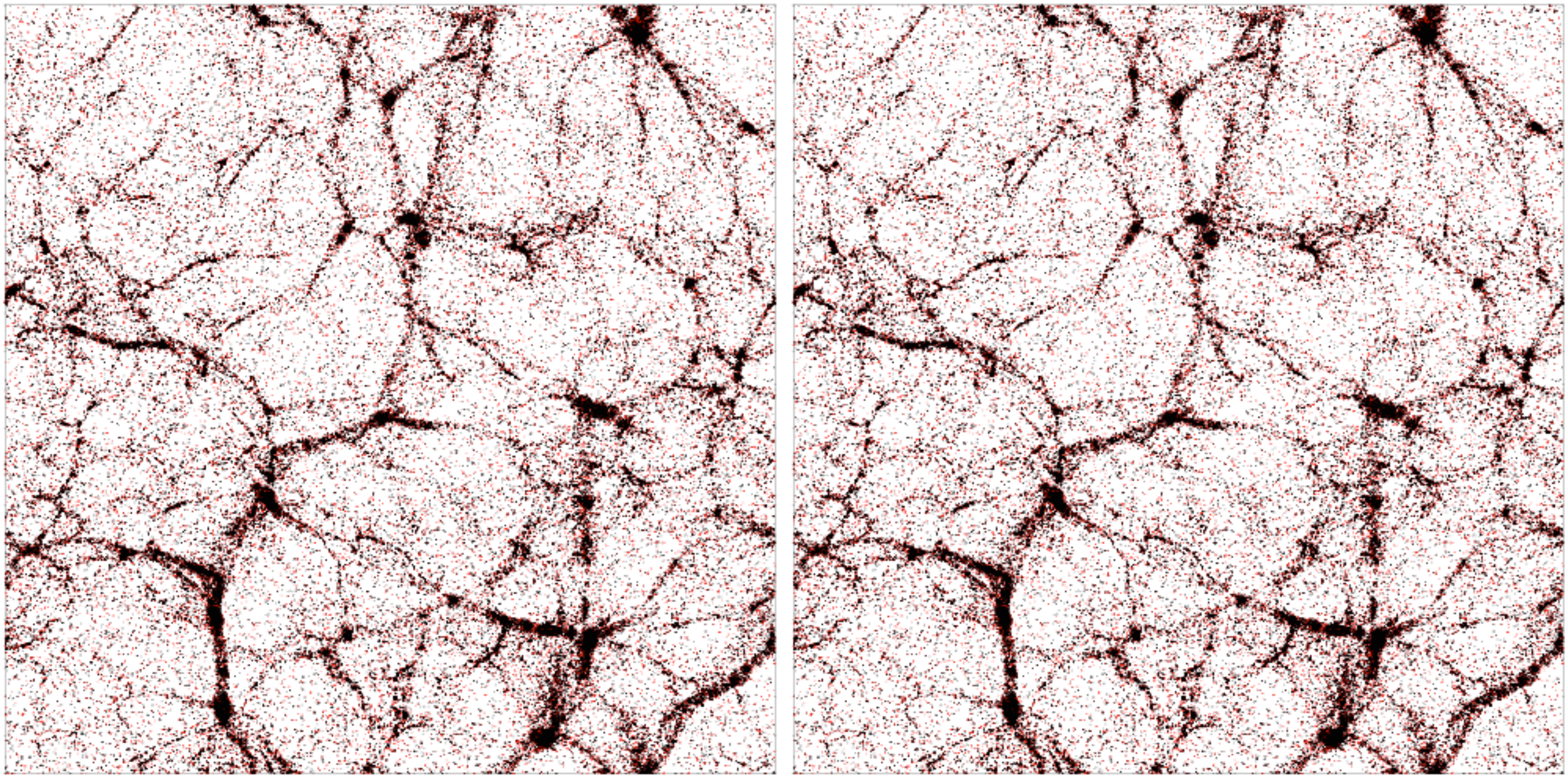

$\beta = 0$ $\beta = 1/2$

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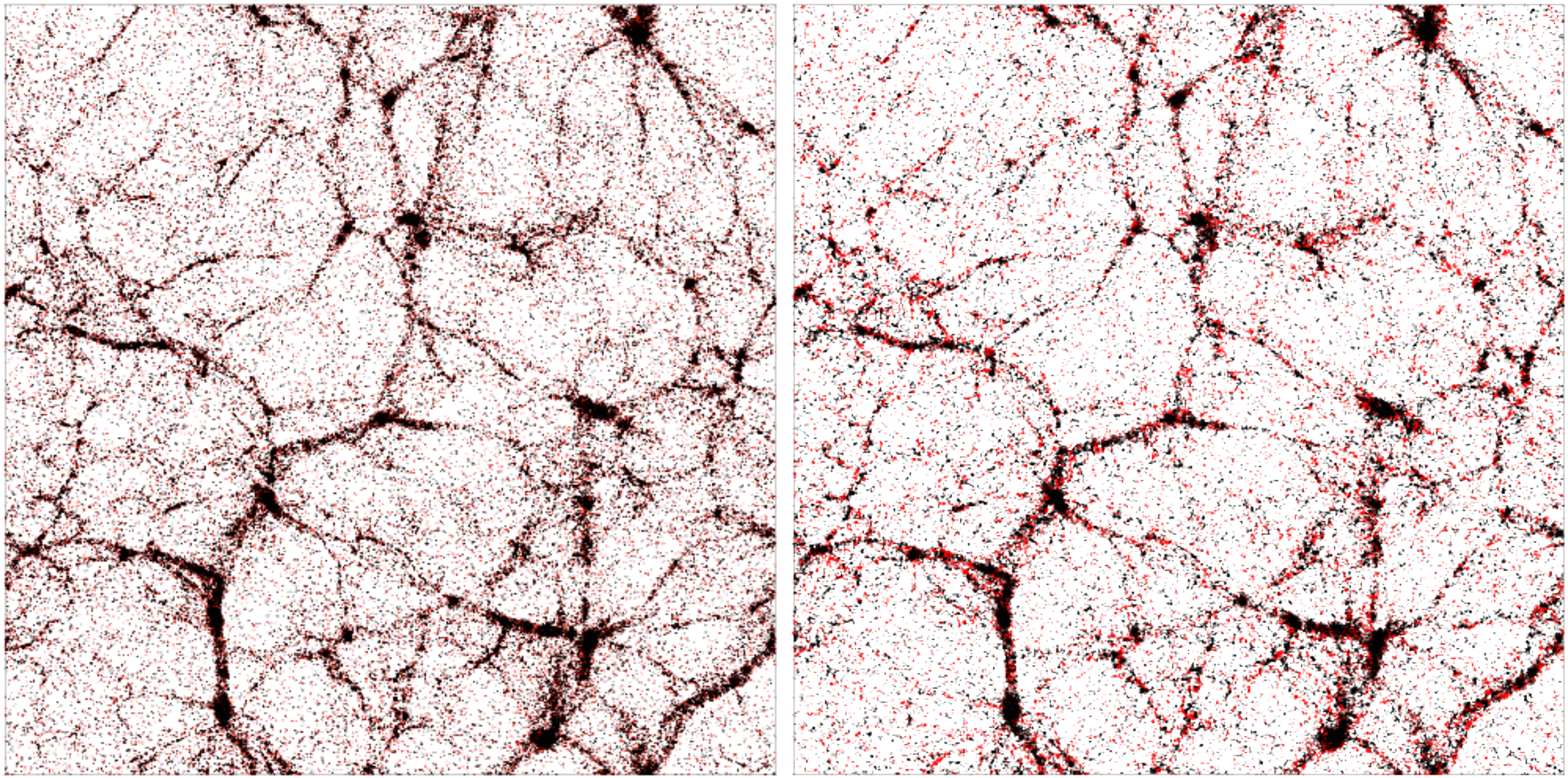

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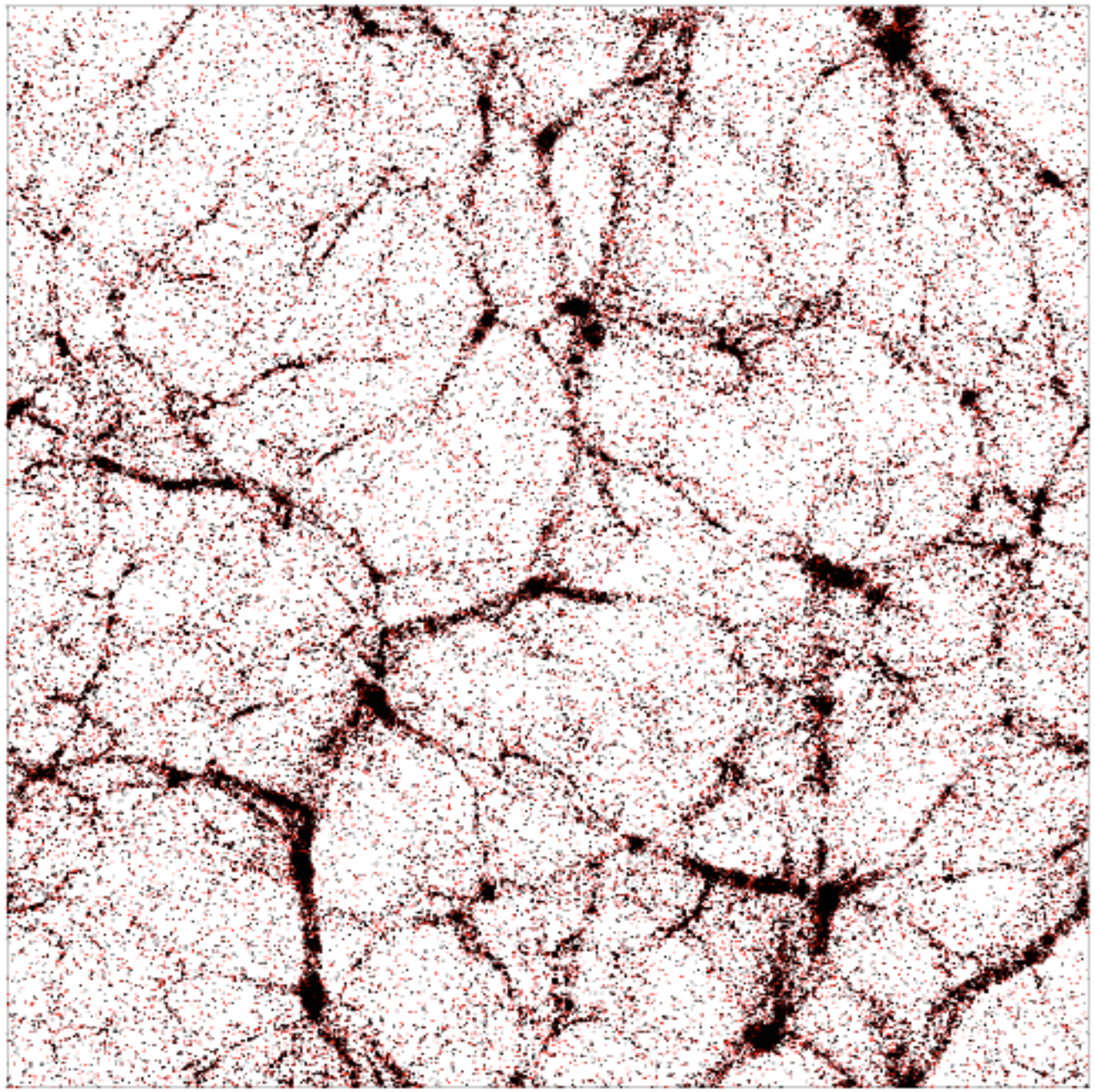

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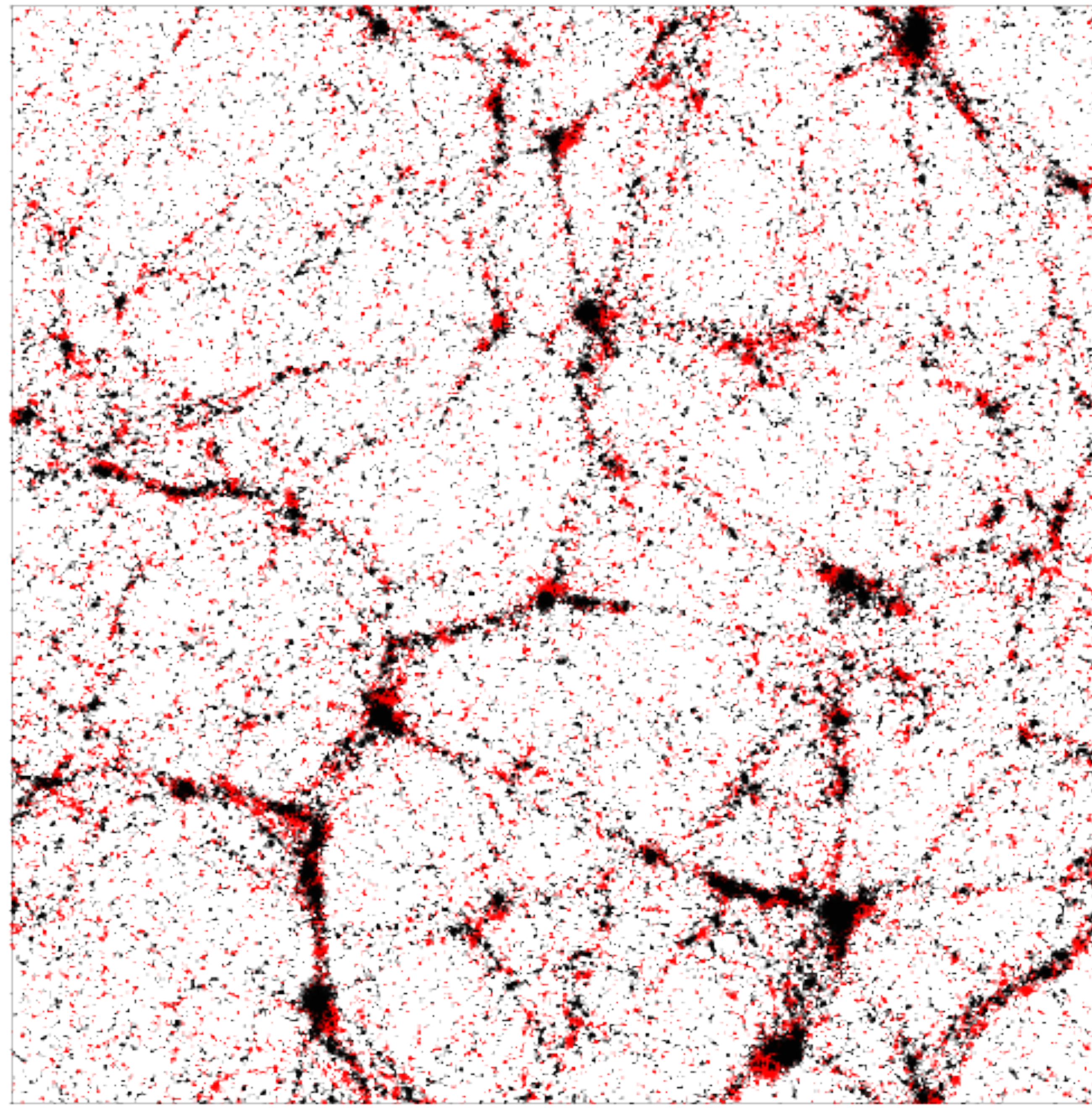

$\beta = 1$

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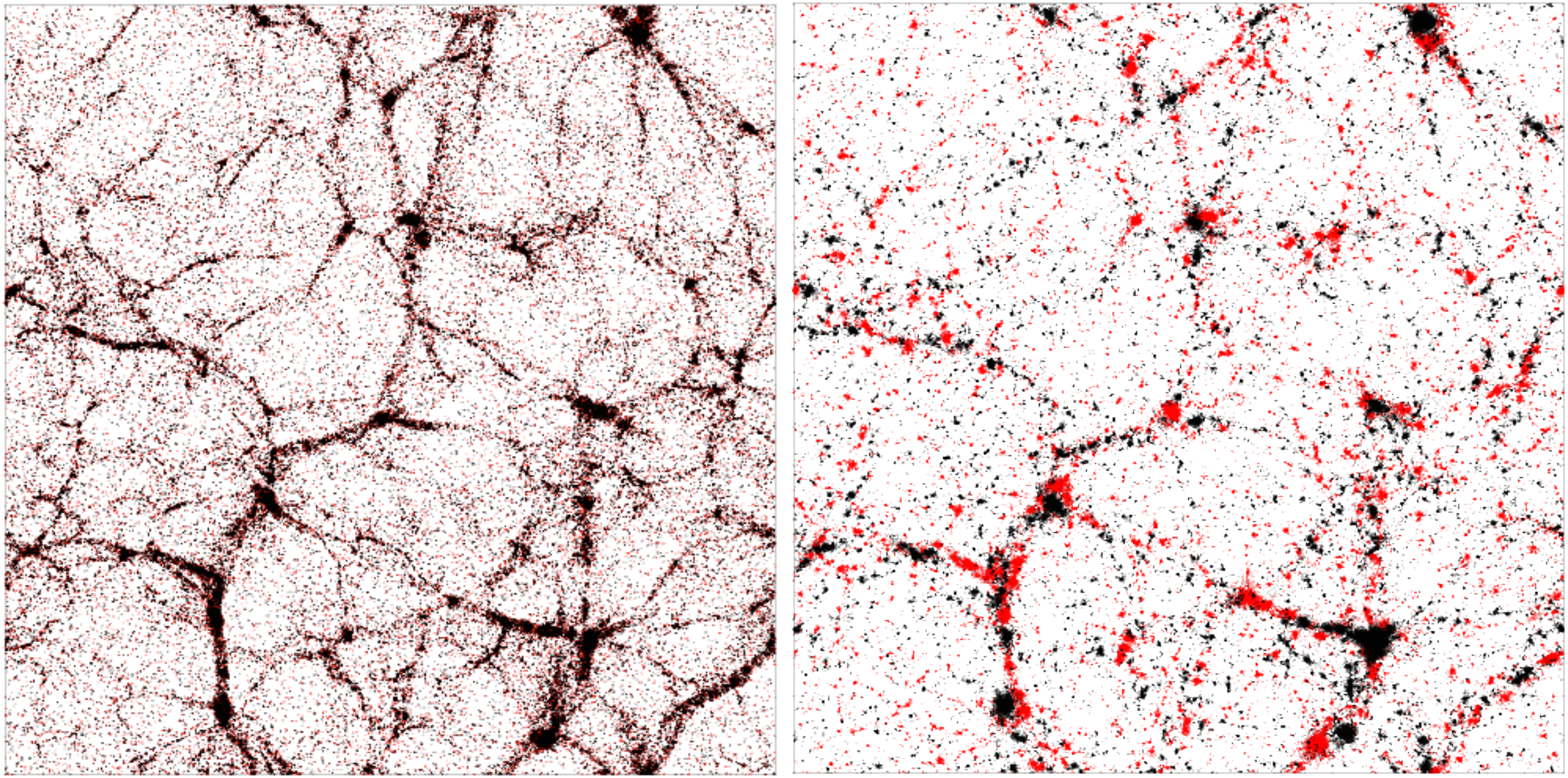

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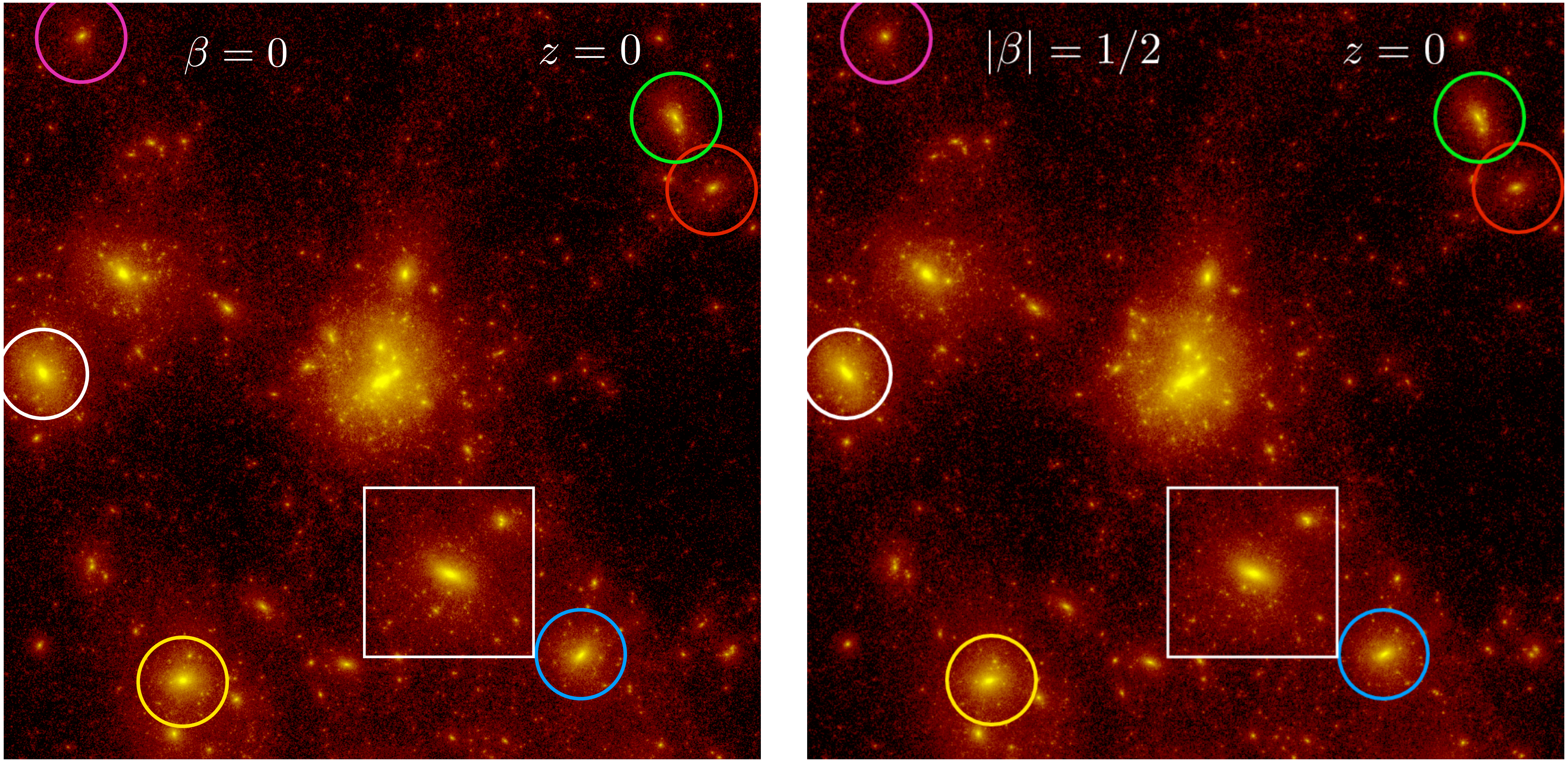
With higher resolution simulations [MB, PDU 2014] it was possible to observe for the first time the fragmentation of individual halos in McDE



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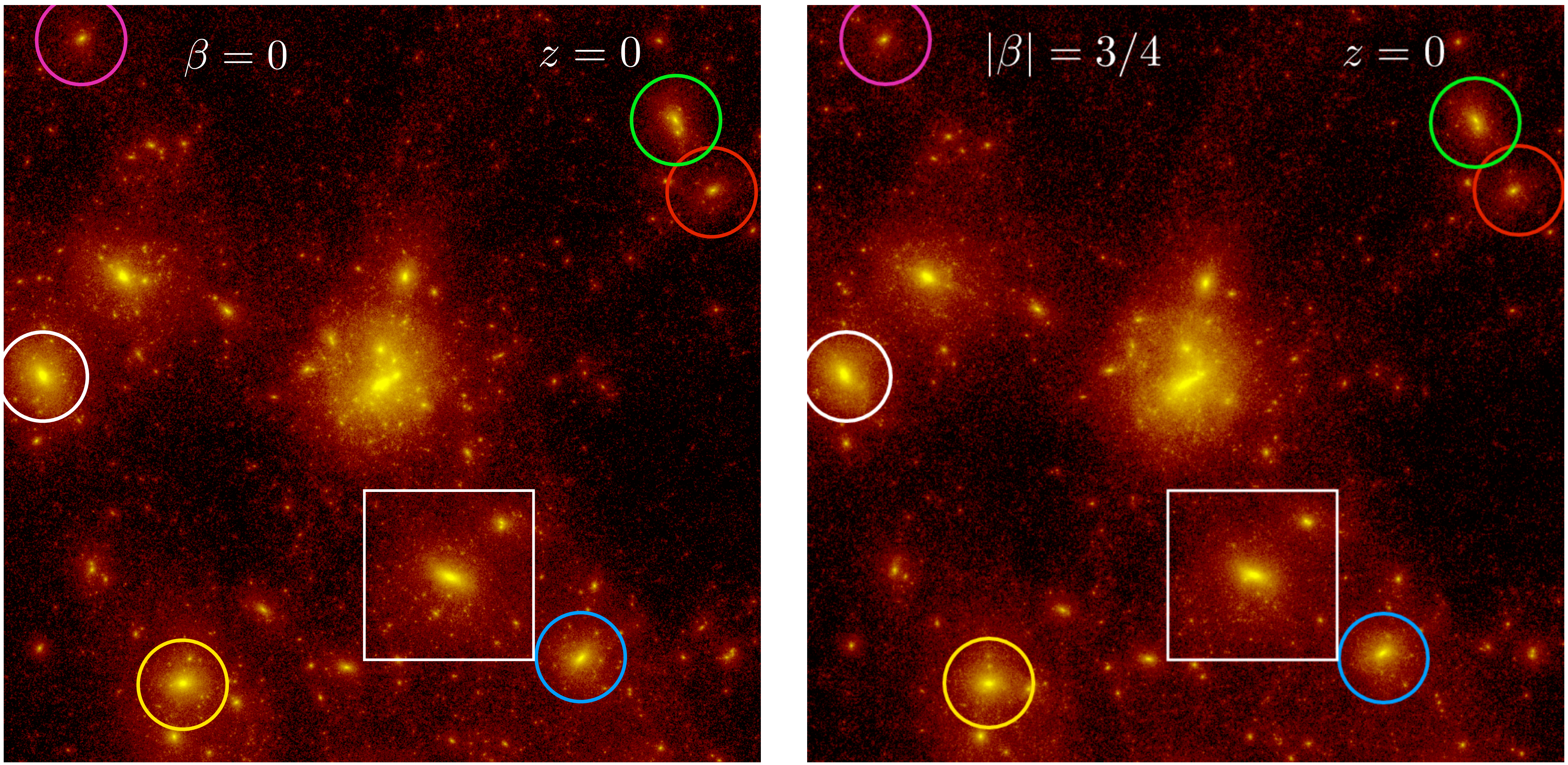

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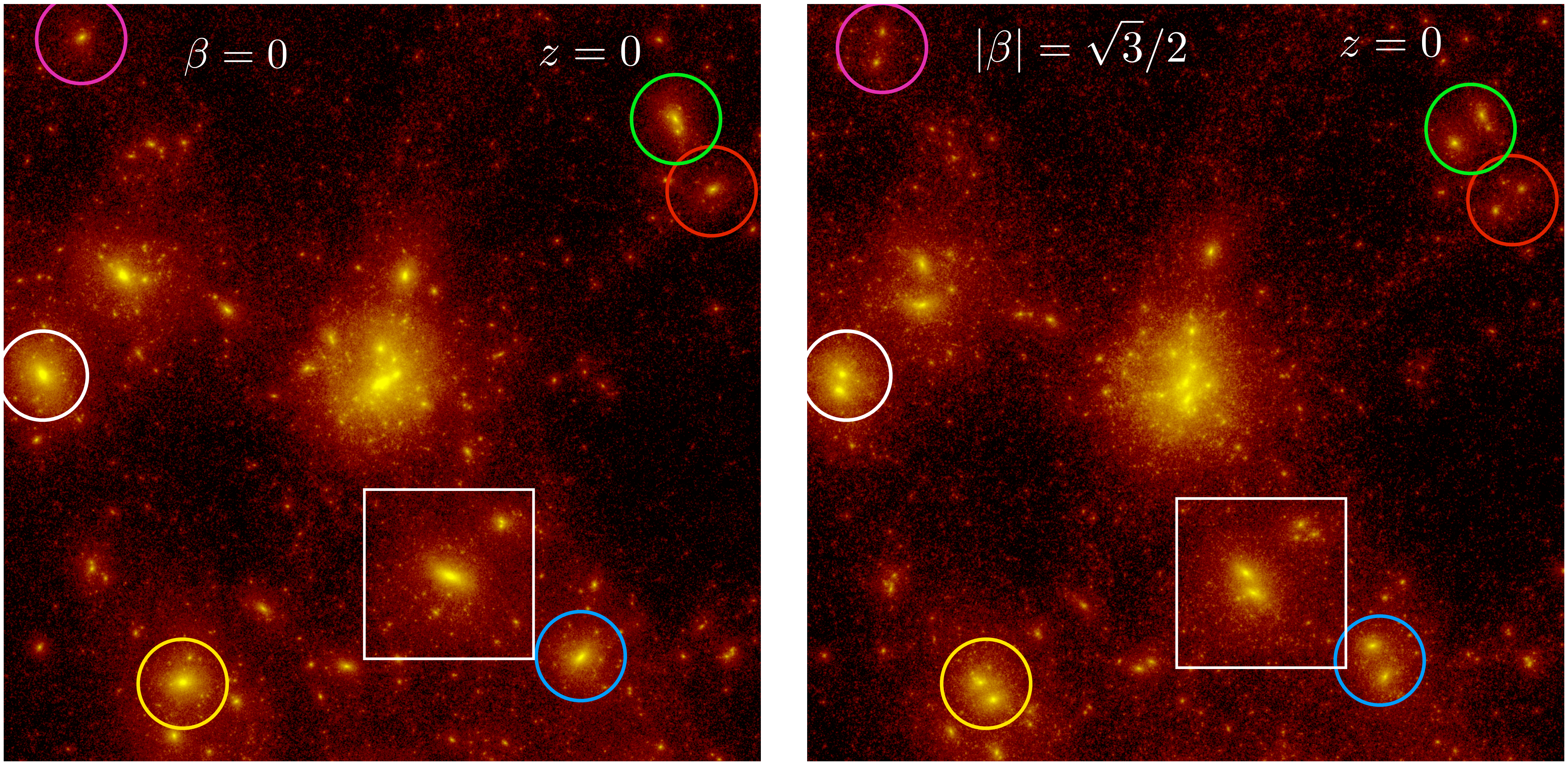

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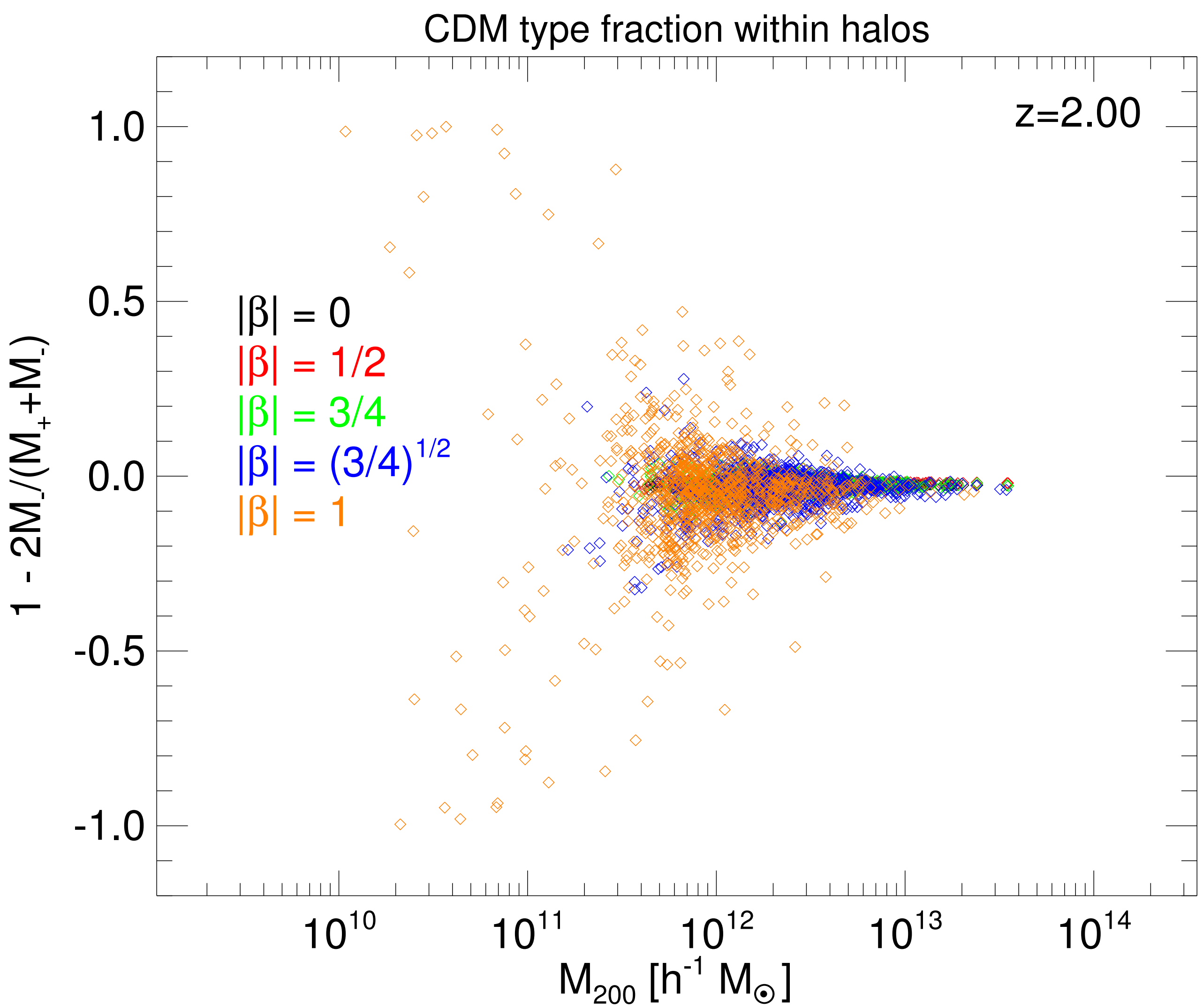
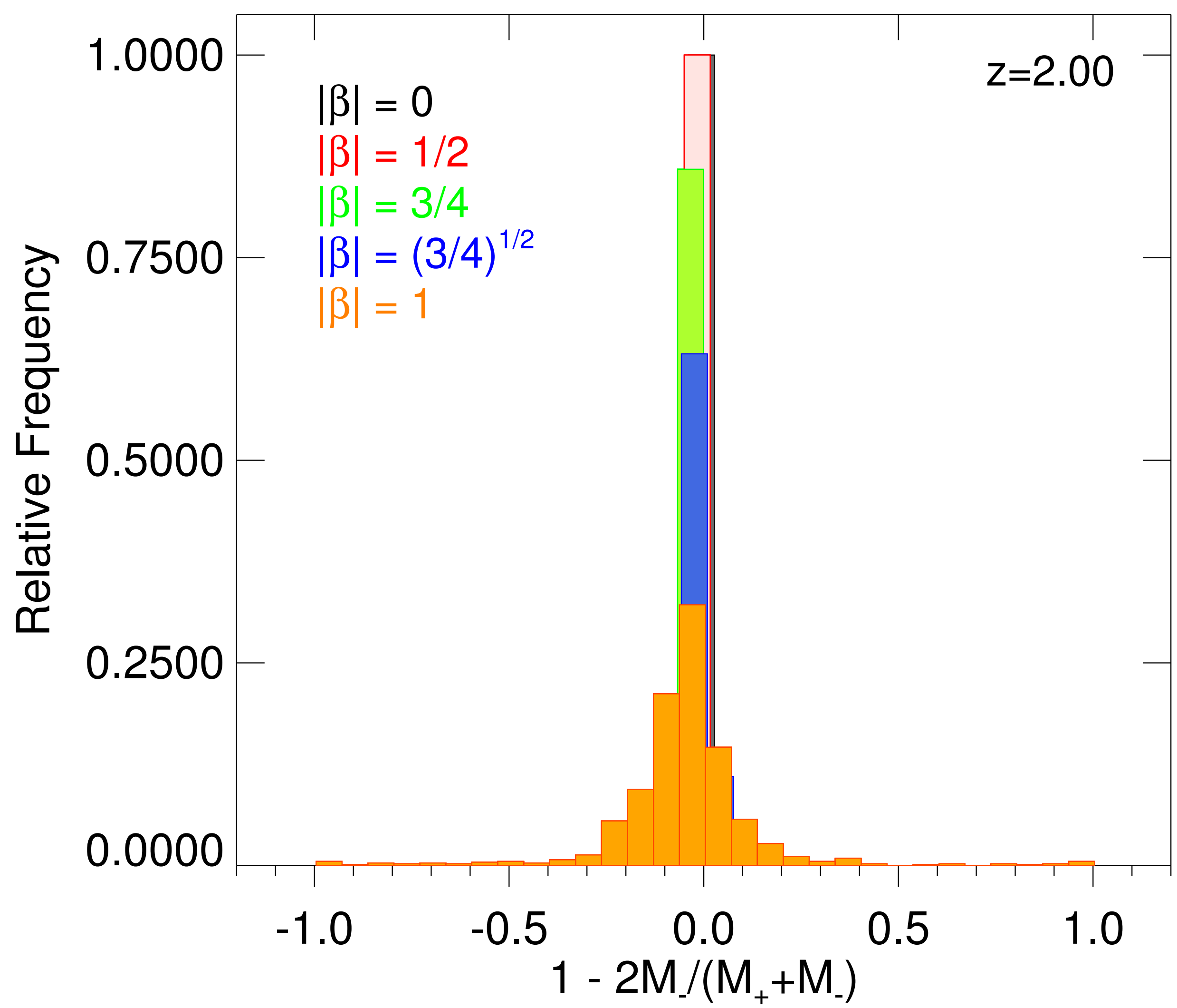
$$1 - \frac{2M_-}{M_+ + M_-} \qquad M_\pm \equiv M(\text{type}, < R_{200})$$





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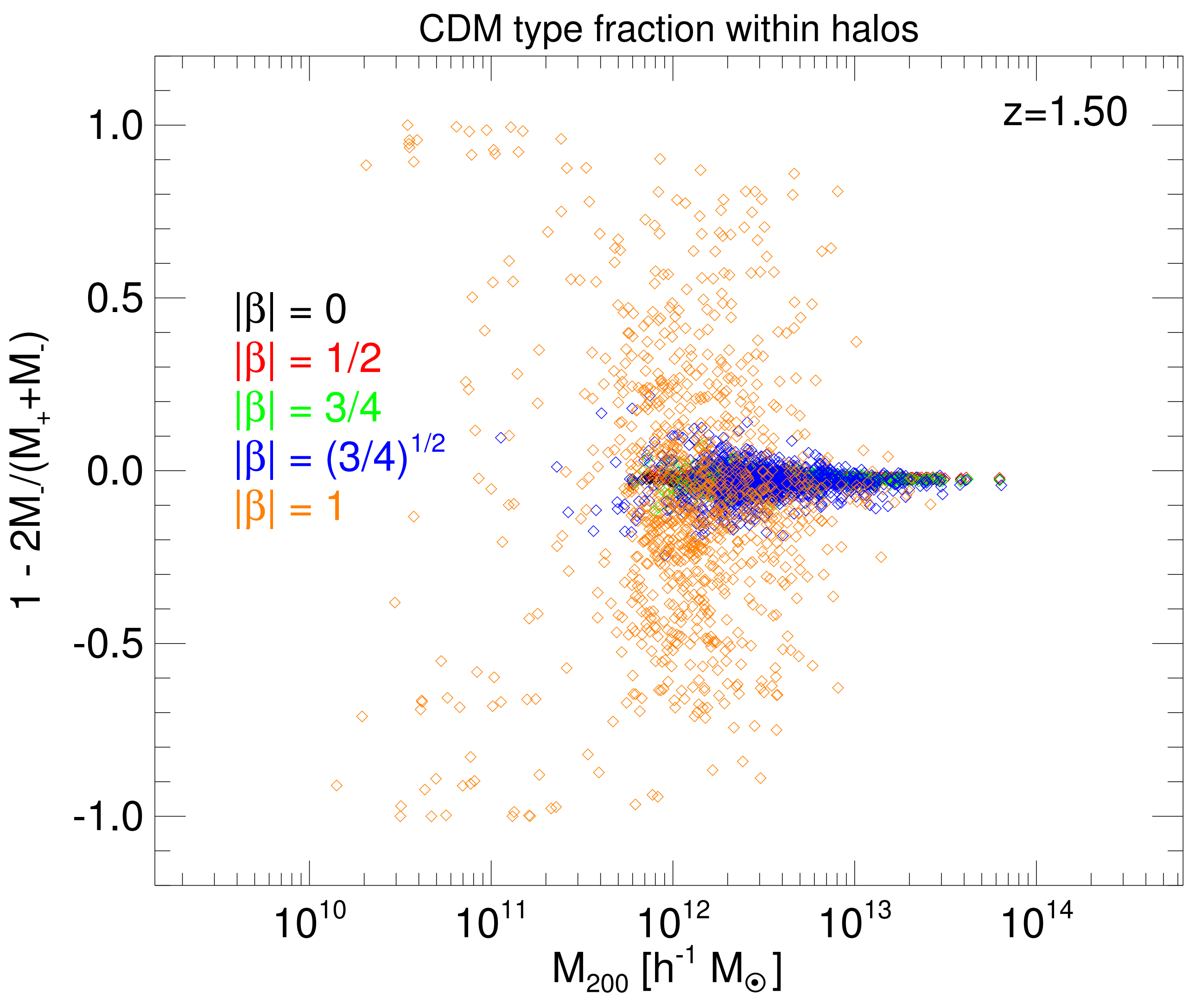

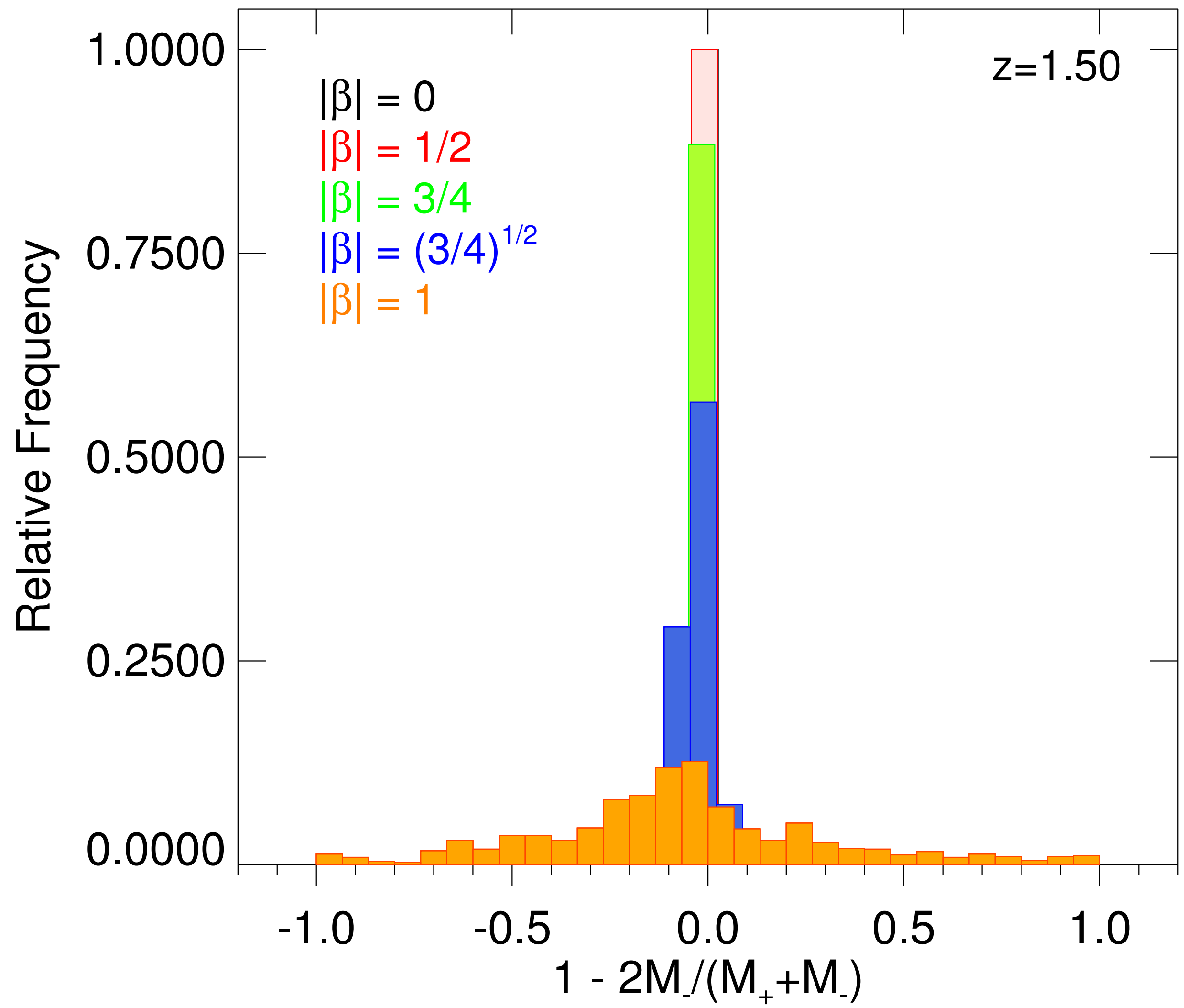

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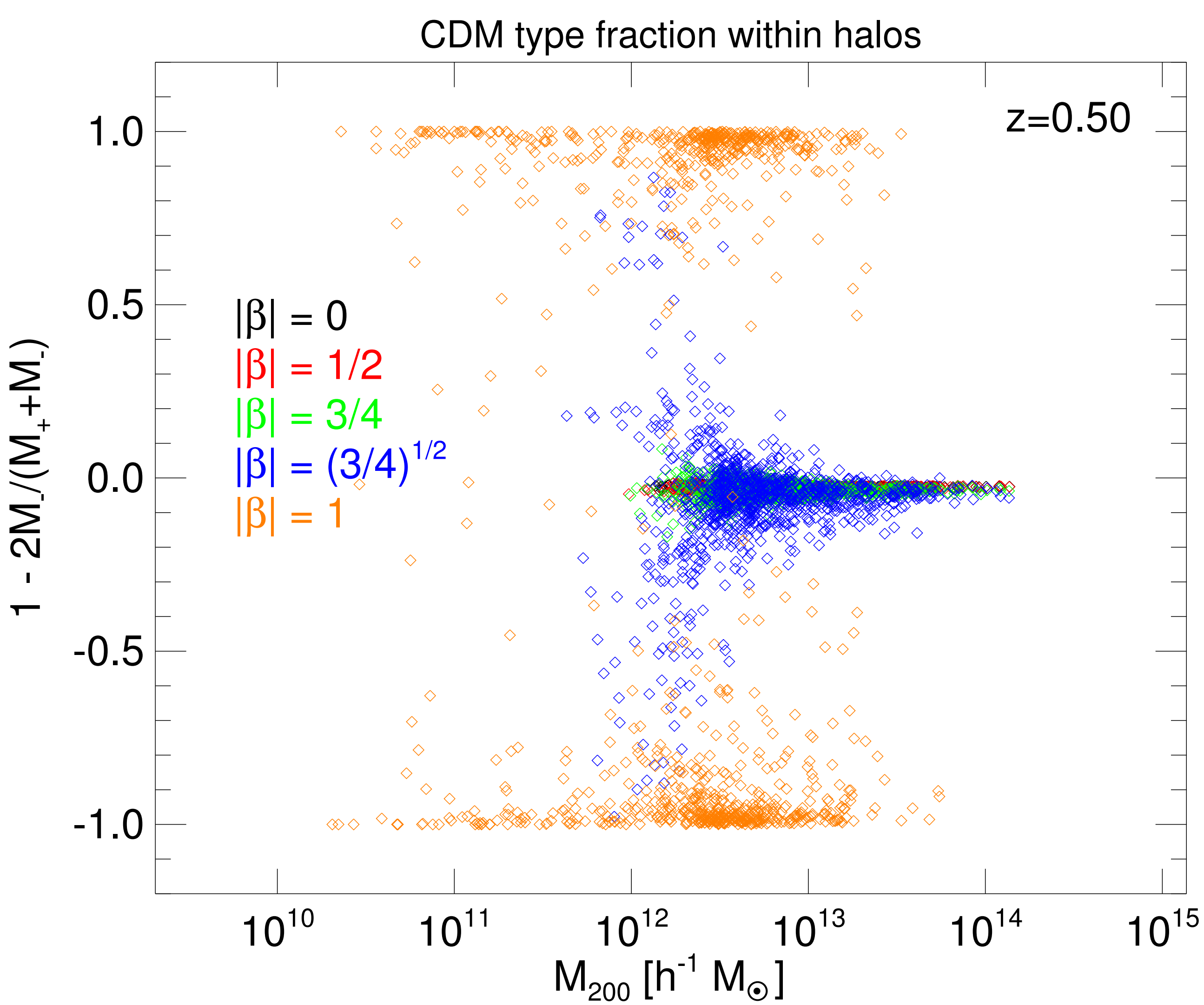

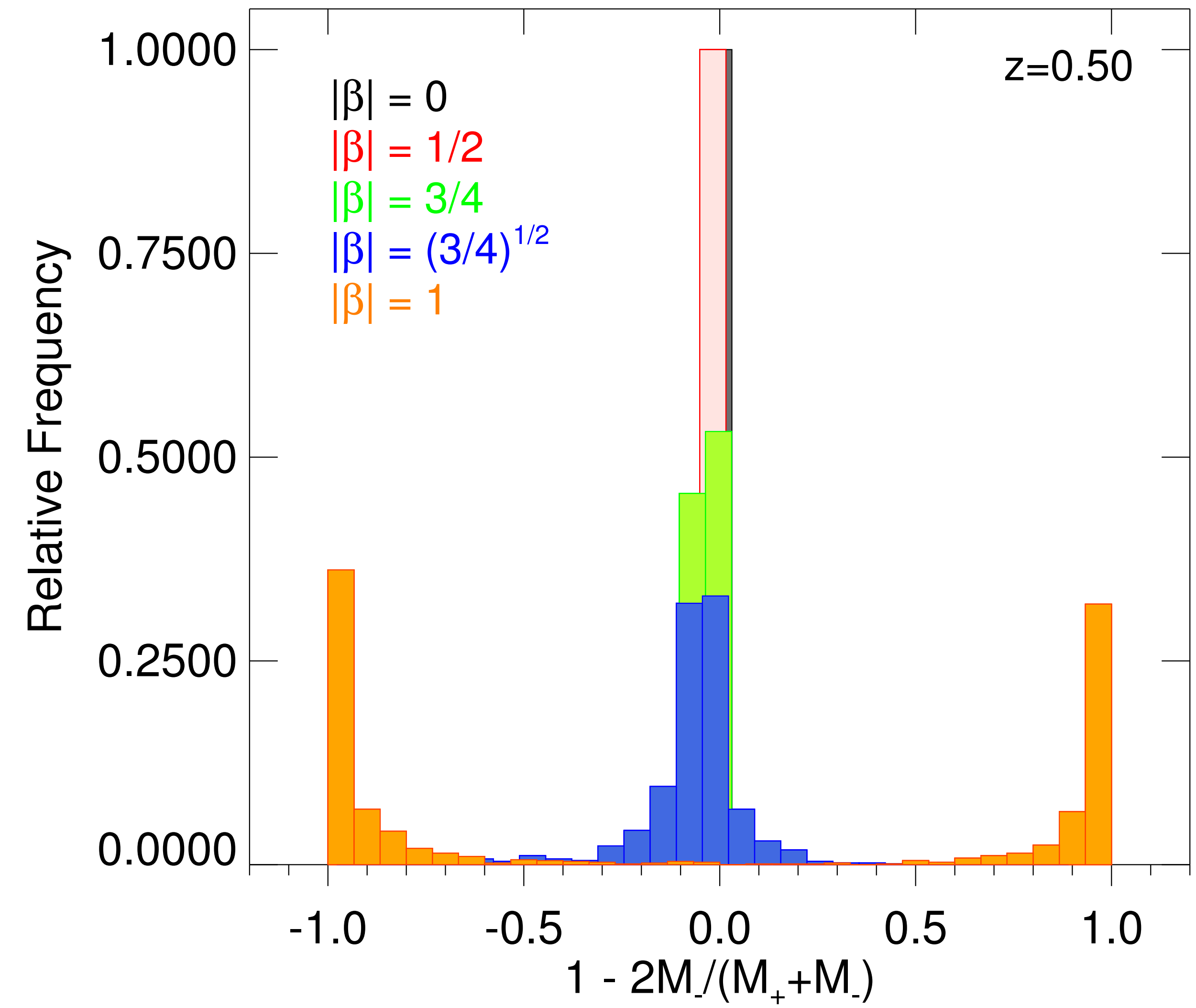

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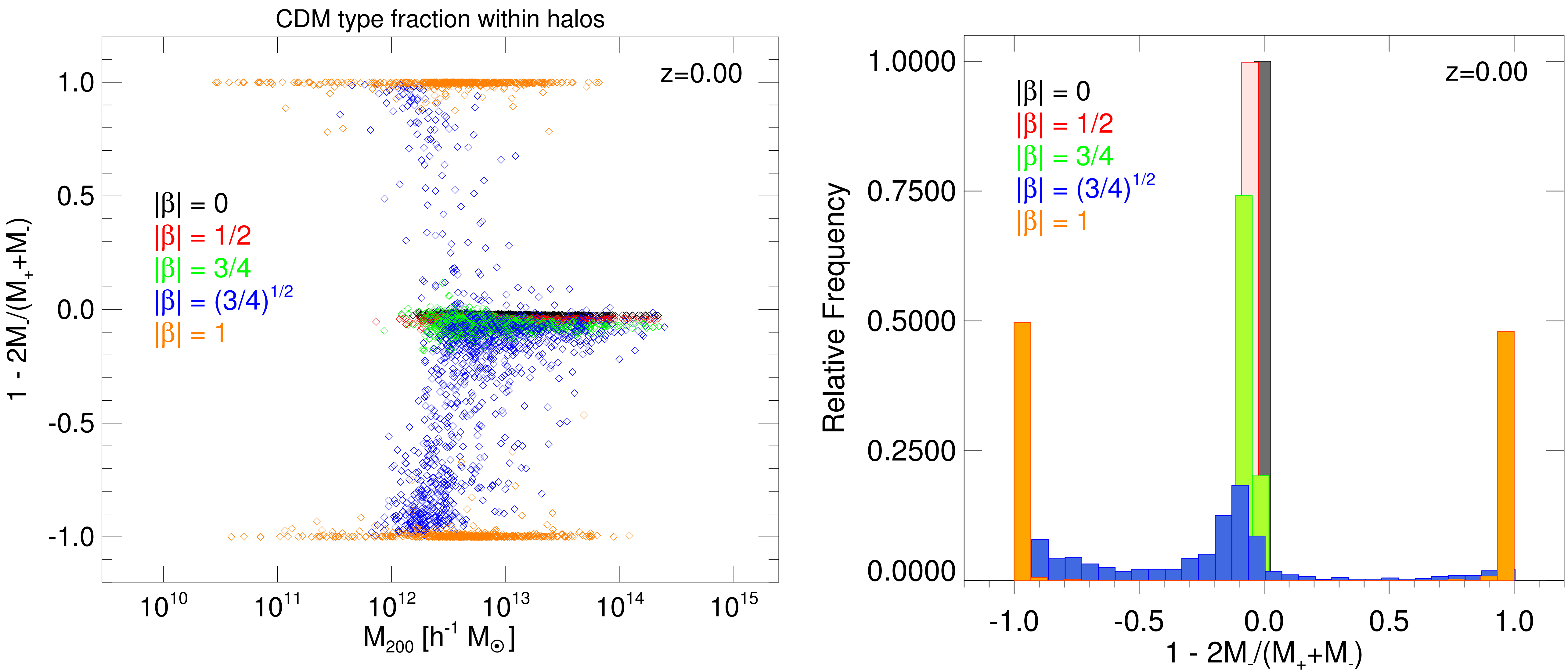

Zoom-in simulations: the ZInCo code

ZInCo (Zoomed Initial Conditions) is a new MPI-parallel code for generating **multi-resolution and multy-particle type** ICs for zoomed simulations

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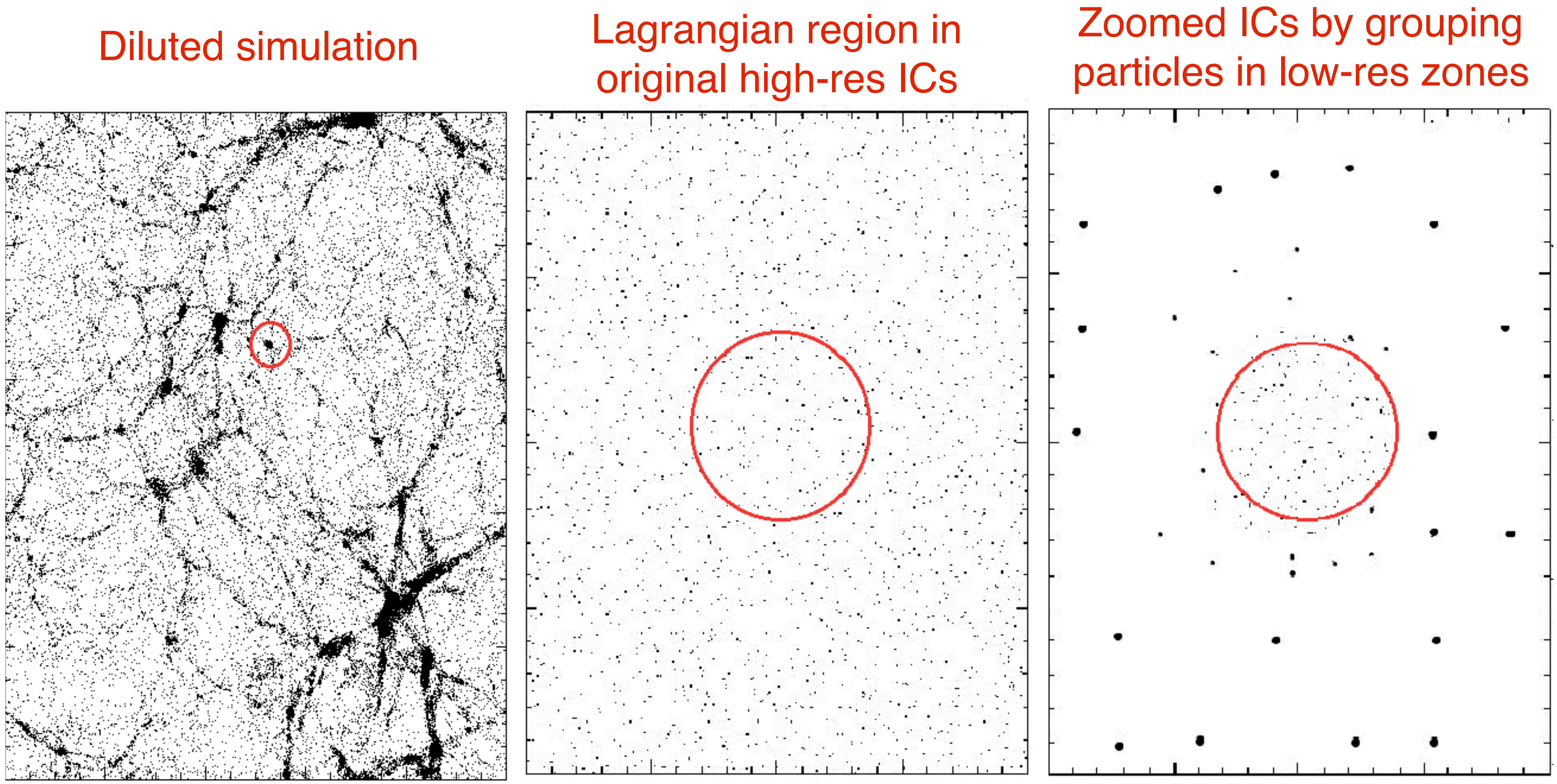

Garaldi et al. 2016

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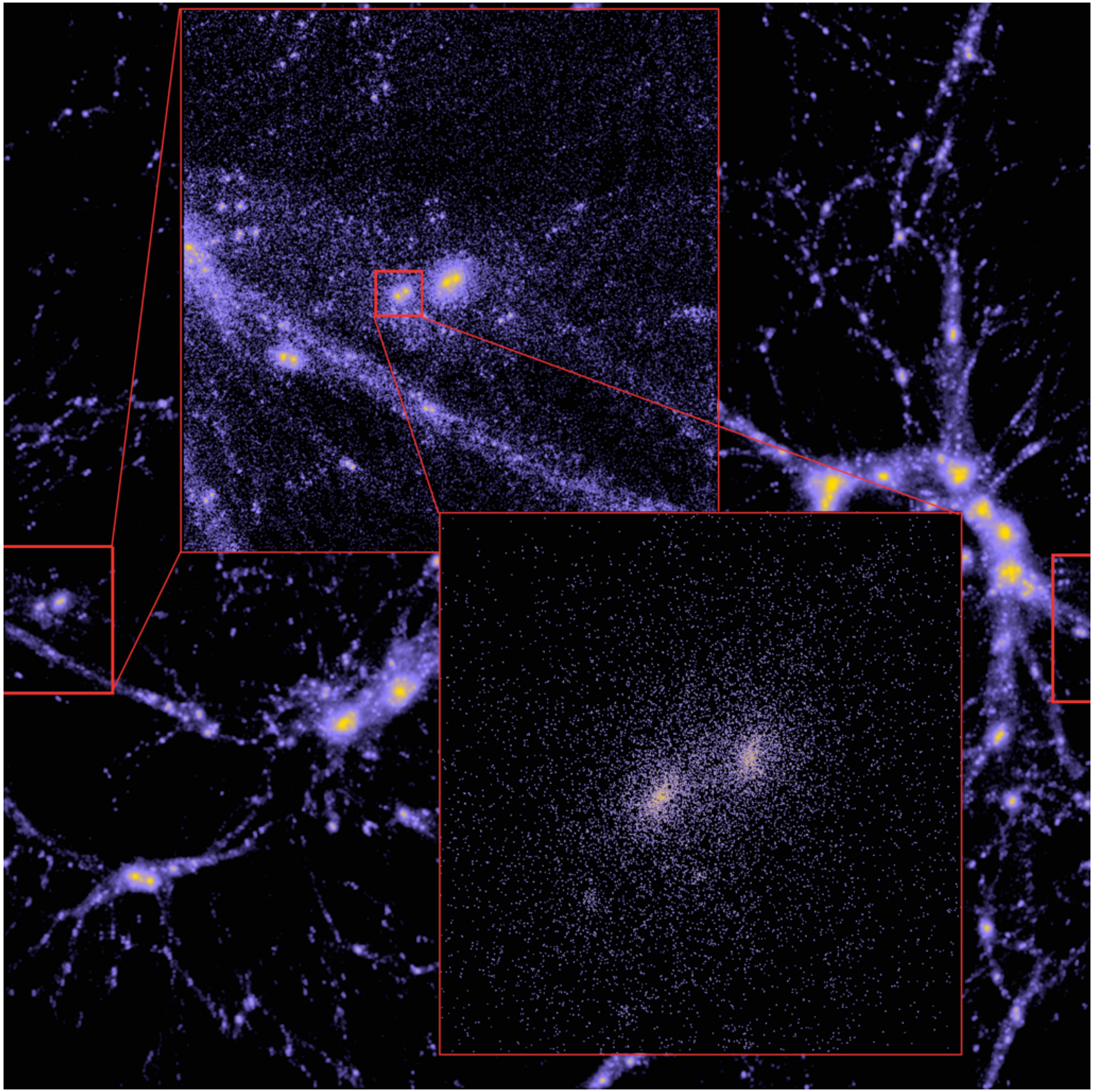
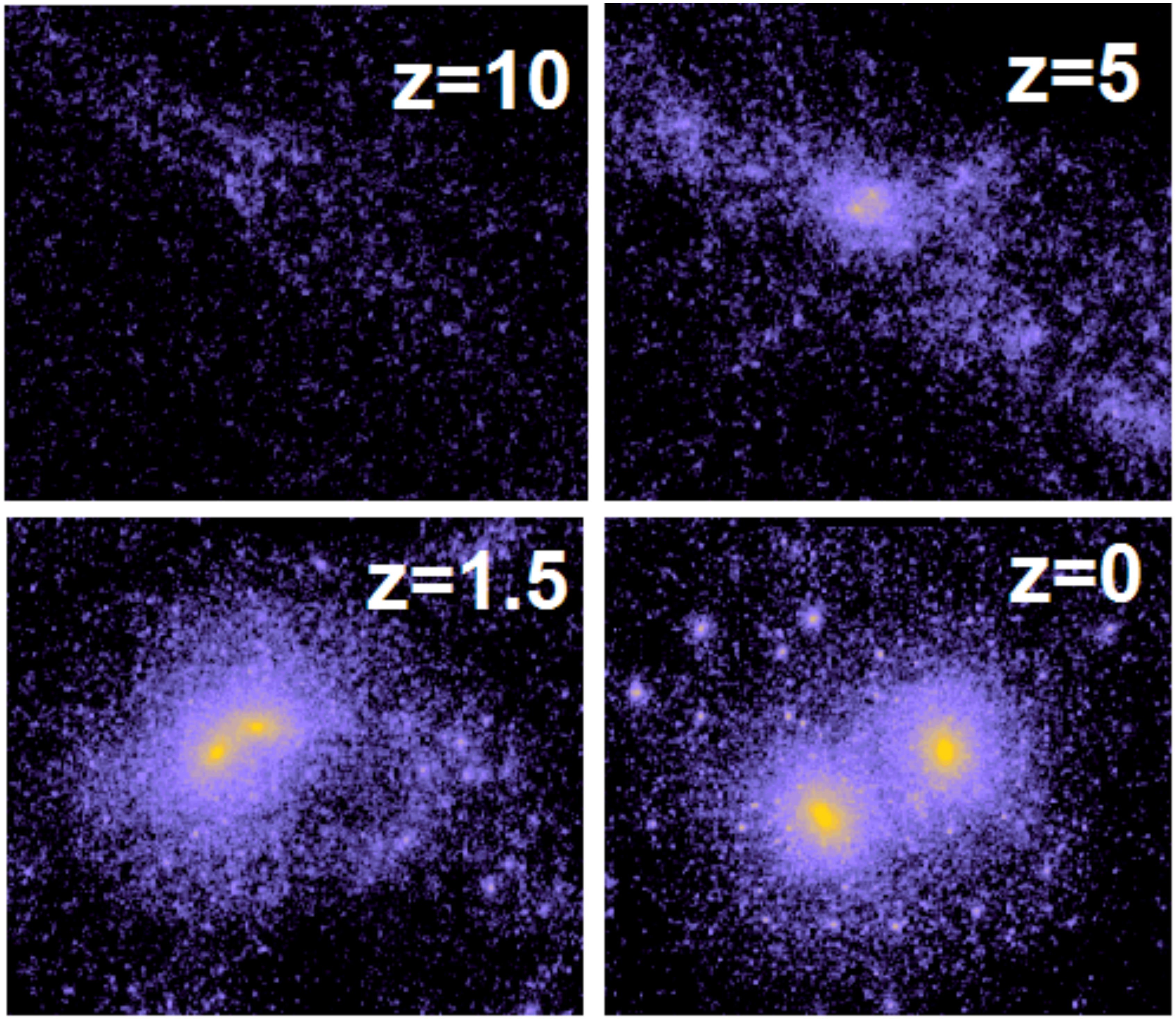


Garaldi et al. 2016

Zoom-in simulations: results

First result: early segregation.

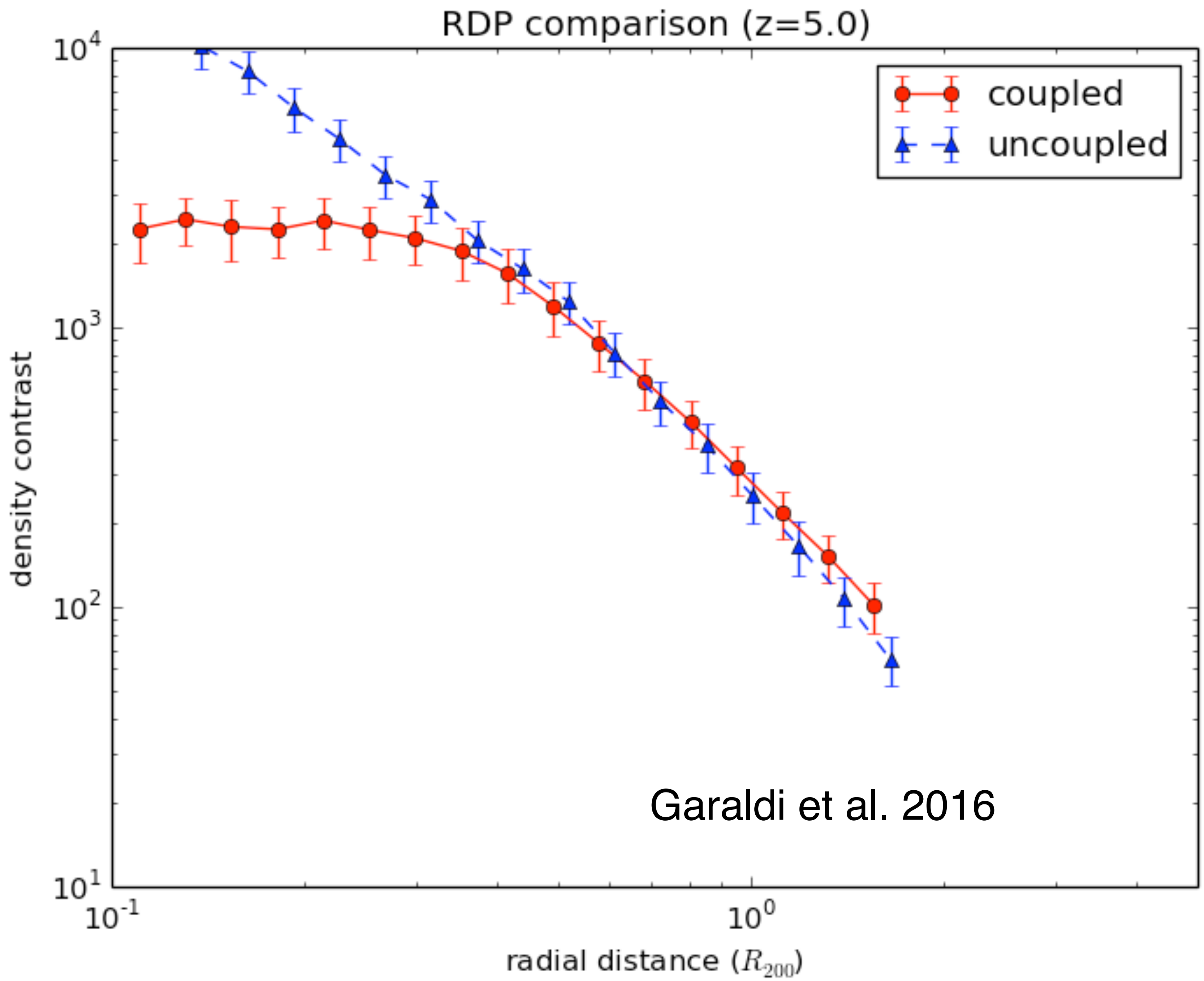
Already at z~5 there are two clear distinct density peaks



Garaldi et al 2016

Zoom-in simulations: results

Second result: formation of density cores in the total DM density



Modified Gravity

Modified Gravity

Universal couplings: Extended Quintessence, f(R), Symmetron, Dilaton, et al.

$$\nabla^2 \delta\phi = F(\delta\phi) + \beta(\phi)\delta\rho_M$$

where F is a nonlinear function: a nonlinear Poisson equation to solve!!

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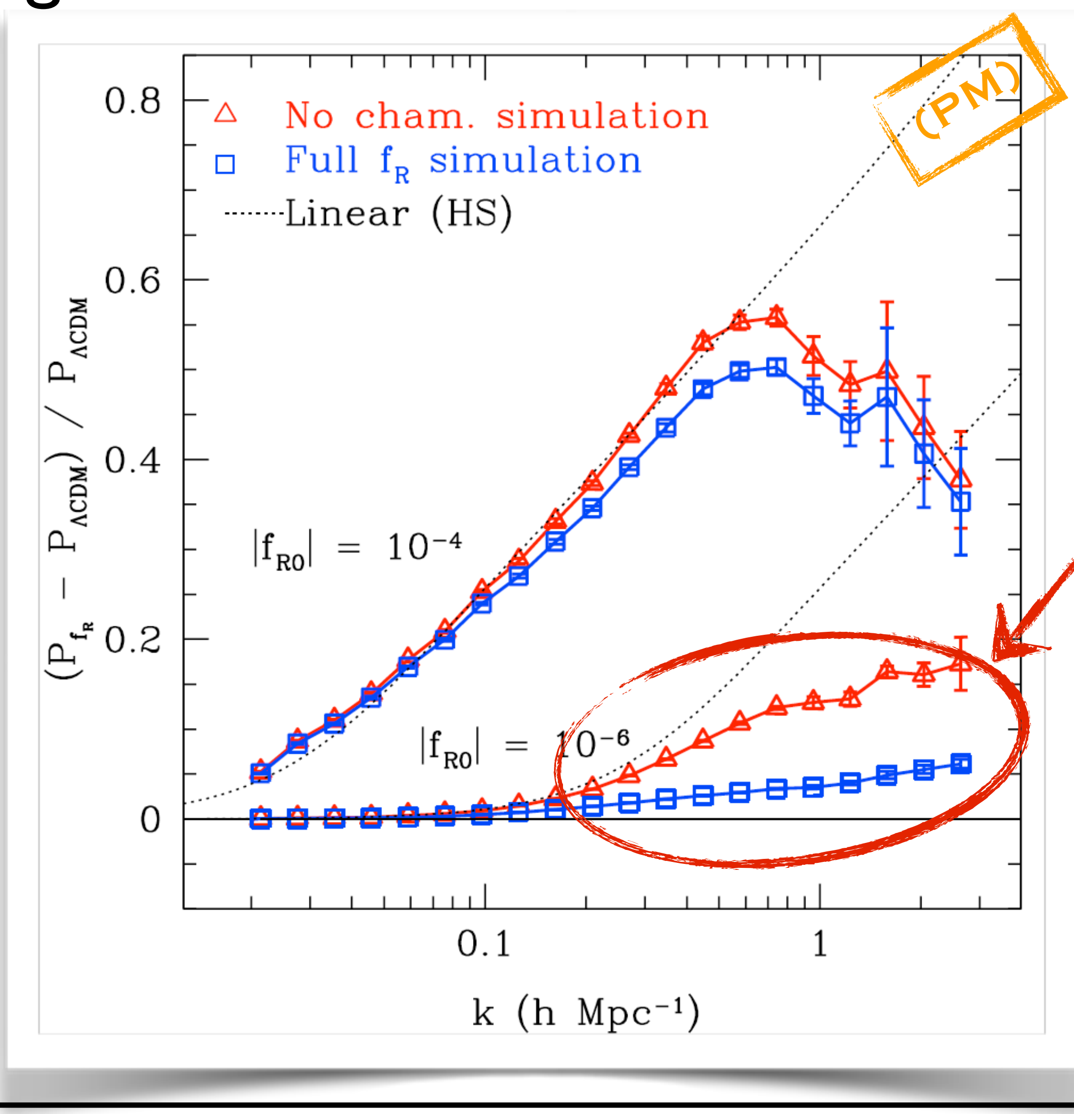
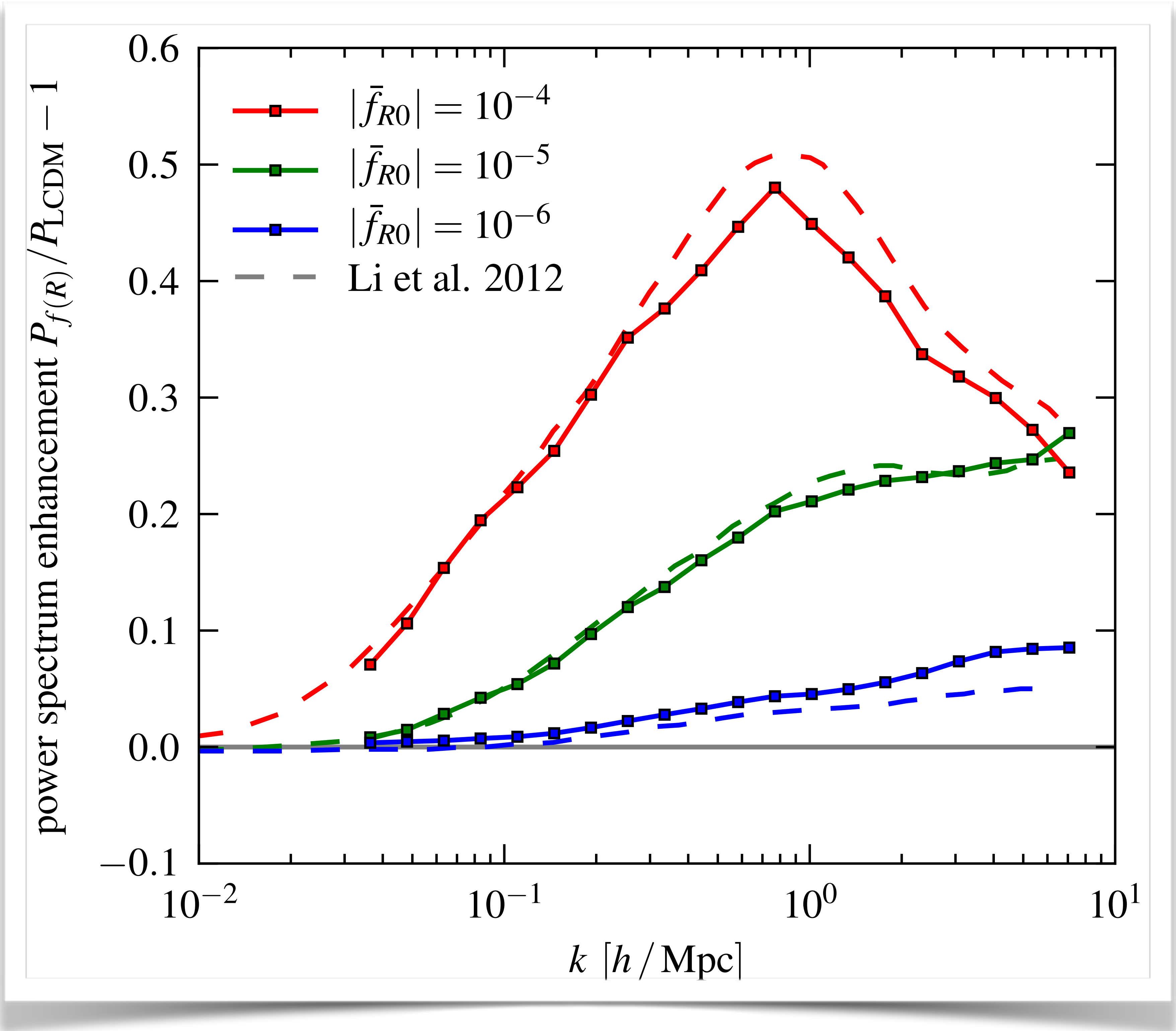
First simulations by Oyaizu 2008; Oyaizu, Lima, Hu 2008; Schmidt et al 2009 using an iterative scheme within a fix-grid PM code



The scalar fifth-force is suppressed in high-density regions according to the solution of the nonlinear Poisson equation for δϕ. The screening mechanism (in this case a Chameleon effect) is more efficient for lower values of $|f_{R0}|$

Modified Gravity

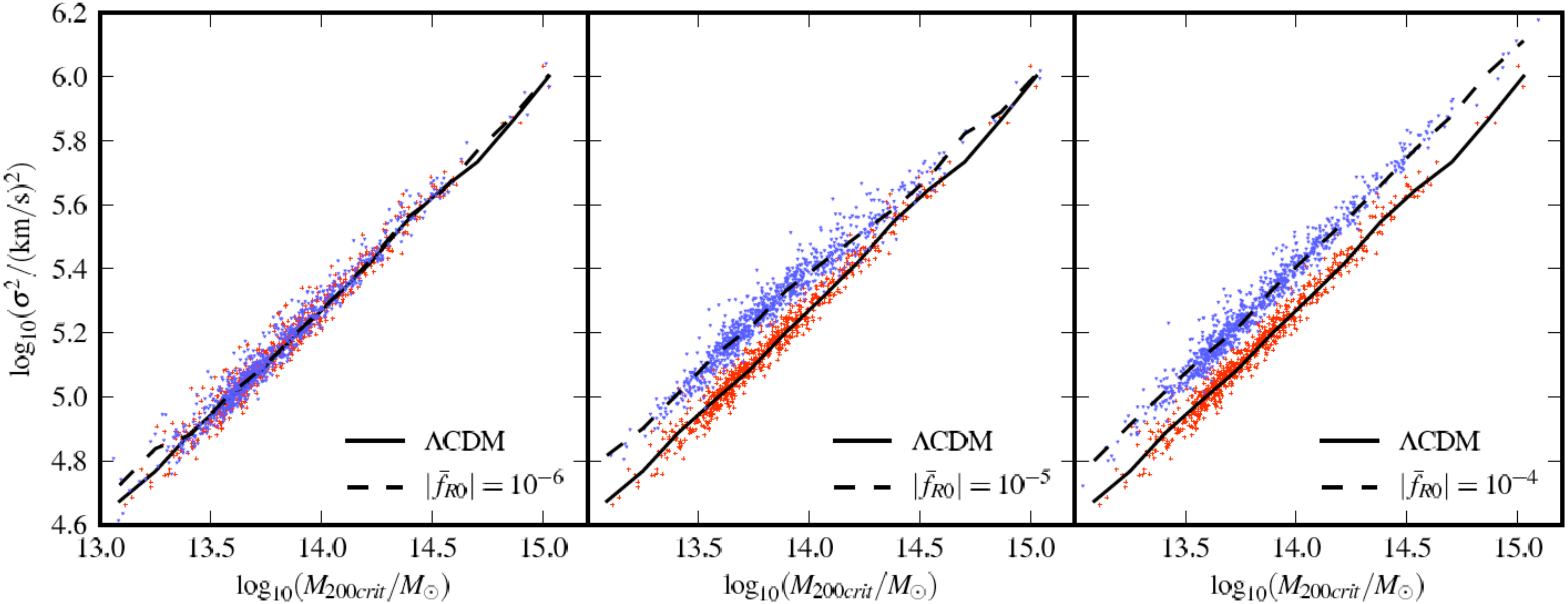
MG-GADGET



Puchwein, MB, Springel, 2013

Modified Gravity

MG-GADGET



Dynamical vs. true masses of groups and clusters

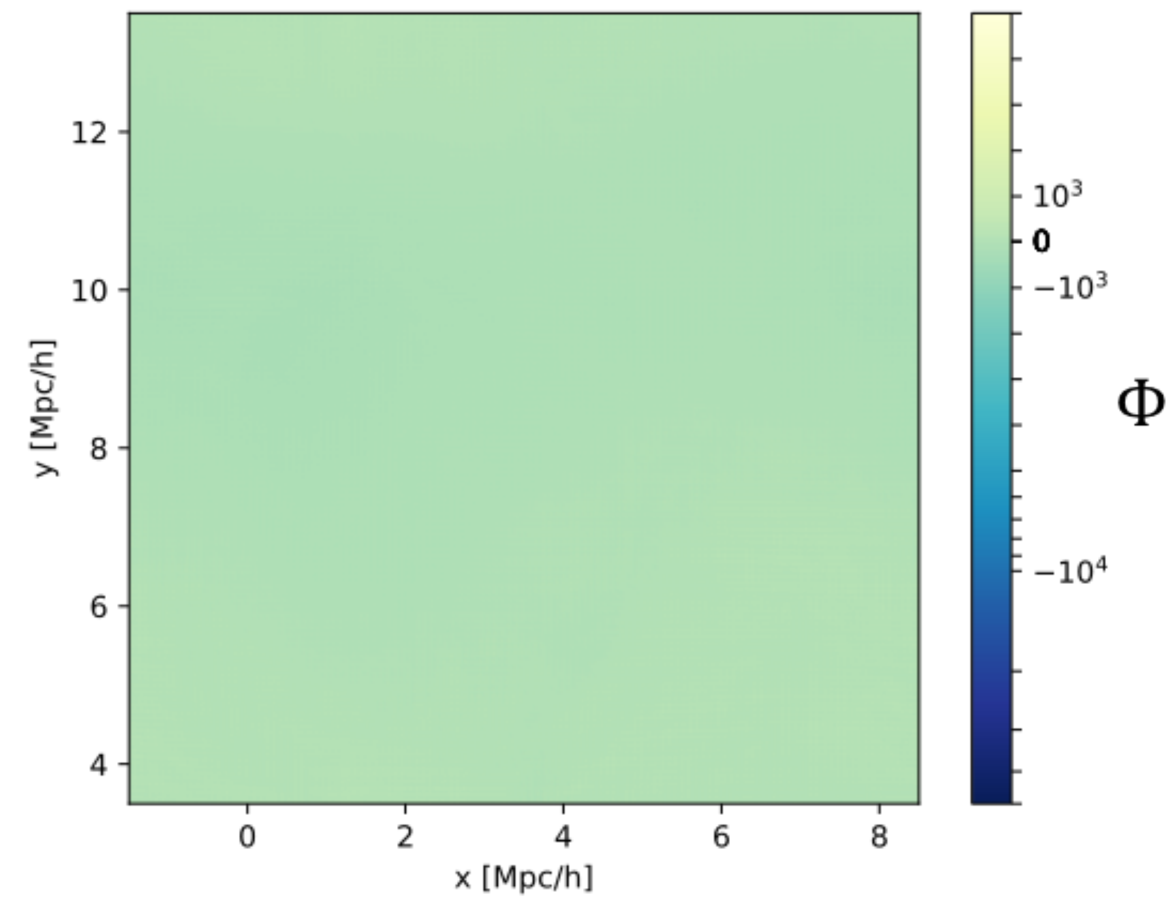
ARNOLD ET AL. 2014

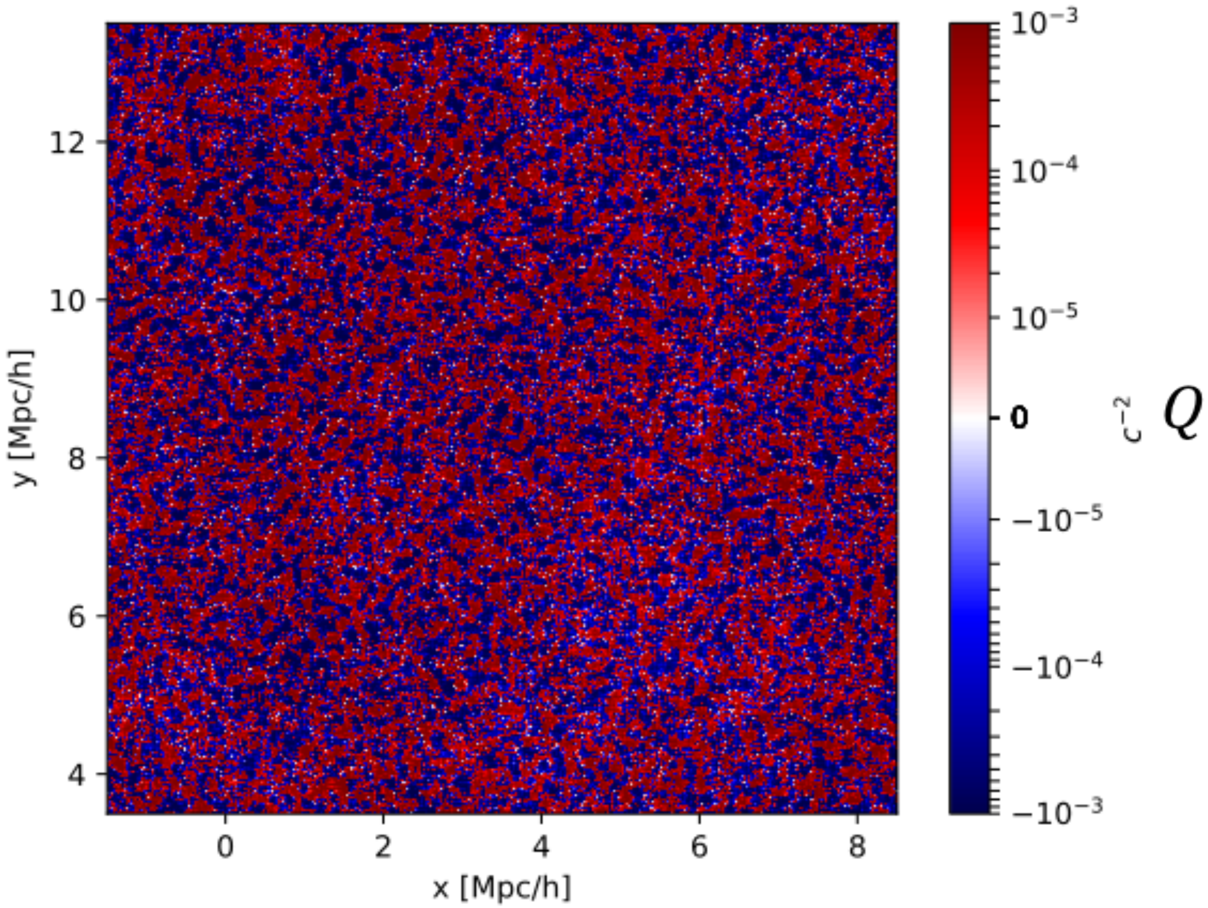
Axion Dark Matter

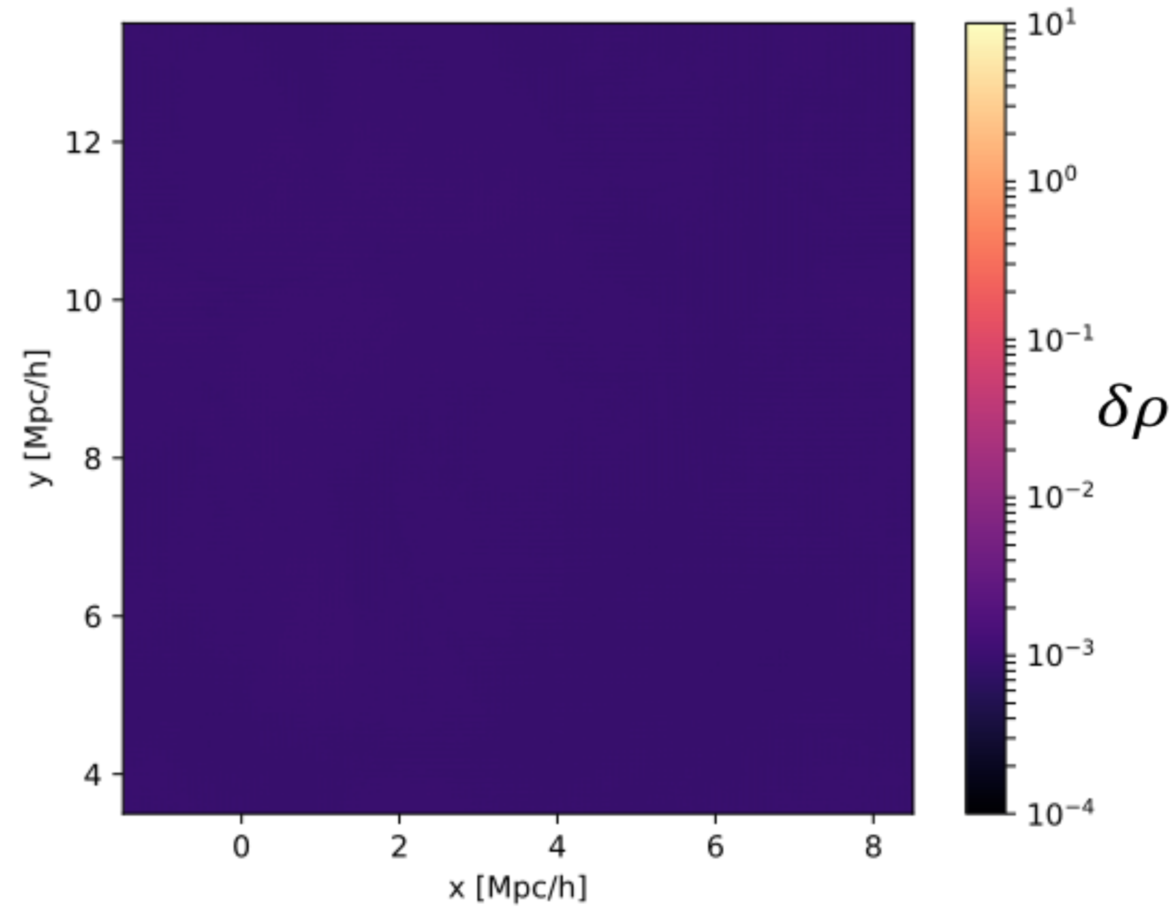
Simulation

10 Mpc side box
256^3 particles

$z = 99$



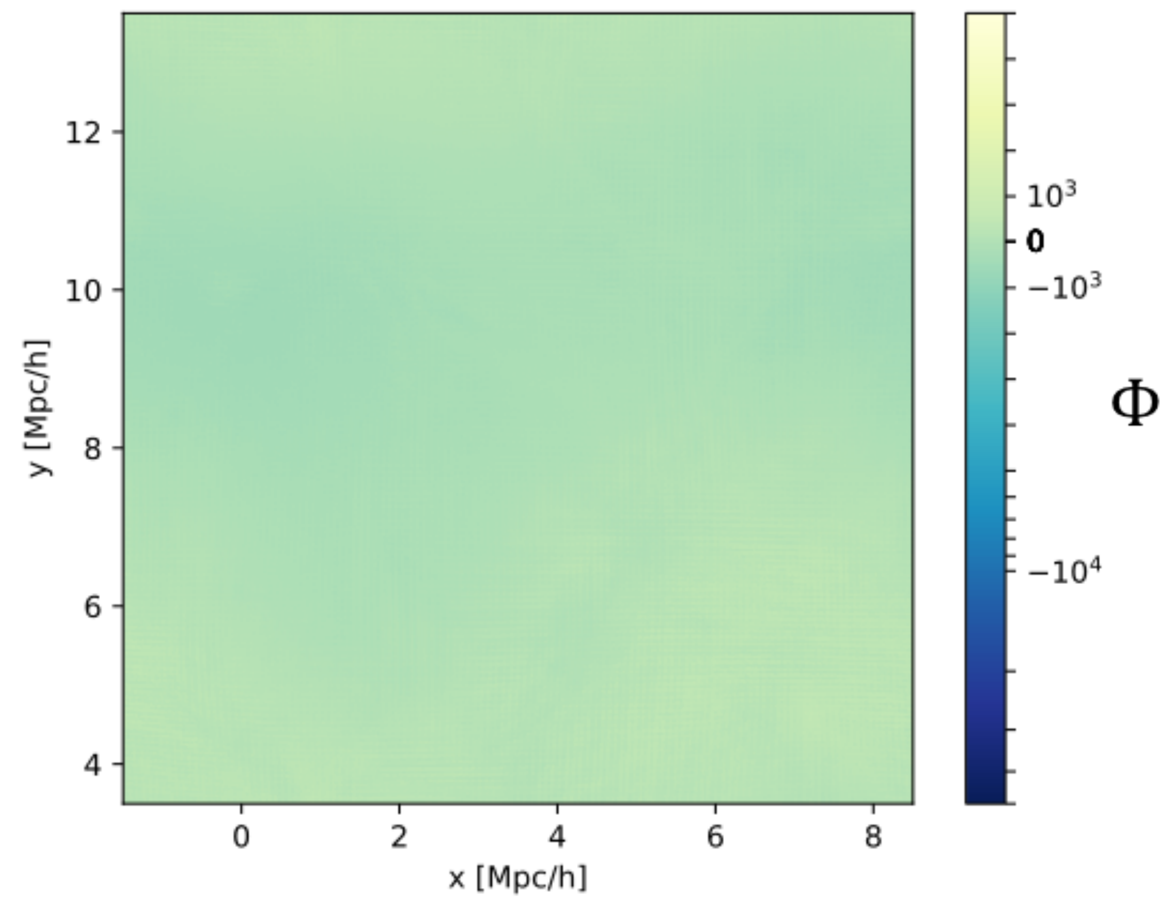


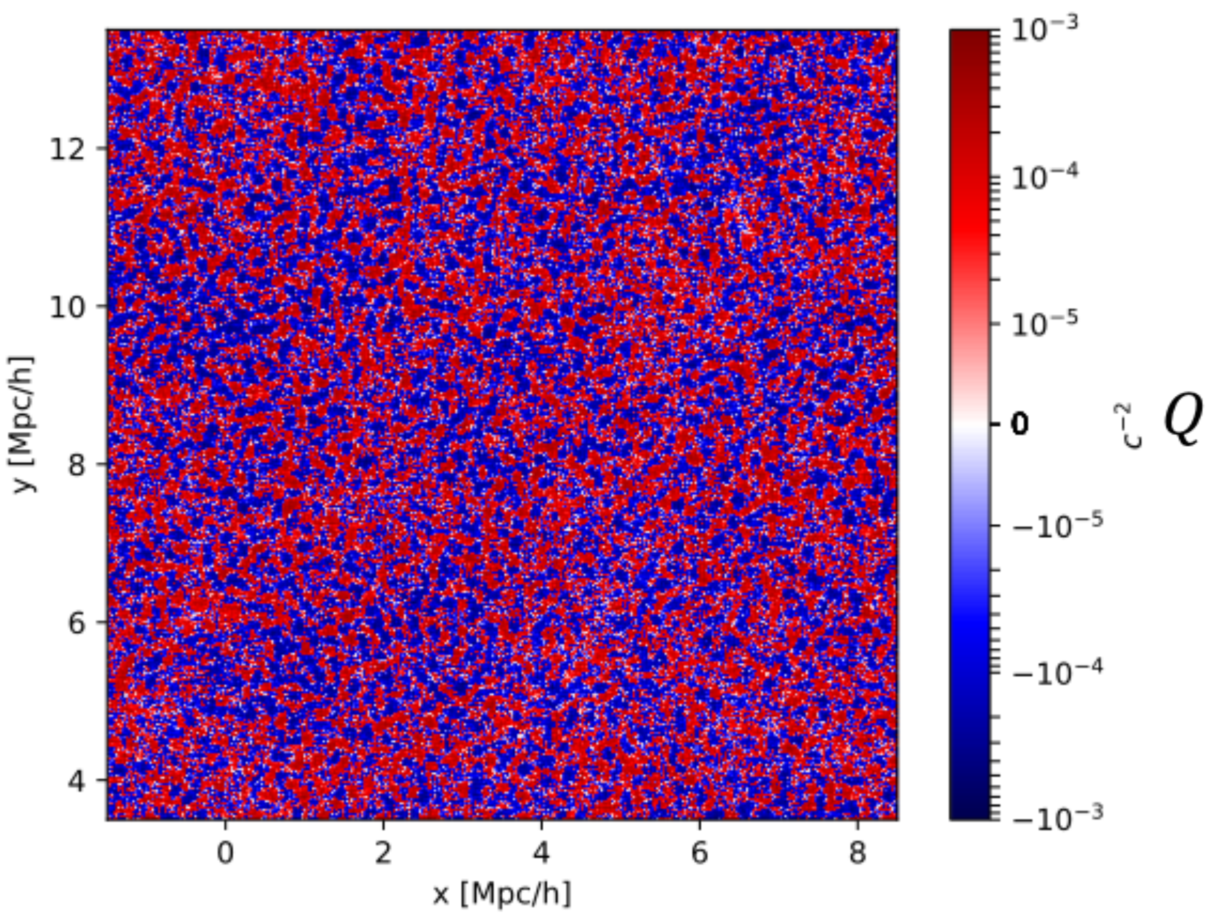


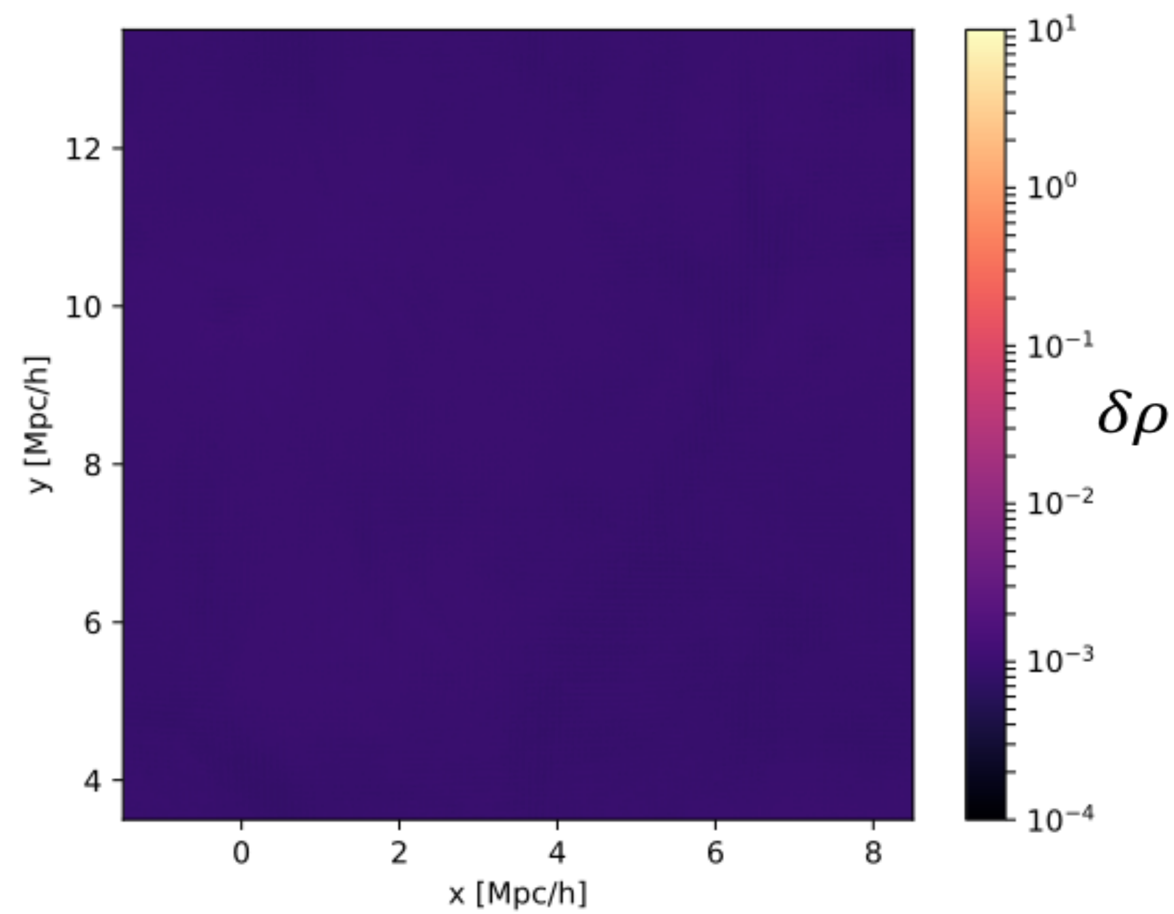
Simulation

10 Mpc side box
256^3 particles

$z = 49$



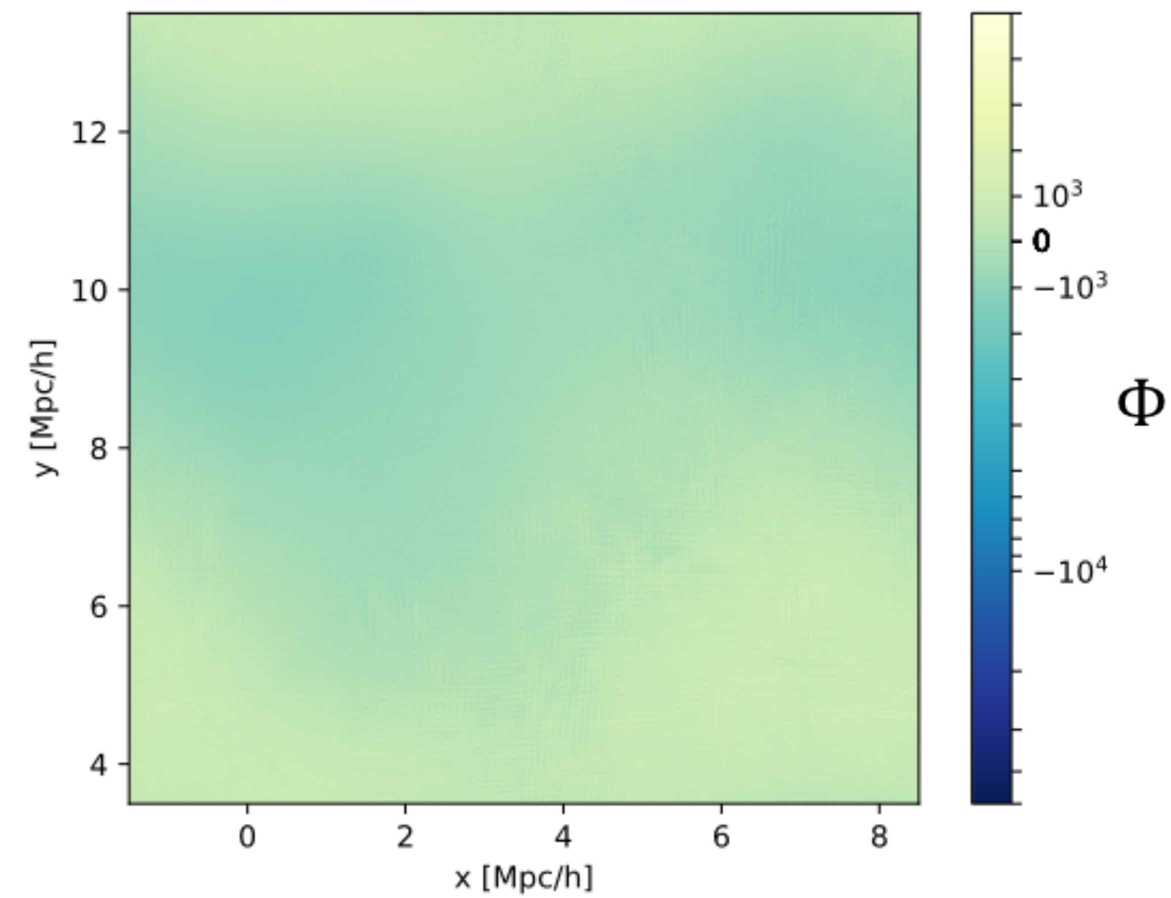


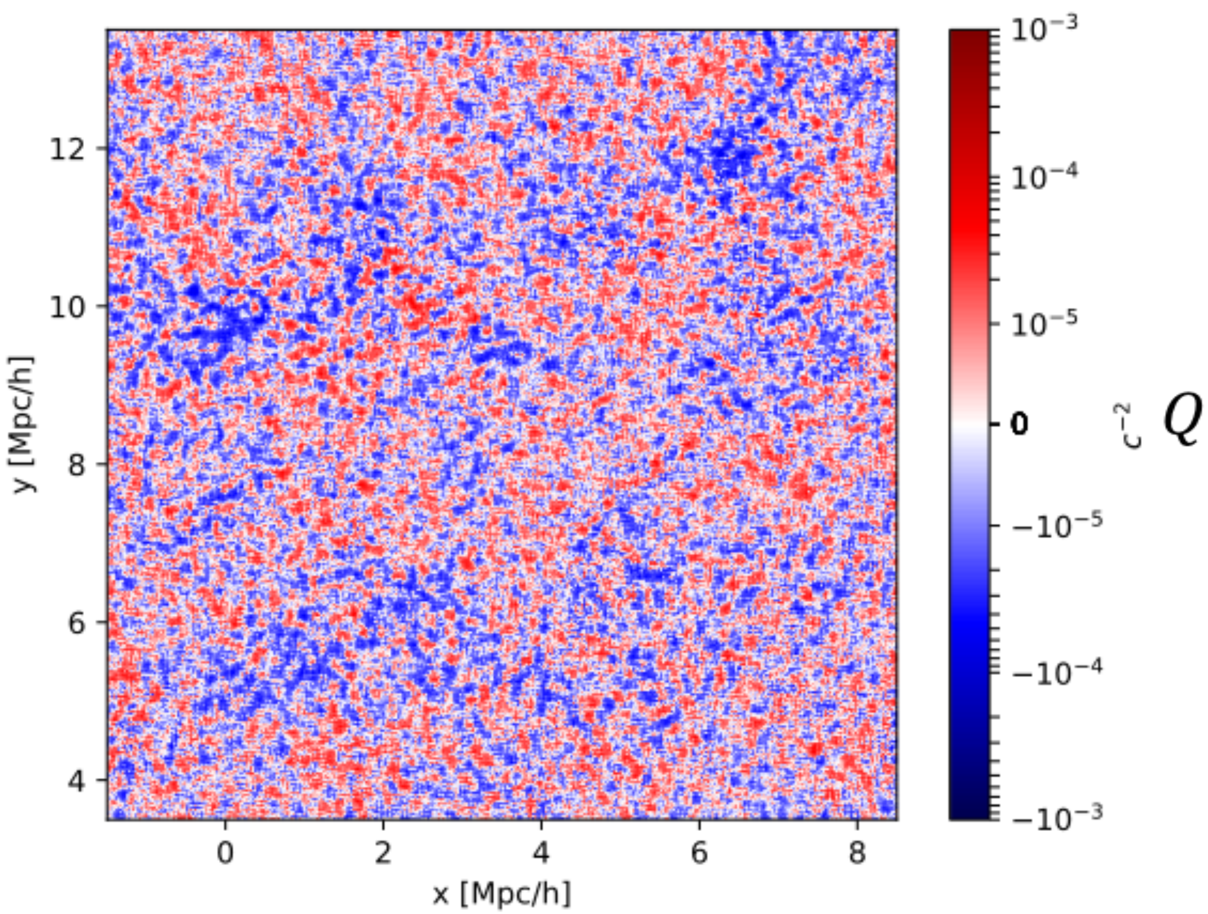


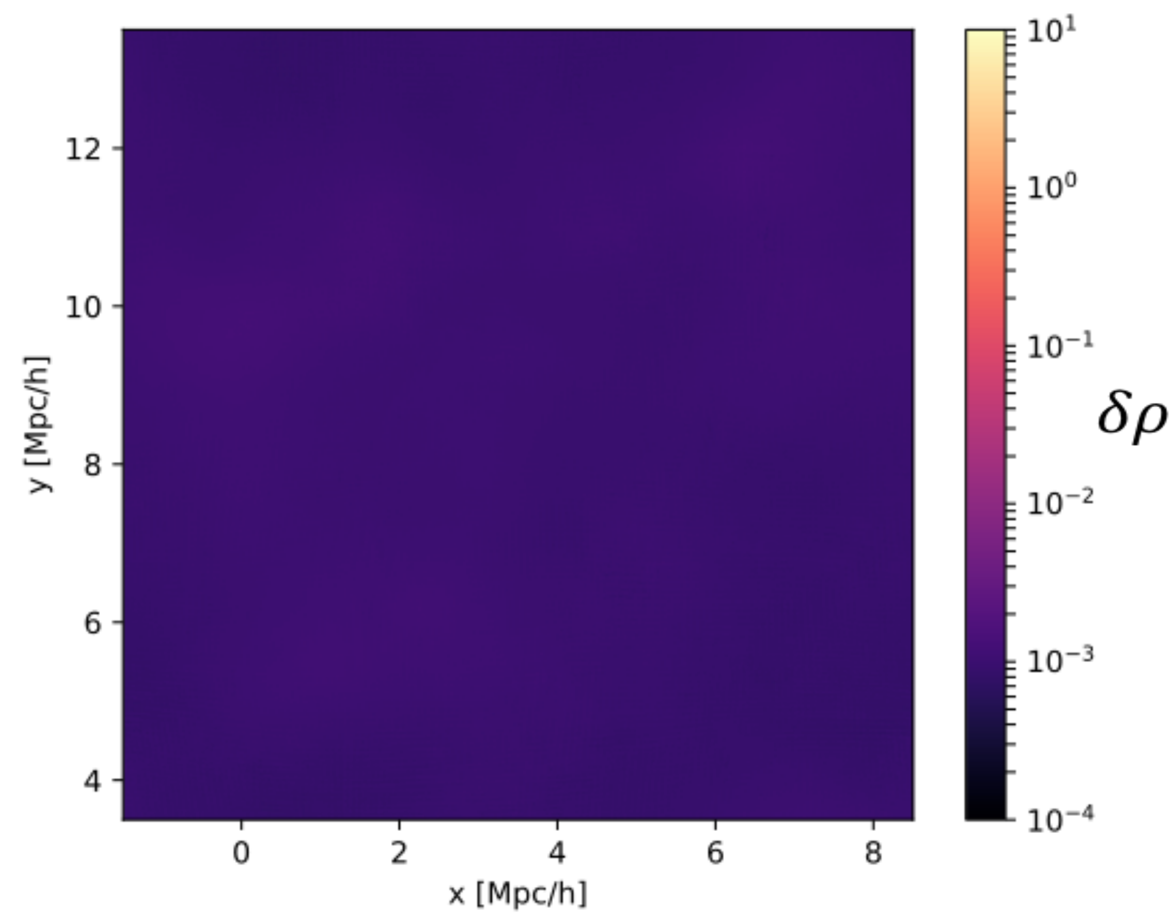
Simulation

$10\ Mpc$ side box
256^3 particles

$z = 19$



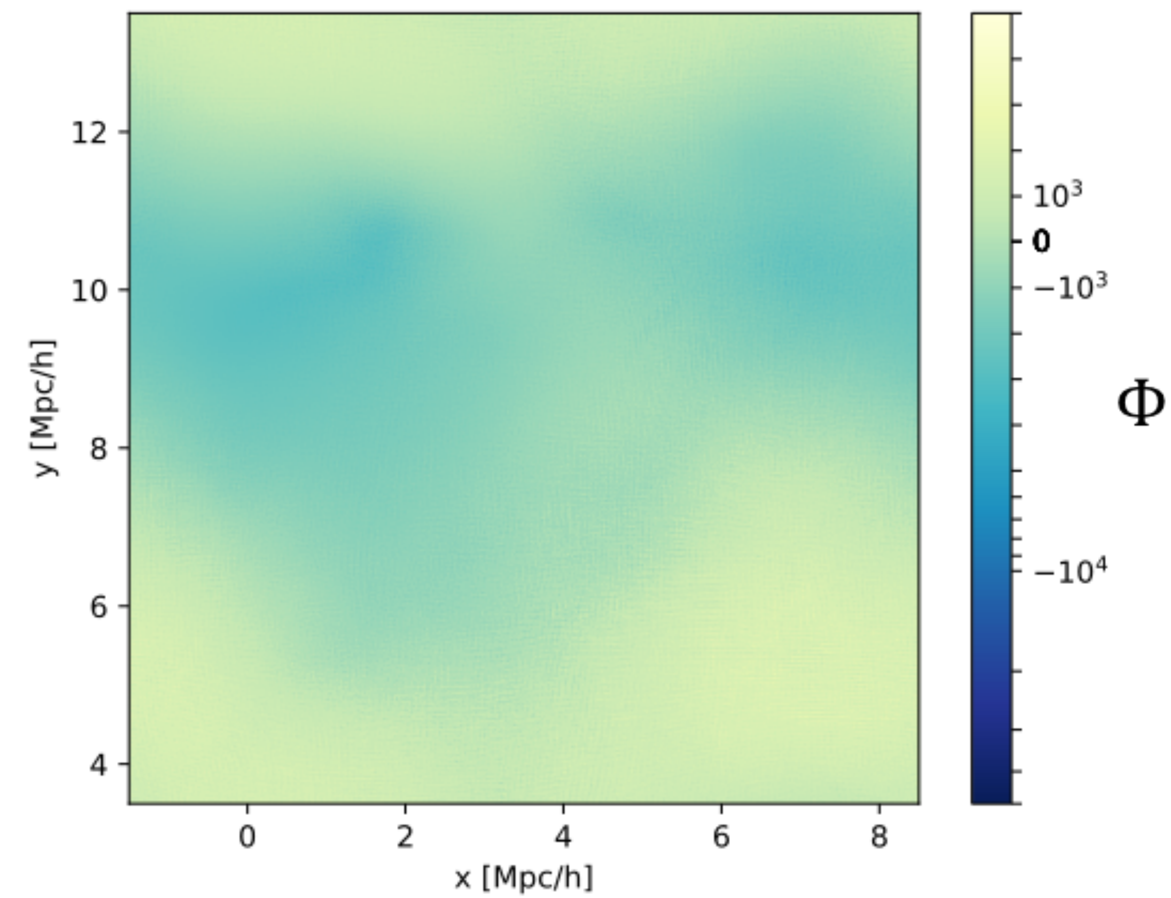


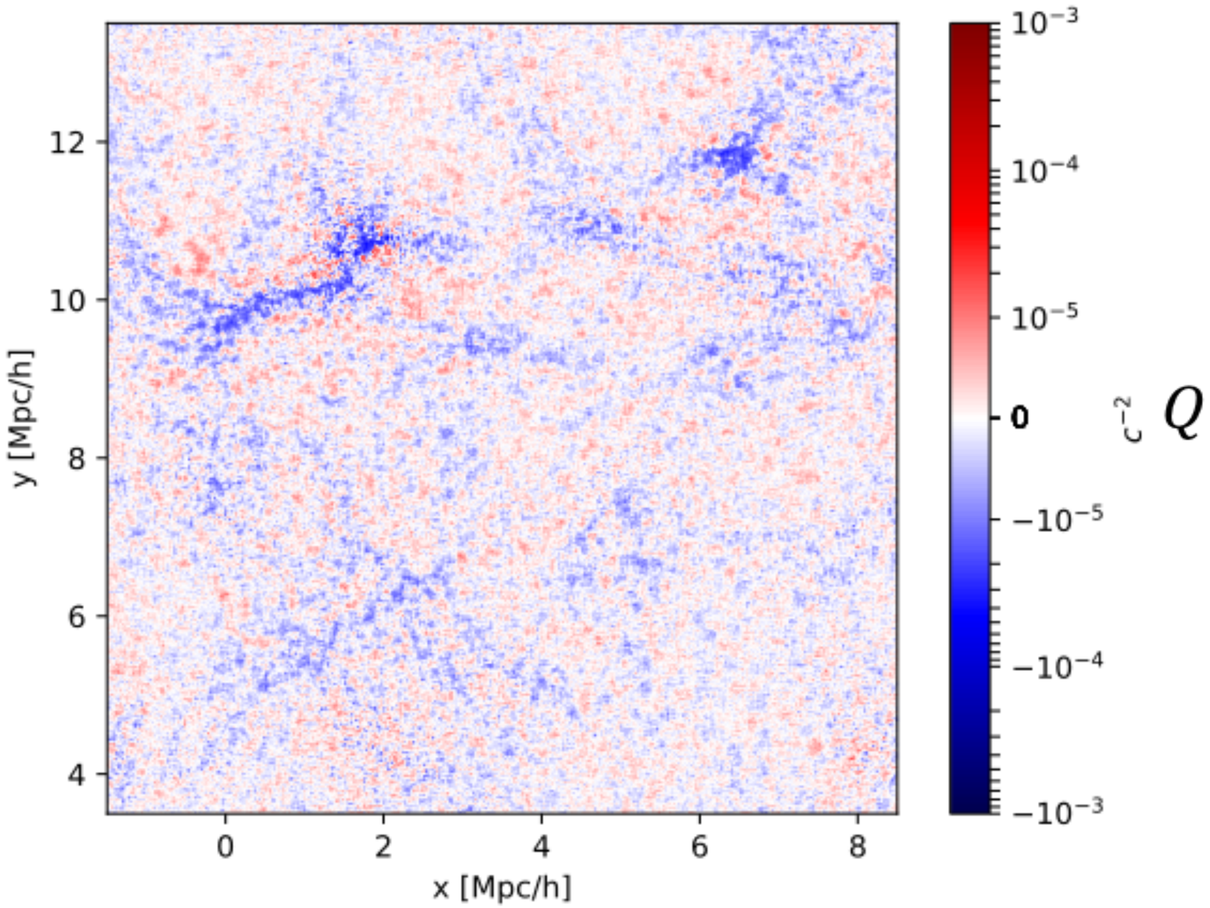


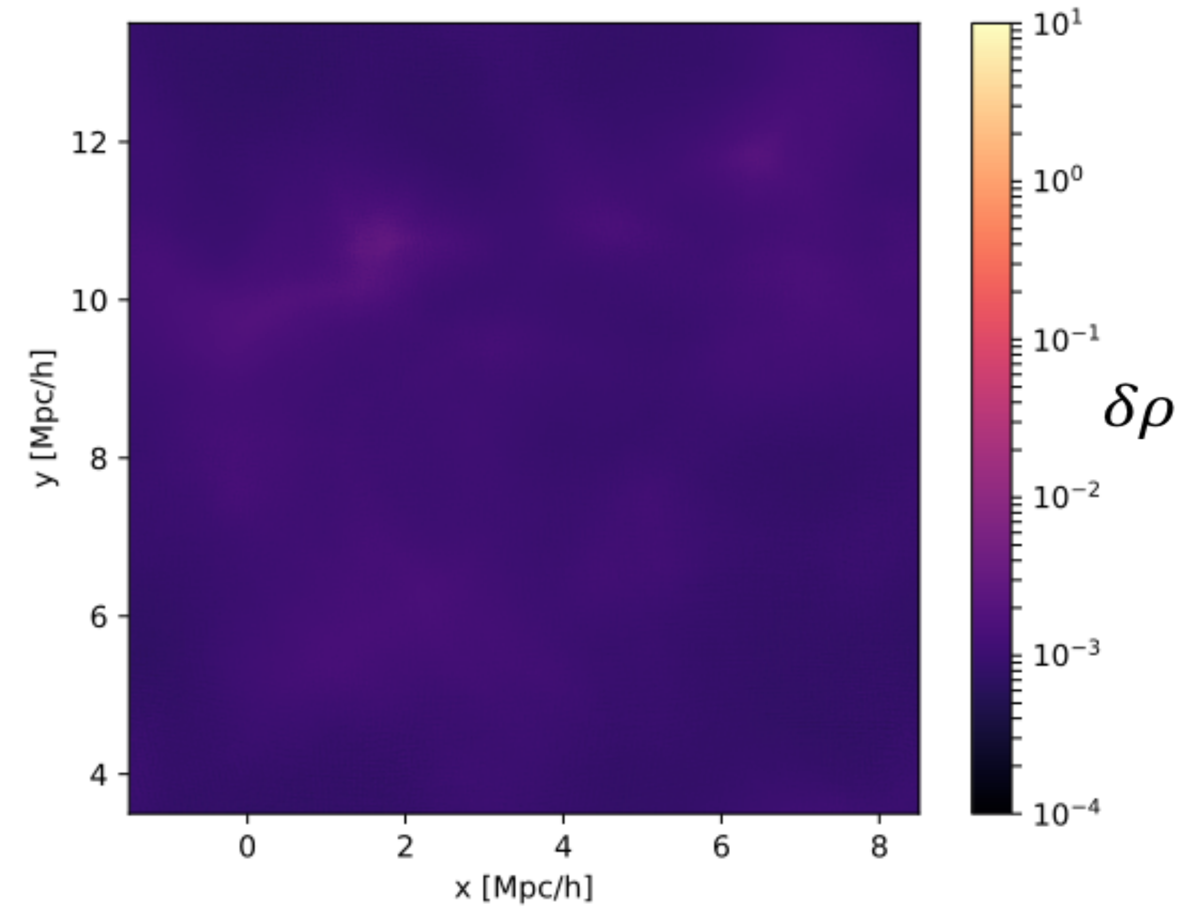
Simulation

10 Mpc side box
256^3 particles

$z = 9$



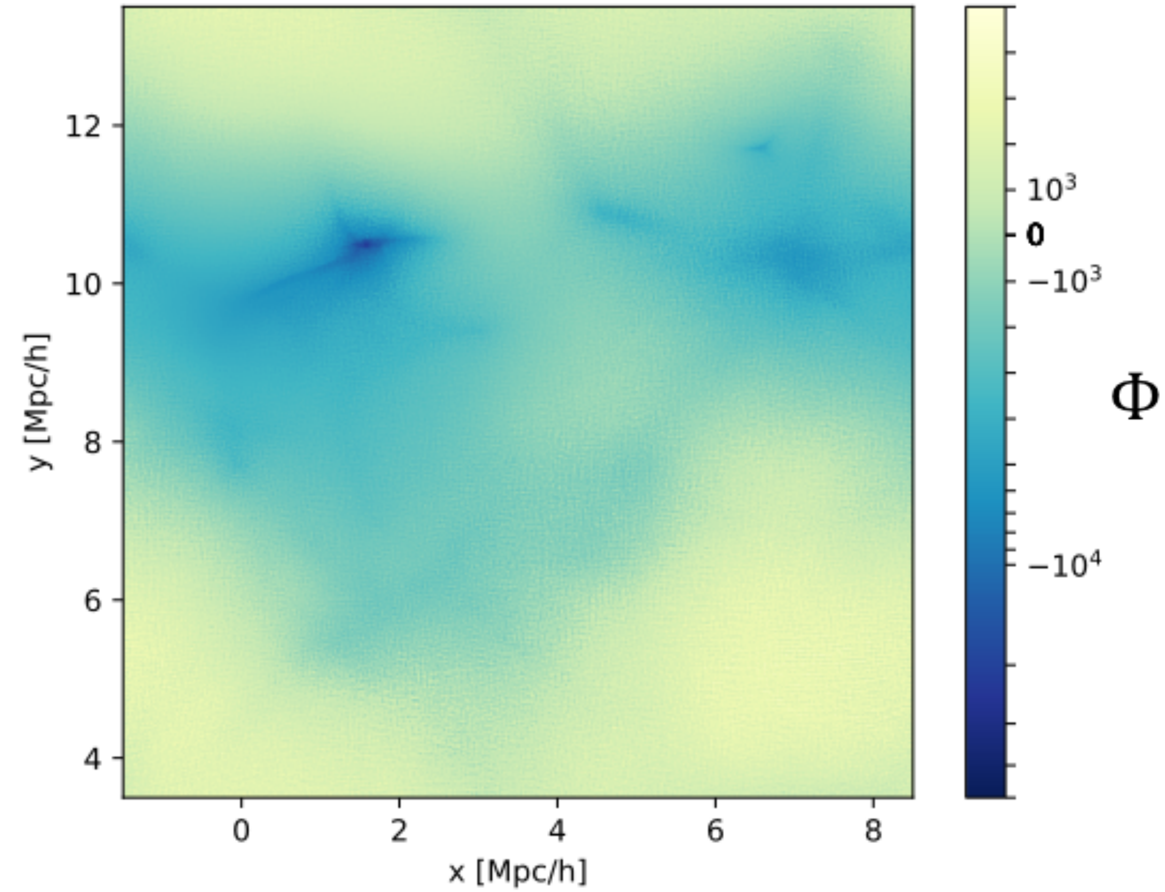


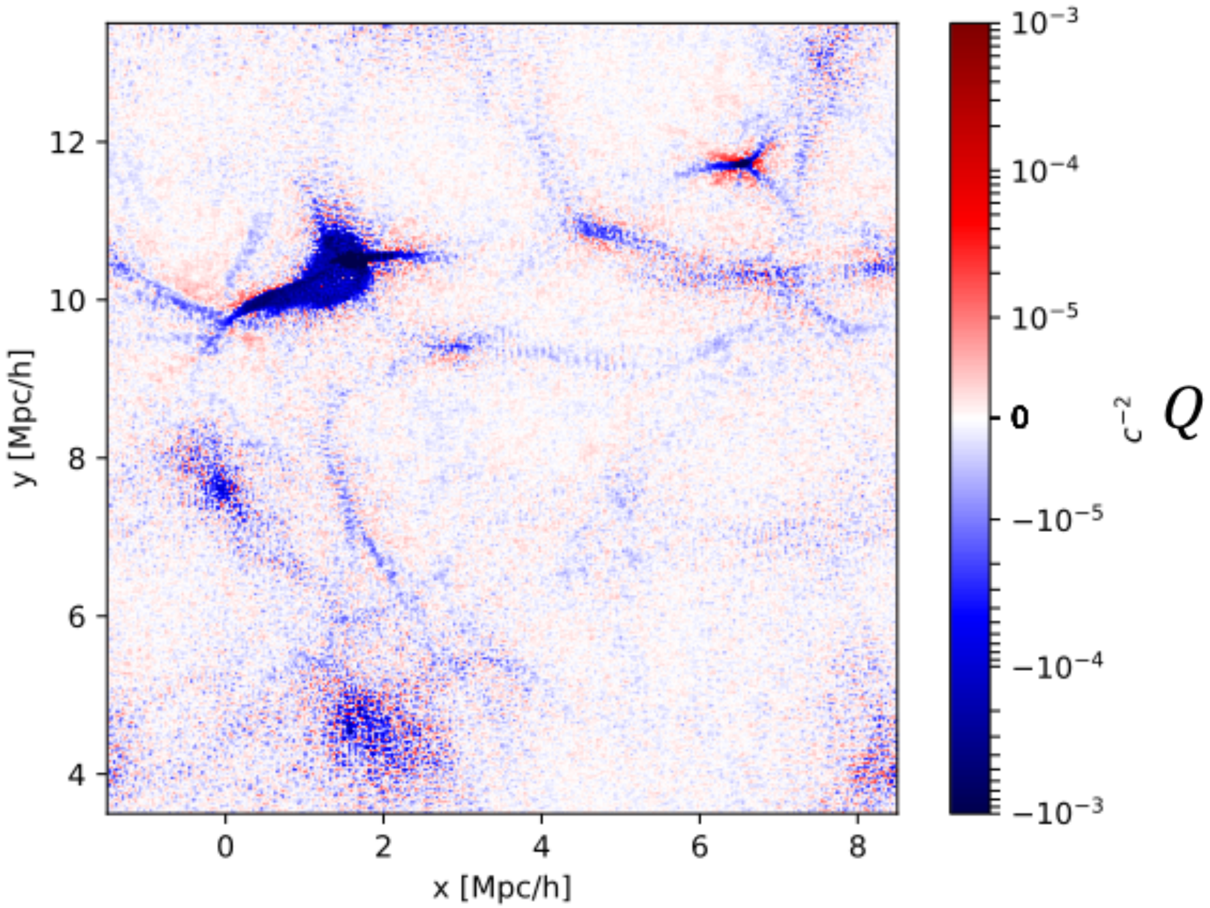


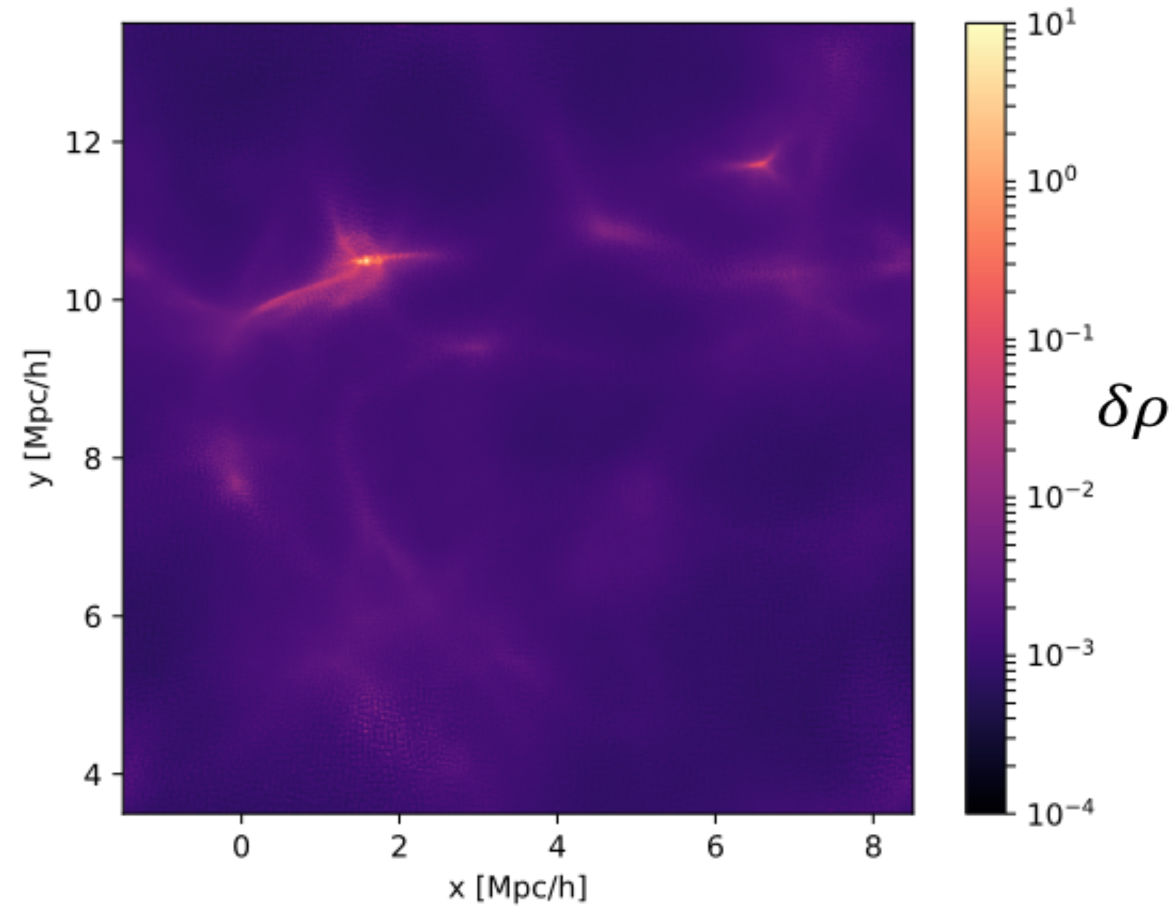
Simulation

$10\ Mpc$ side box
256^3 particles

$z = 5$



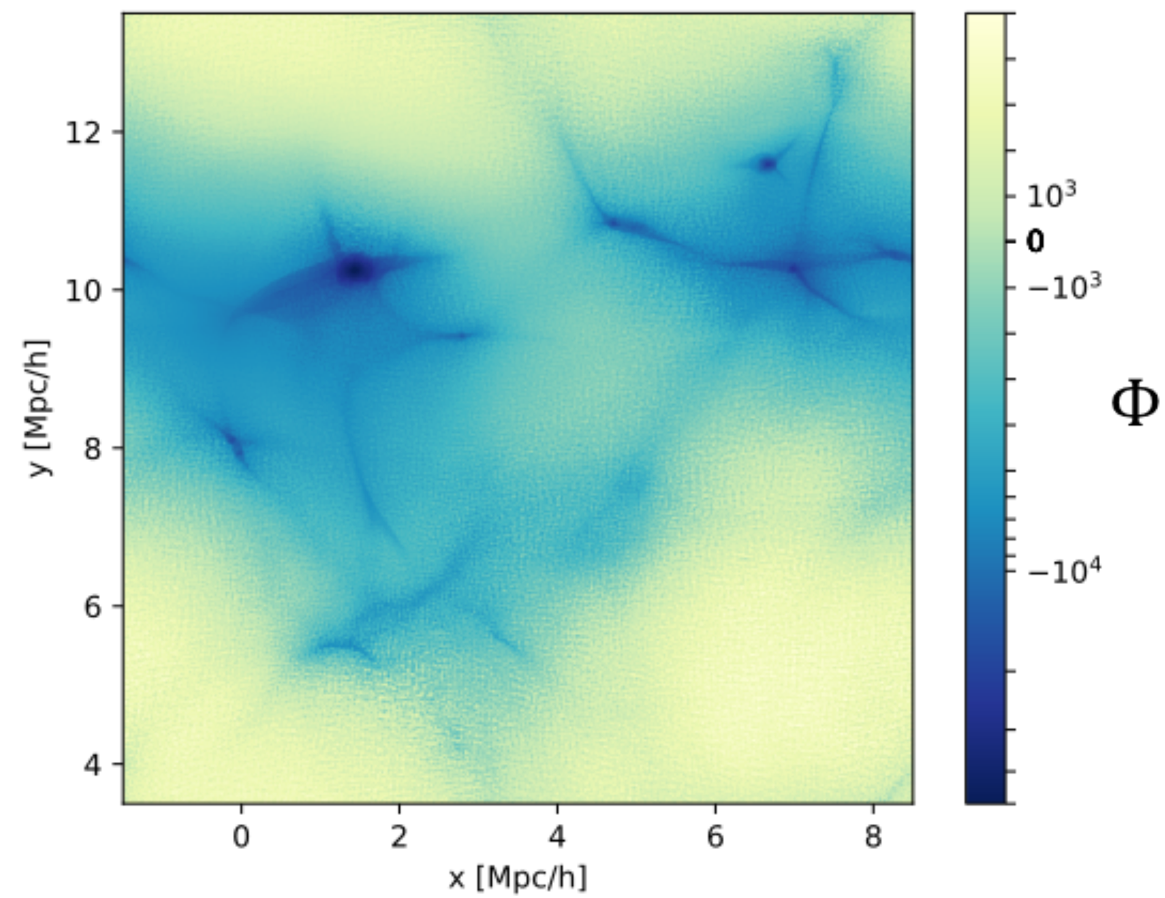


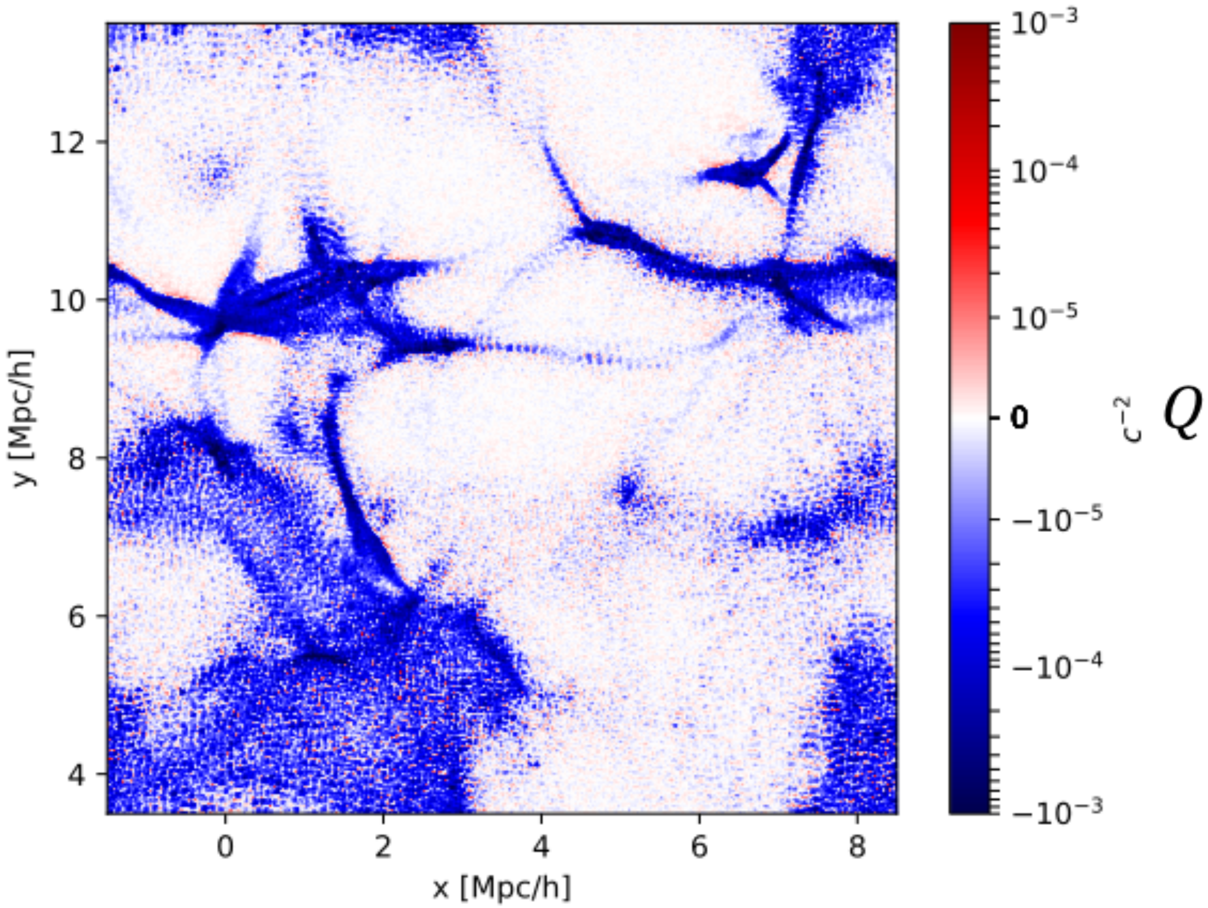


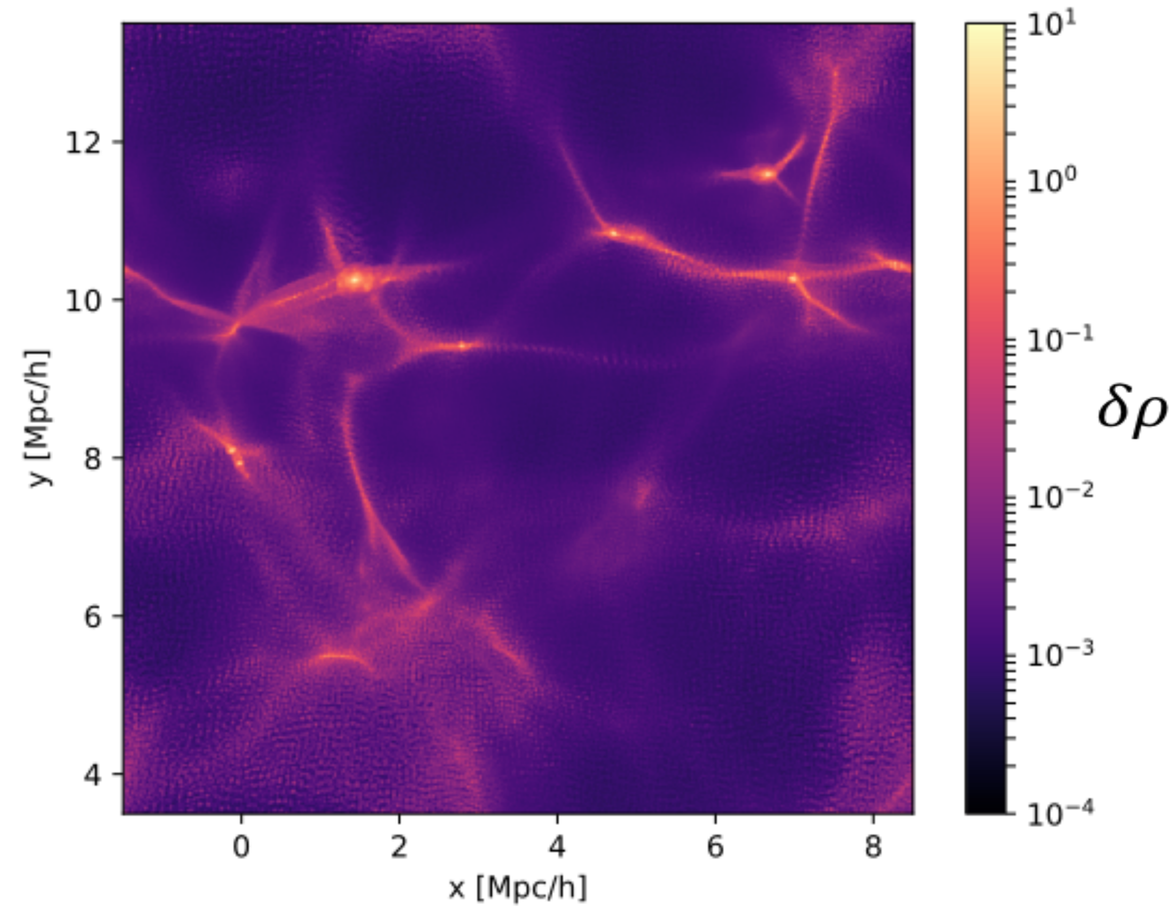
Simulation

$10\ Mpc$ side box
256^3 particles

$z = 3$



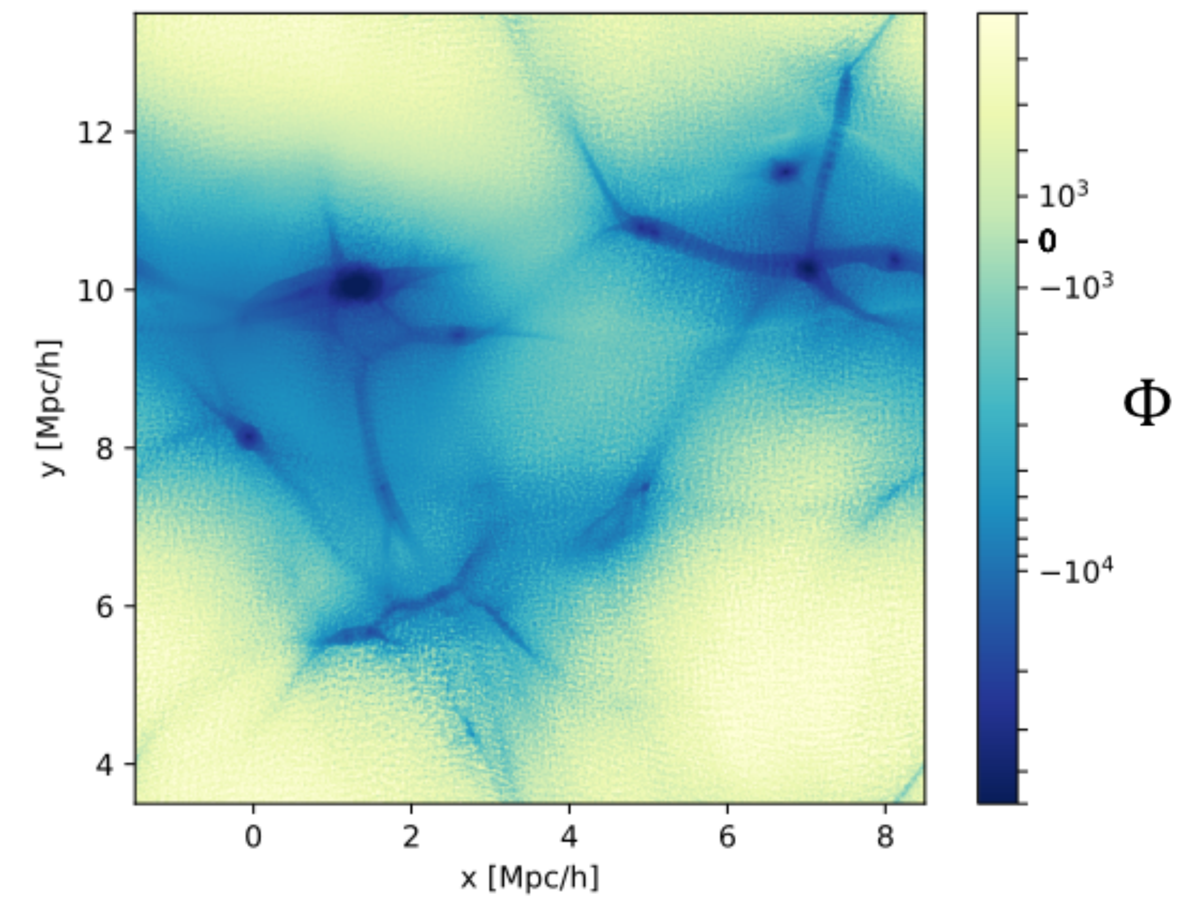


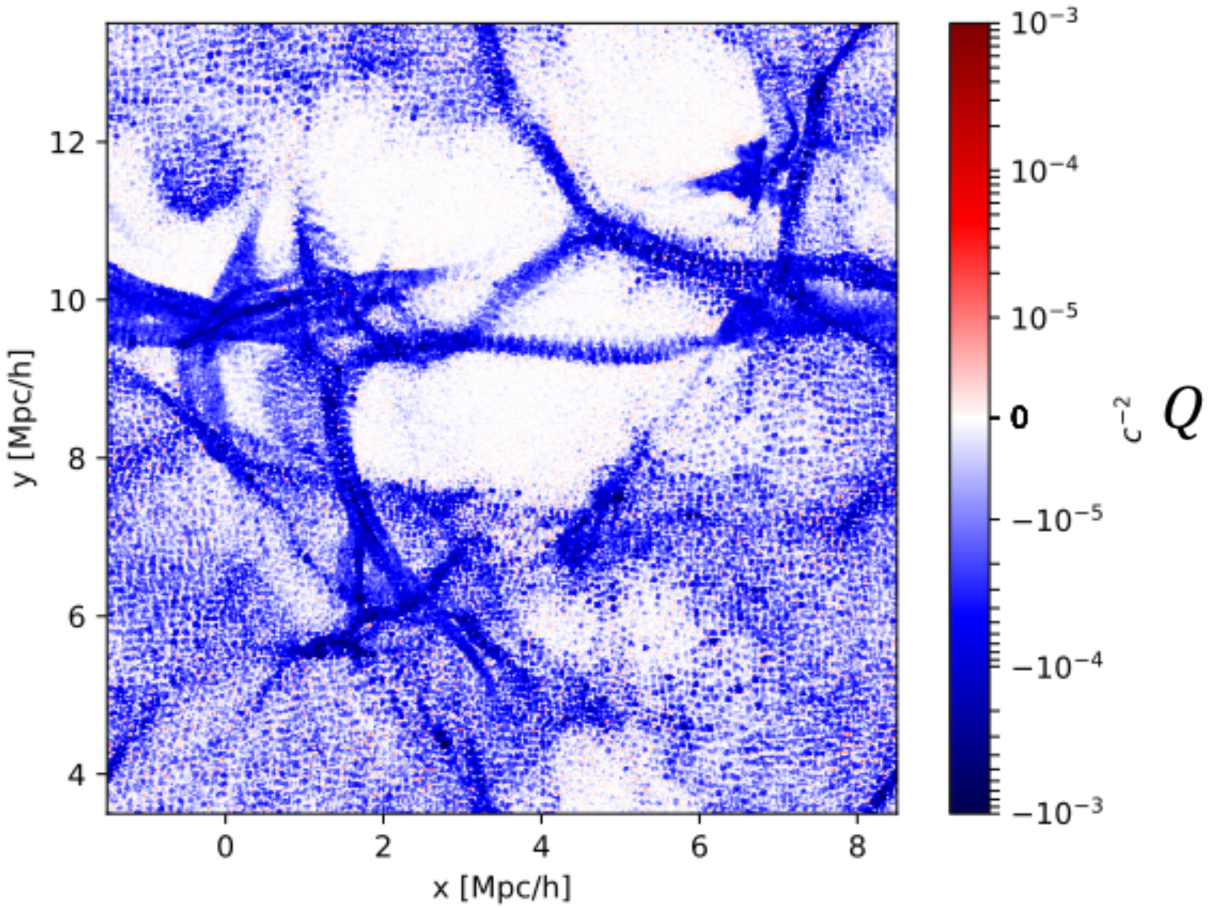


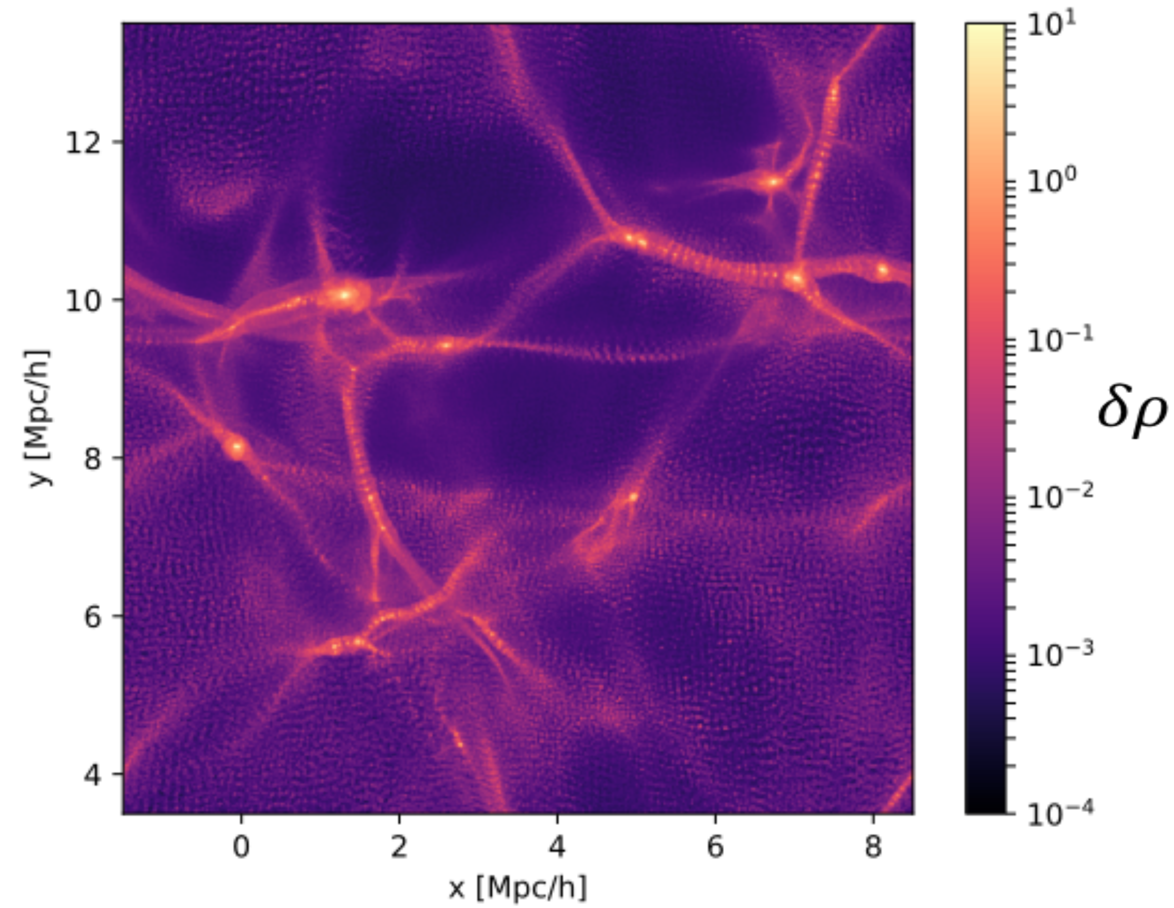
Simulation

$10\ Mpc$ side box
256^3 particles

$z = 2$



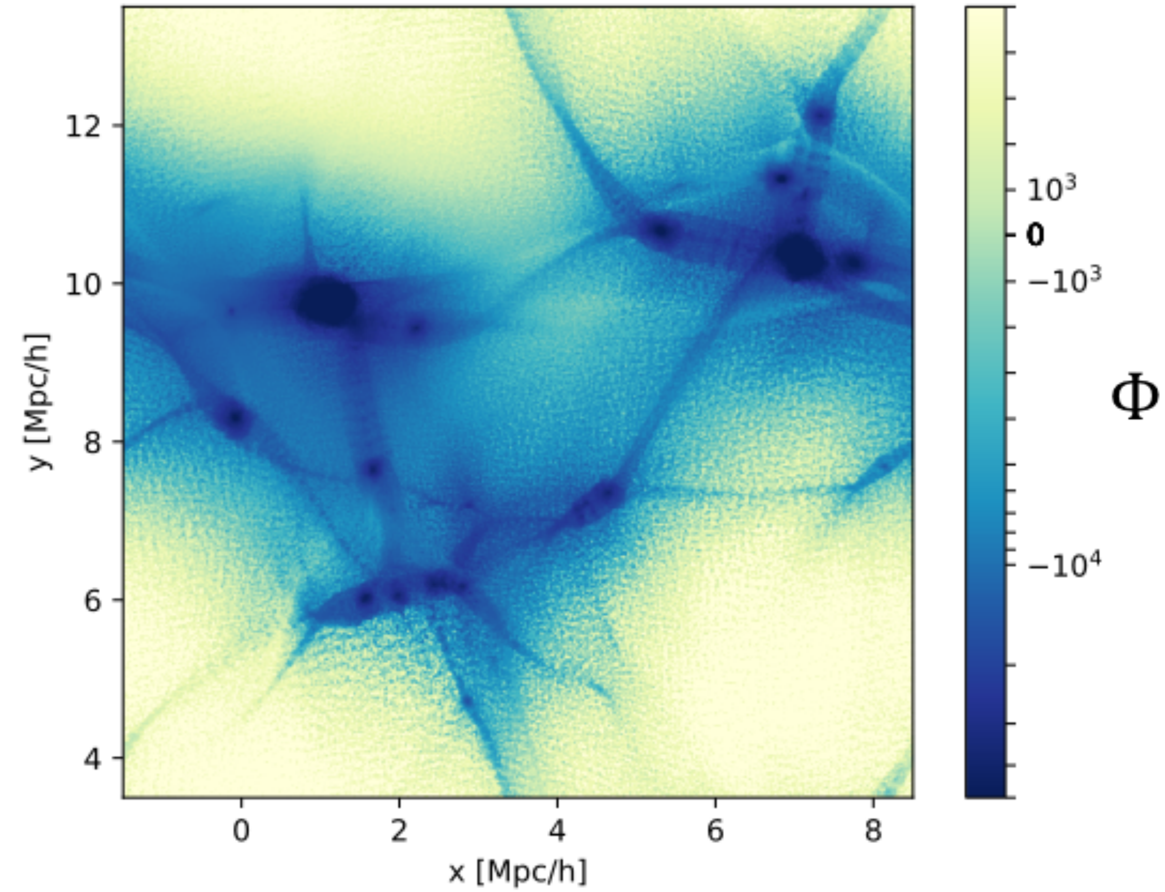


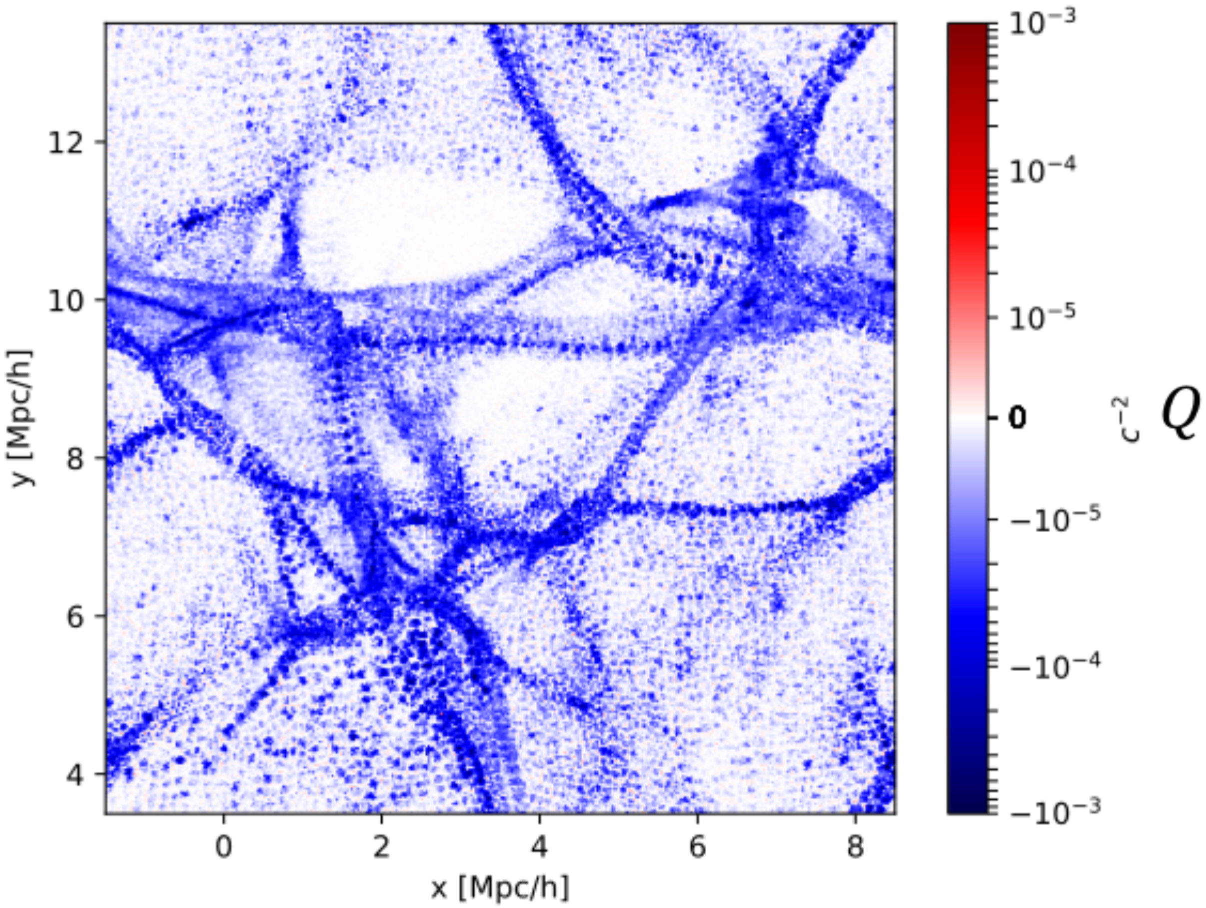


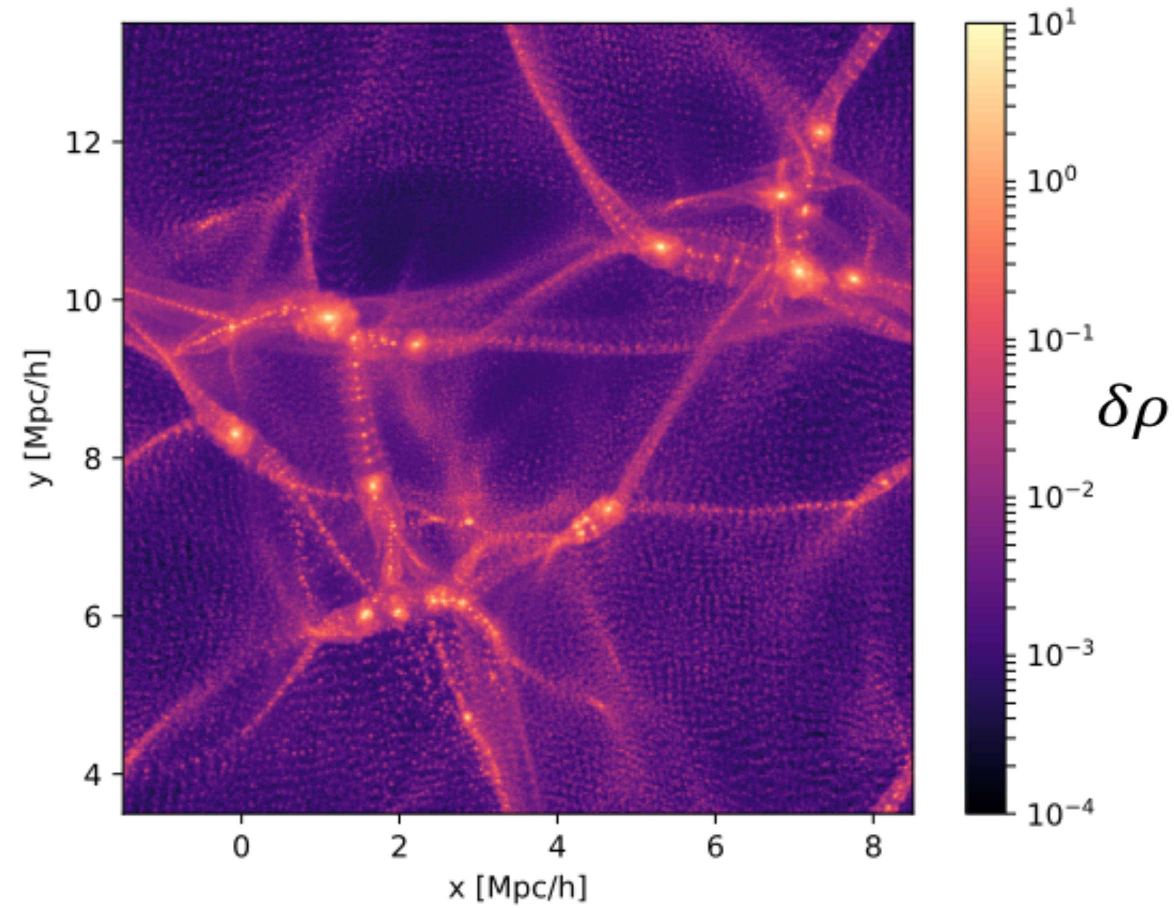
Simulation

$10\ Mpc$ side box
256^3 particles

$z = 1$



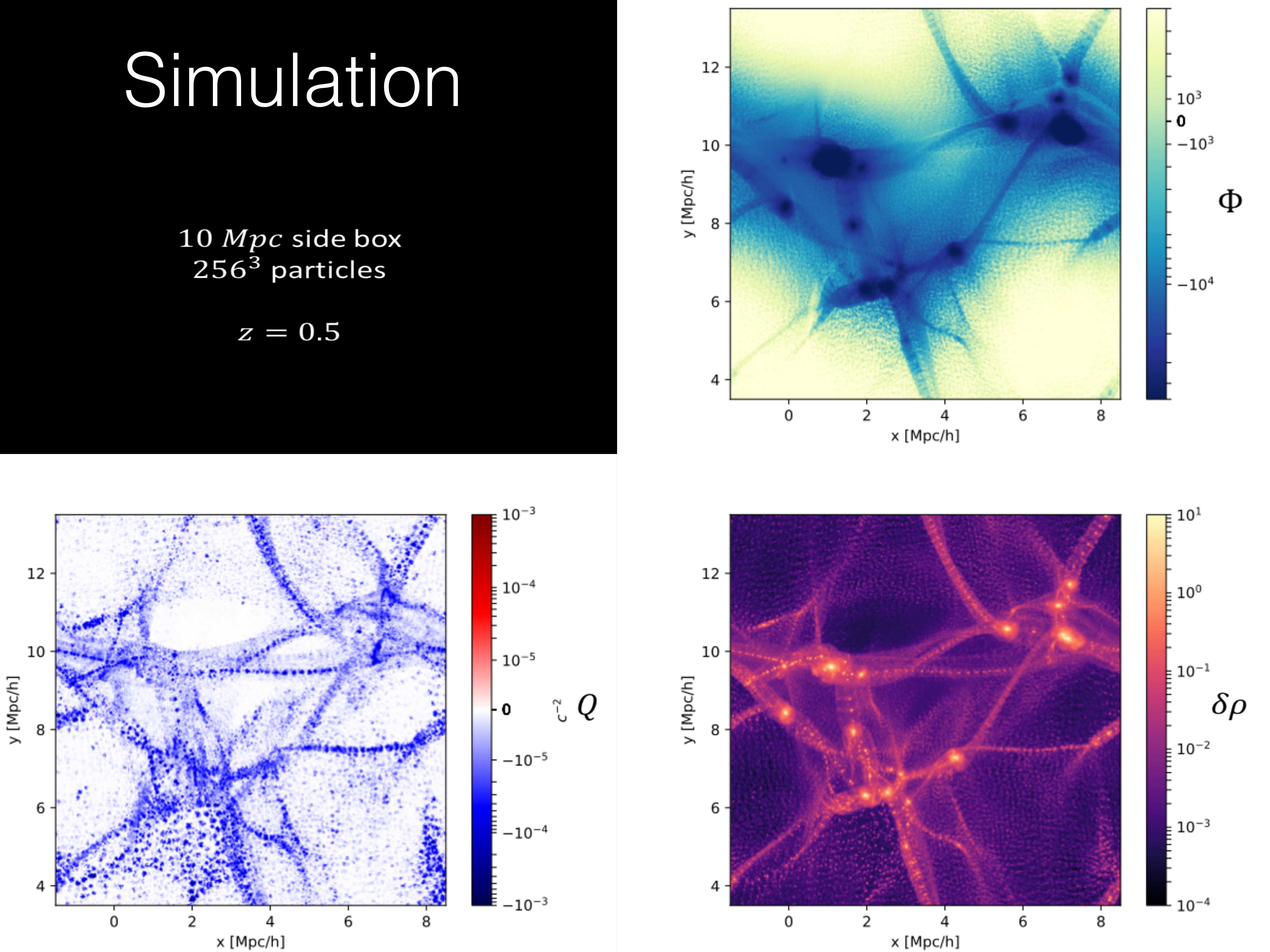


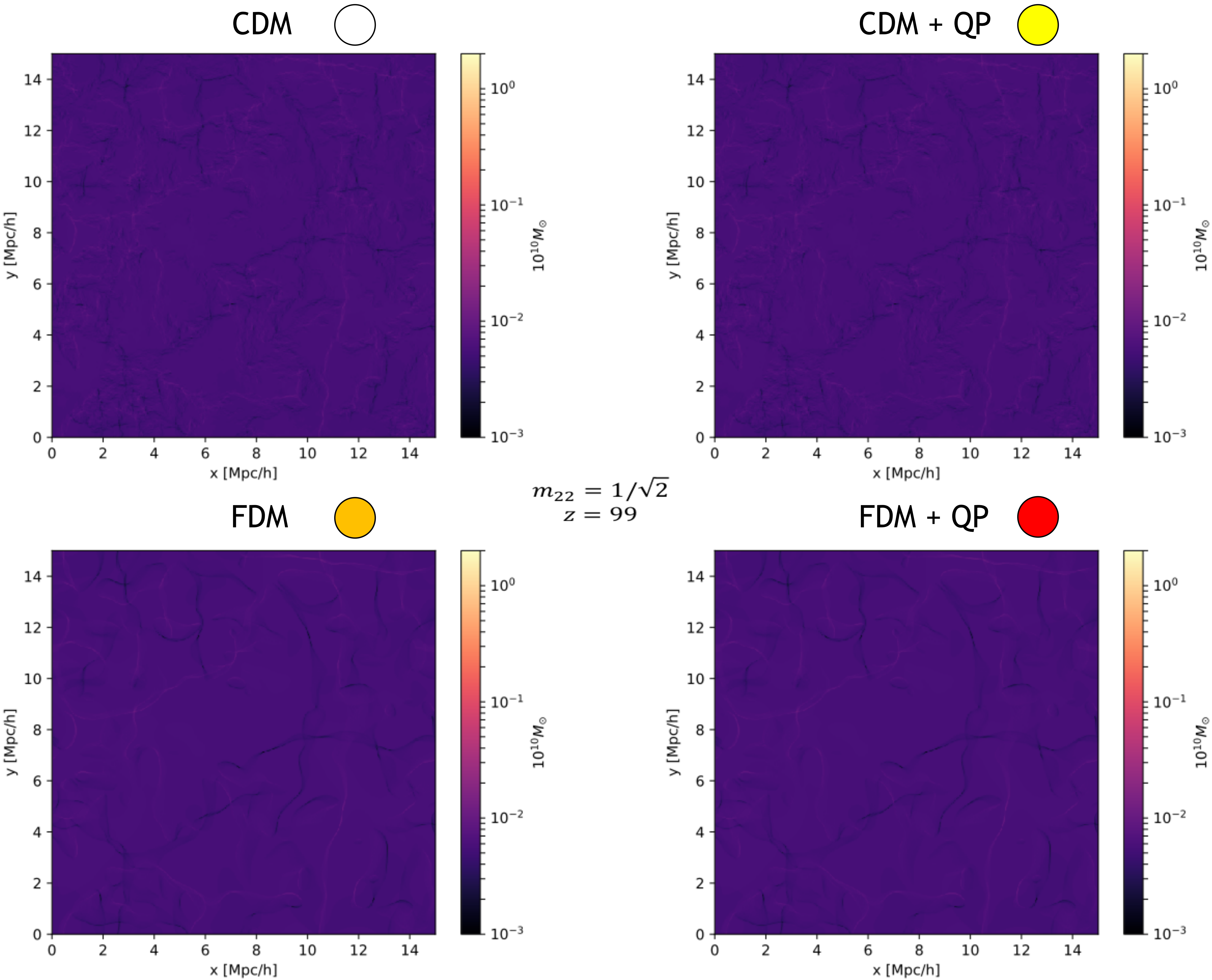


Simulation

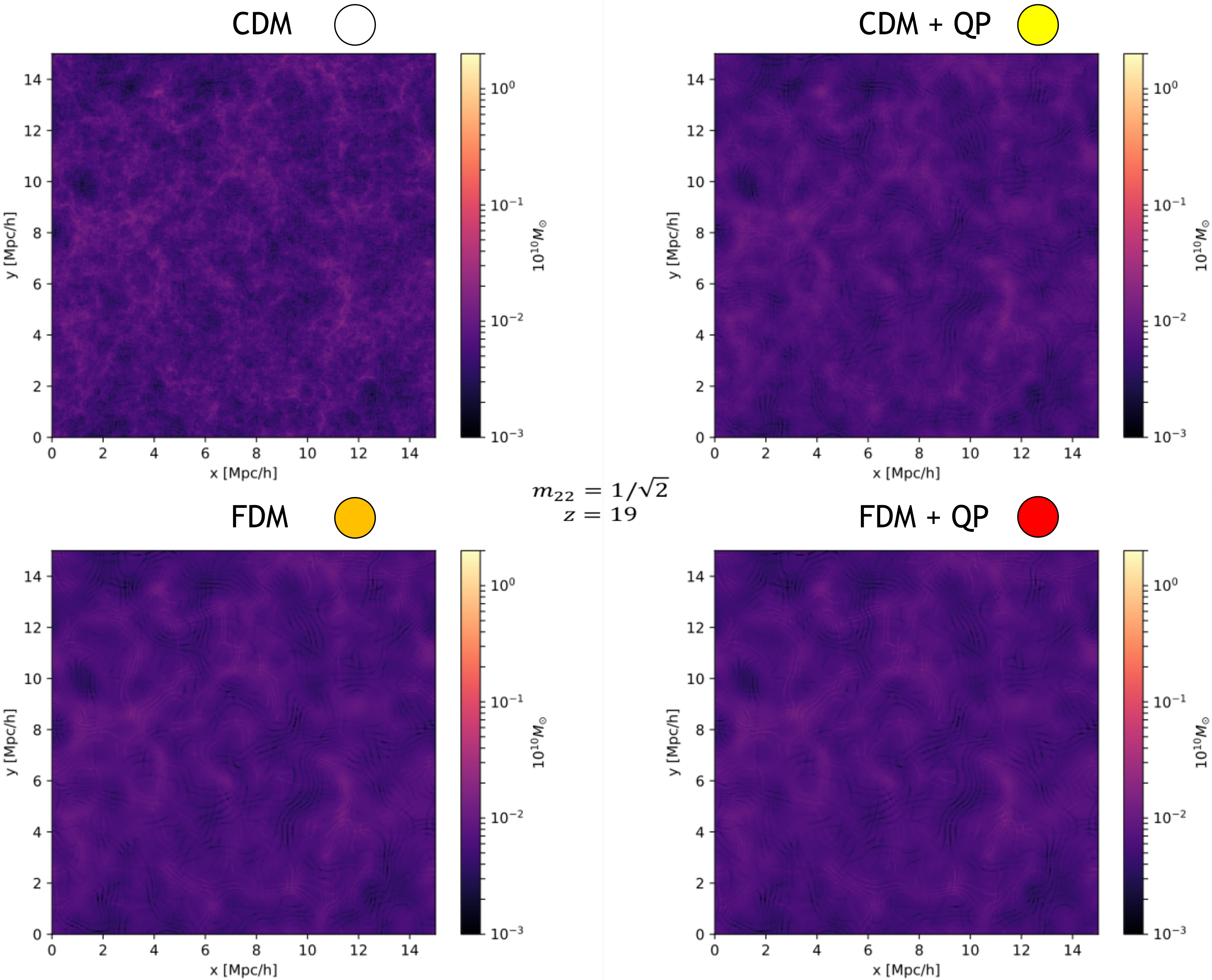
10 Mpc side box
256^3 particles

$z = 0.5$

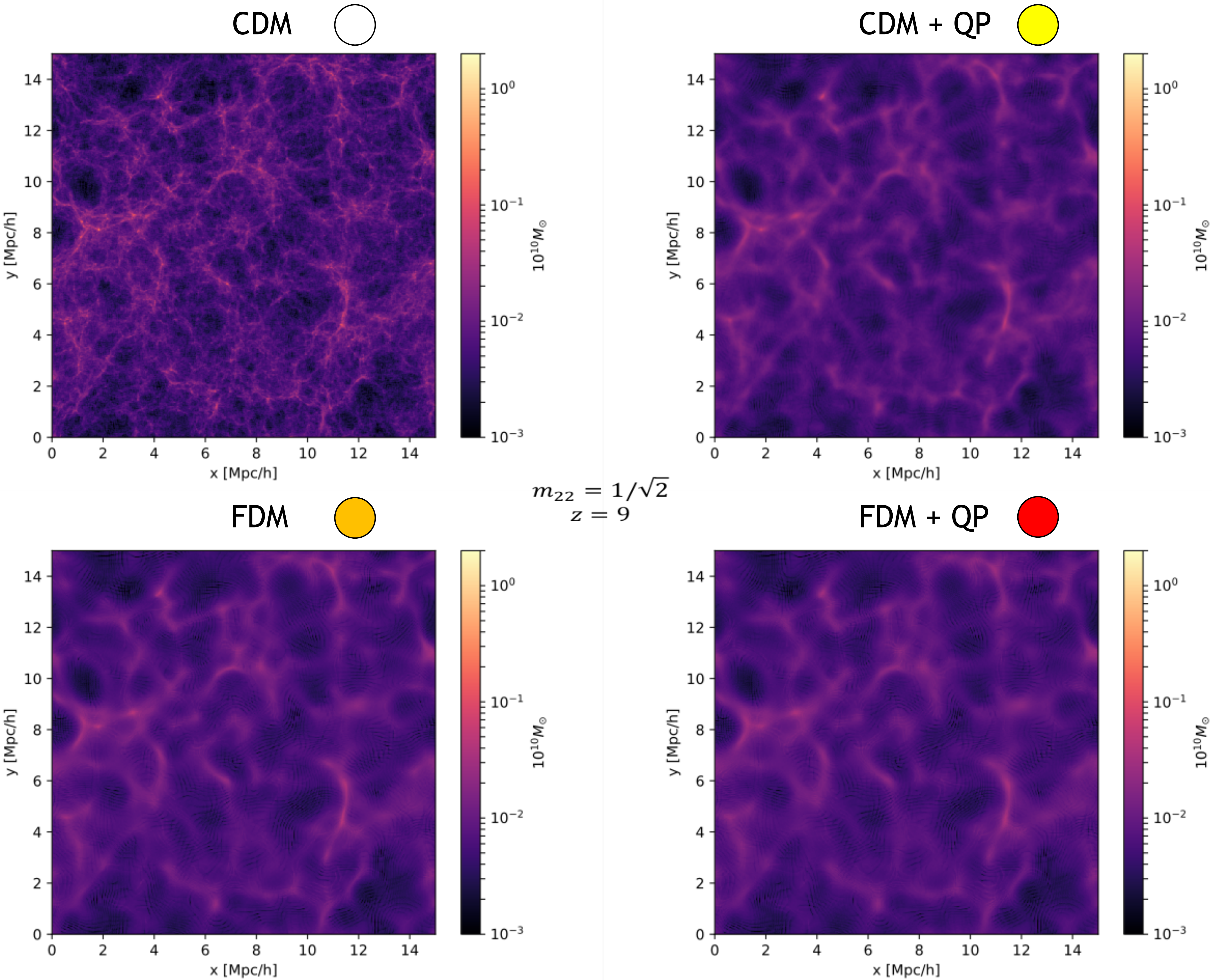




CDM
CDM + QP
FDM
FDM + QP
$m_{22} = 1/\sqrt{2}$
$z = 99$
x [Mpc/h]
y [Mpc/h]
$10^{10} M_\odot$
10^{0}
10^{-1}
10^{-2}
10^{-3}



CDM
CDM + QP
FDM
FDM + QP
$m_{22} = 1/\sqrt{2}$
$z = 19$
x [Mpc/h]
y [Mpc/h]
$10^{10} M_\odot$
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
2
4
6
8
10
12
14



CDM
CDM + QP
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FDM + QP
$m_{22} = 1/\sqrt{2}$
$z = 9$
x [Mpc/h]
y [Mpc/h]
$10^{10} M_\odot$
10^{0}
10^{-1}
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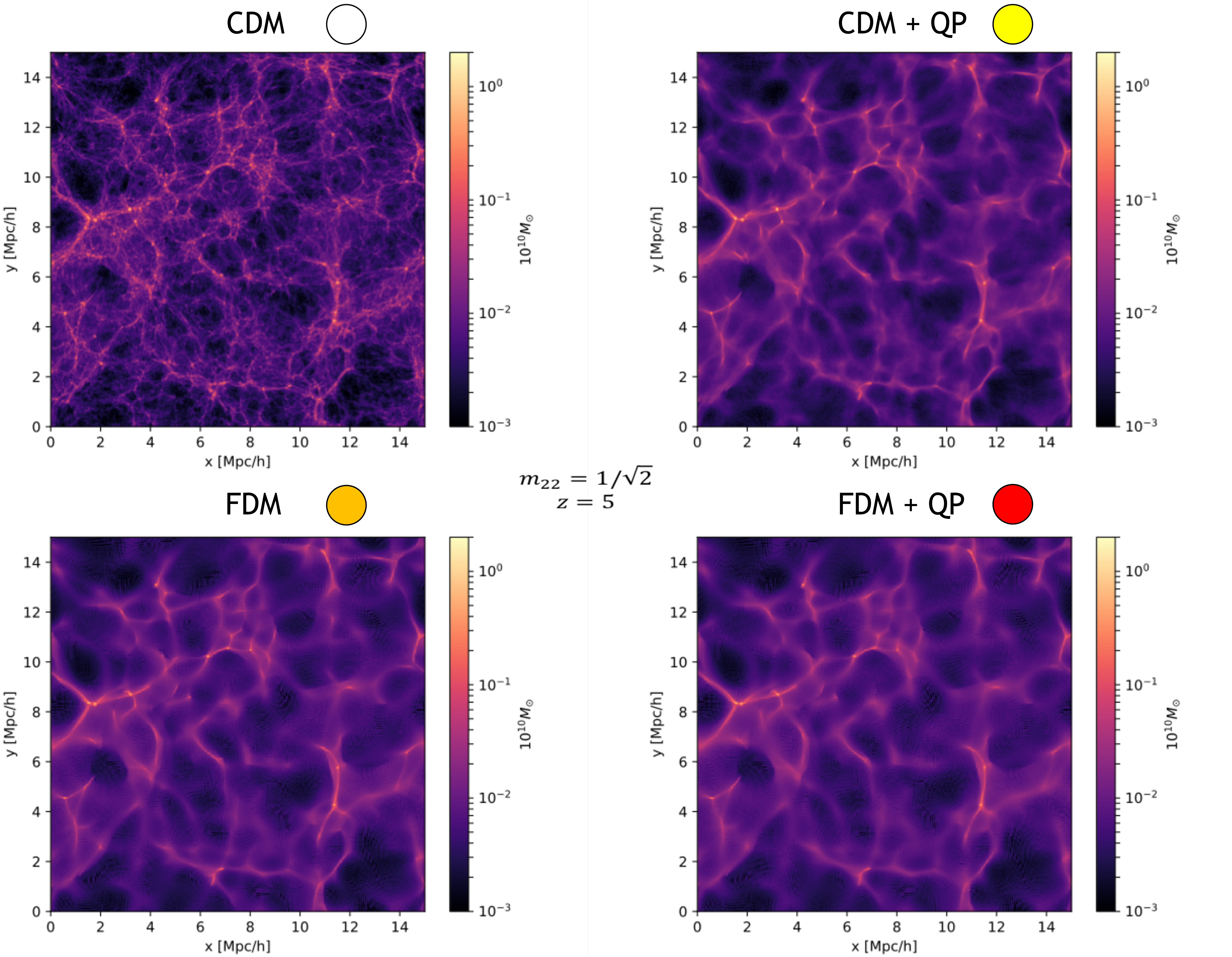
CDM
CDM + QP
FDM
FDM + QP
$m_{22} = 1/\sqrt{2}$
$z = 5$
x [Mpc/h]
y [Mpc/h]
$10^{10} M_{\odot}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
2
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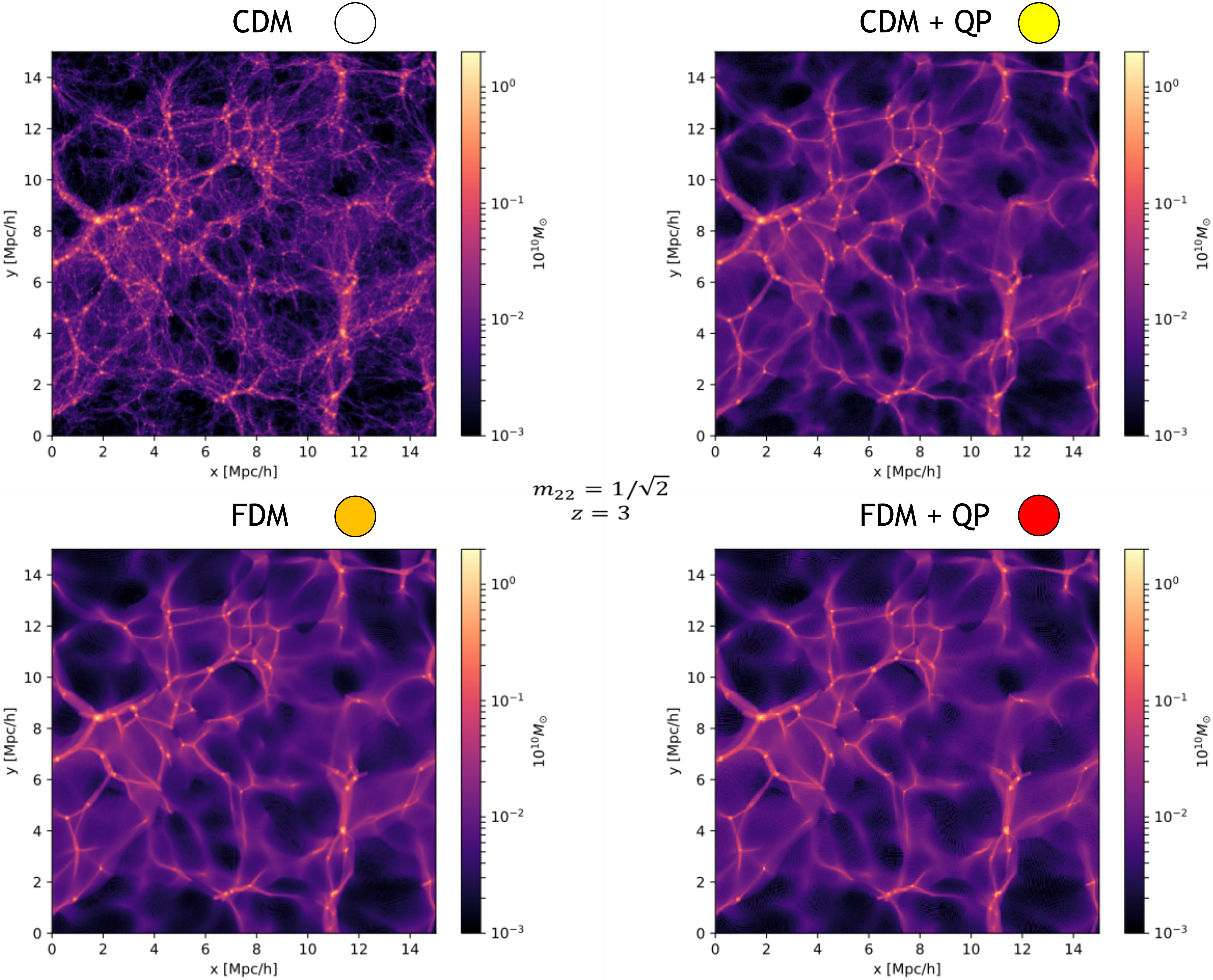
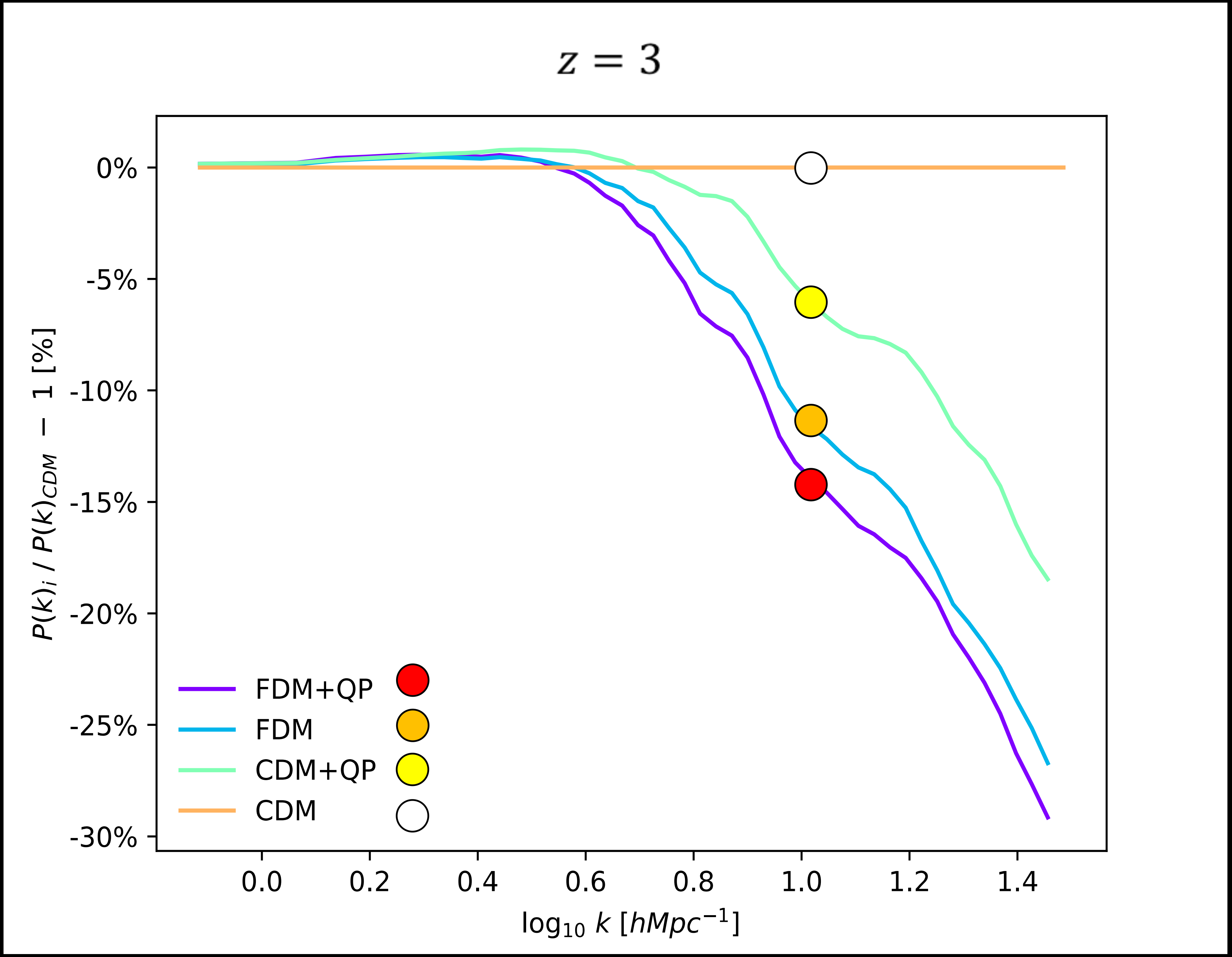
6
8
10
12
14


CDM
CDM + QP
FDM
FDM + QP
$m_{22} = 1/\sqrt{2}$
$z = 3$
y [Mpc/h]
x [Mpc/h]
$10^{10} M_\odot$
10^{0}
10^{-1}
10^{-2}
10^{-3}
0 2 4 6 8 10 12 14

Initial Conditions



CONCLUSIONS

The next era of Precision Cosmology **needs large and accurate N-body simulations** to test data analysis pipelines, to perform cosmological model selection, and to constrain cosmological parameters

Many competing models still on the market means many simulations to be performed, for many values of the related parameters… **computational cost is an issue.**

SUGGESTED READINGS

Cosmology and structure formation

A. Liddle, An introduction to modern cosmology (Wiley)
J. Peacock, Cosmological Physics (Cambridge University Press)
S. Weinberg, Gravitation and Cosmology (Wiley)
S. Dodelson, Modern Cosmology (Elsevier)

N-body simulations

R. Hockney and J. Eastwood, Computer simulations using particles (Taylor & Francis)
S. Aarseth, Gravitational N-body simulations (Cambridge)
V. Springel 2005, The cosmological simulation code GADGET-2 (MNRAS)
M. Kuhlen et al. 2012, Numerical Simulations of the Dark Universe (Phys. Dark Univ.)

Non-standard cosmological models

L. Amendola and S. Tsujikawa: Dark Energy, Theory and Observations (Cambridge)
The Euclid Theory WG Review, Amendola et al. 2013 (Living. Rev. Rel.)

Non-standard cosmological simulations

M. Baldi 2012, Dark Energy simulations (Phys. Dark Univ.)