Challenges and goals of Eulerian MHD in cosmology

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Overview

- Where/why do we need numerics in cosmology?
- Which methods are best?
- How do we compare results?
- Considerations to design a large simulation

Hydro simulations of large-scale structures

gravity → supersonic motions → shocks → cosmic rays

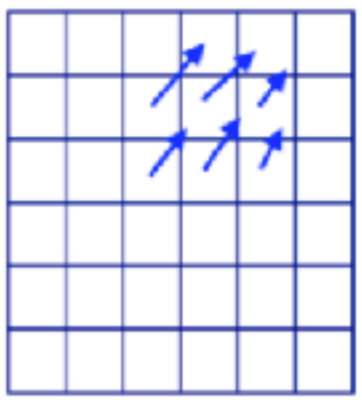
turbulence → B-fields

Gas modelling: do we discretise space or mass?

Eulerian

discretize space

representation on a mesh
(volume elements)



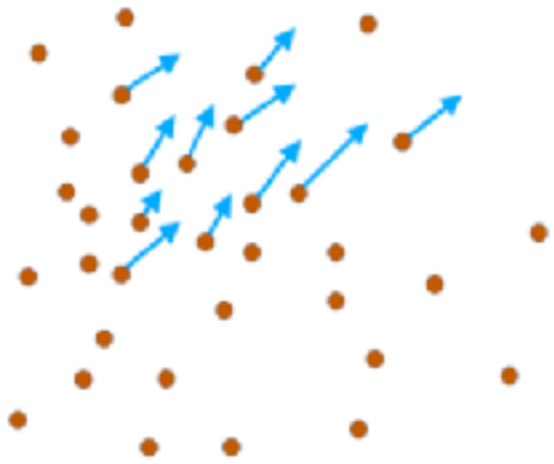
principle advantage:

high accuracy (shock capturing), low
numerical viscosity

Lagrangian

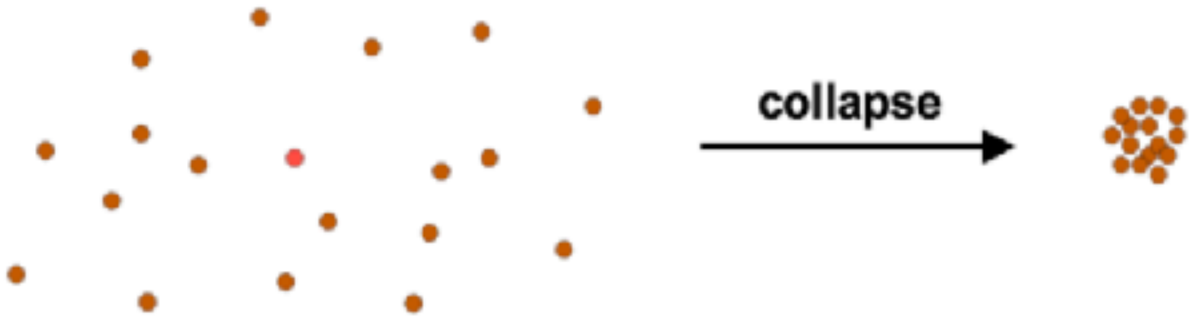
discretize mass

representation by fluid elements
(particles)



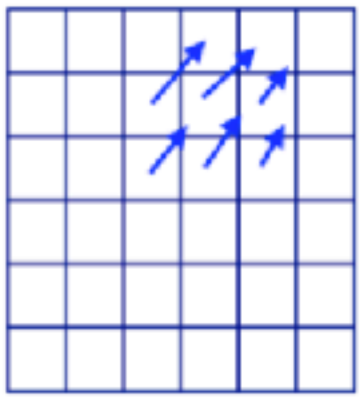
principle advantage:

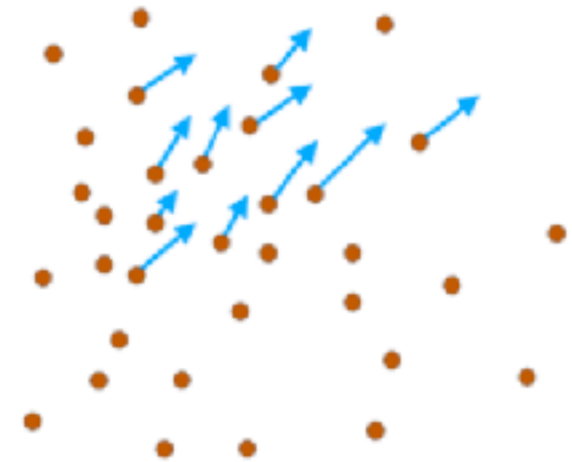
resolutions adjusts
automatically to the flow



Credits: V.Springel

EULERIAN VS LAGRANGIAN





$$\frac{\partial \varrho}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x_j}(\varrho v_j) = 0\,,$$

$$\frac{\partial \varrho v_i}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x_j}(\varrho v_i v_j + p\delta_{i,j}) = -\frac{\dot{a}}{a}\varrho v_i - \frac{1}{a}\varrho\frac{\partial \phi}{\partial x_i}\,,$$

$$\rho_i = \sum_{j=1}^{N} m_j W(|\mathbf{r}_{ij}|, h_i)$$

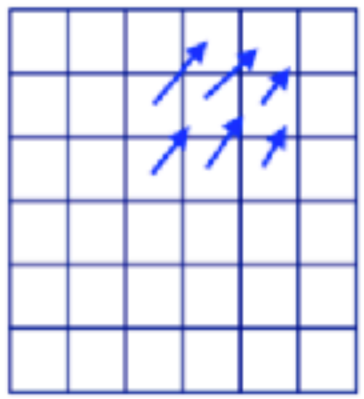
$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \nabla \Phi$$

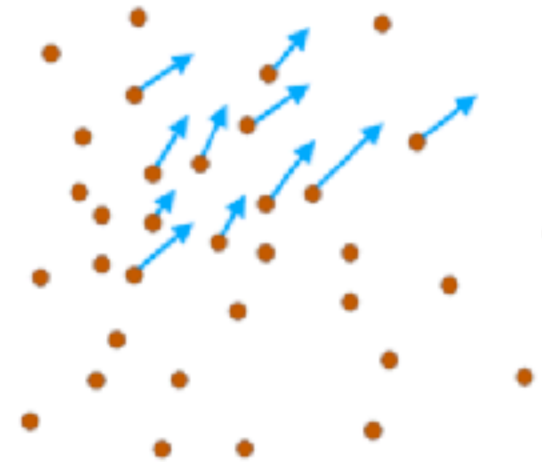
Credits: V.Springel

combine fluxes +
Riemann solver for discontinuities

continuity equation automatically satisfied
artificial viscosity to prevent contacts

EULERIAN VS LAGRANGIAN

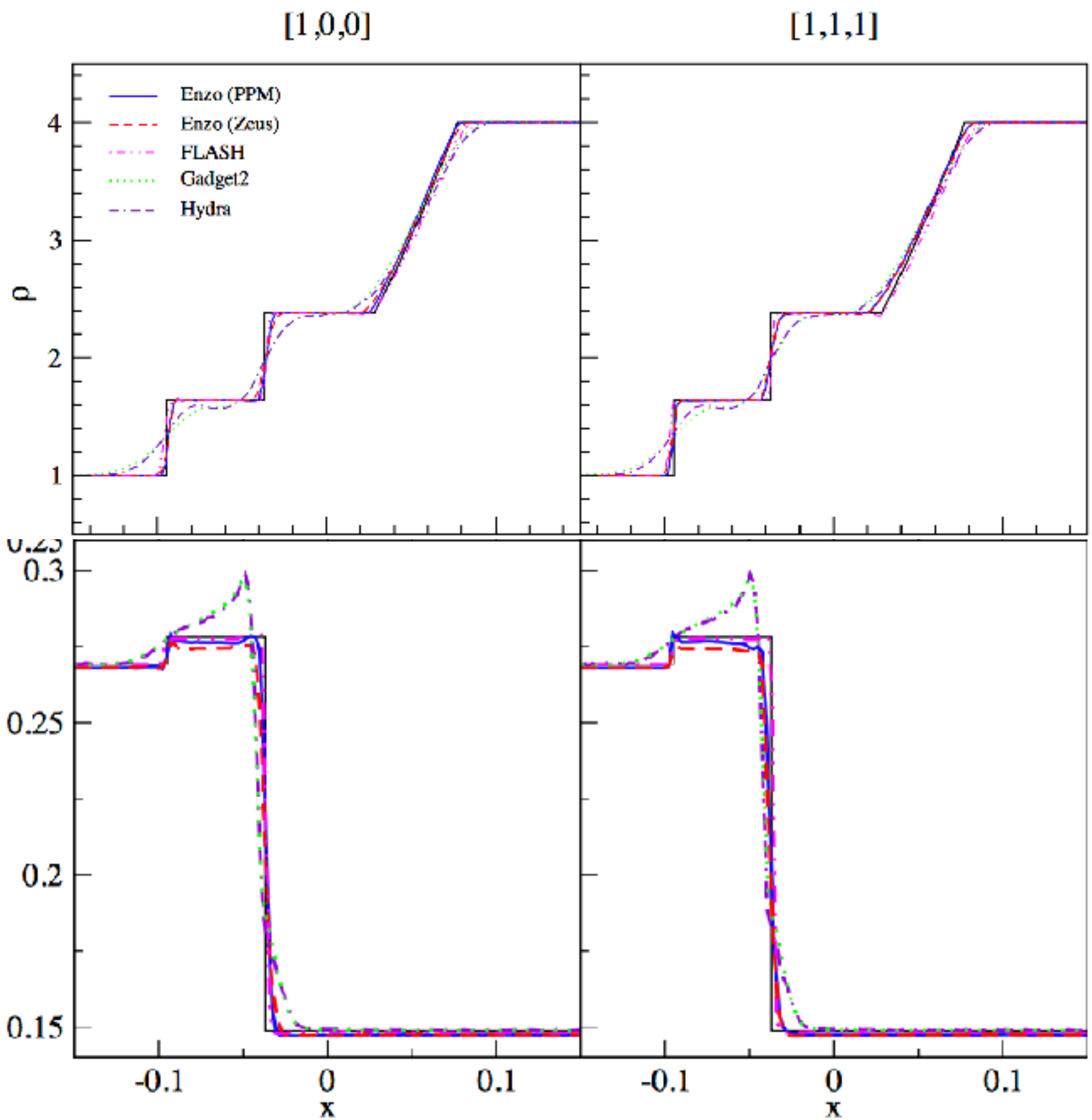




Some important criticalities:

- gravity leads to high density contrasts: $\delta\rho/\rho >> 1000$
- inward advection: things are injected at low density and later advected into high densities (e.g. cosmic rays)
- some phenomena emerge only with a fair sampling of space (e.g. dynamo, turbulent statistics)

Which method is best?



Tasker et al. 2009

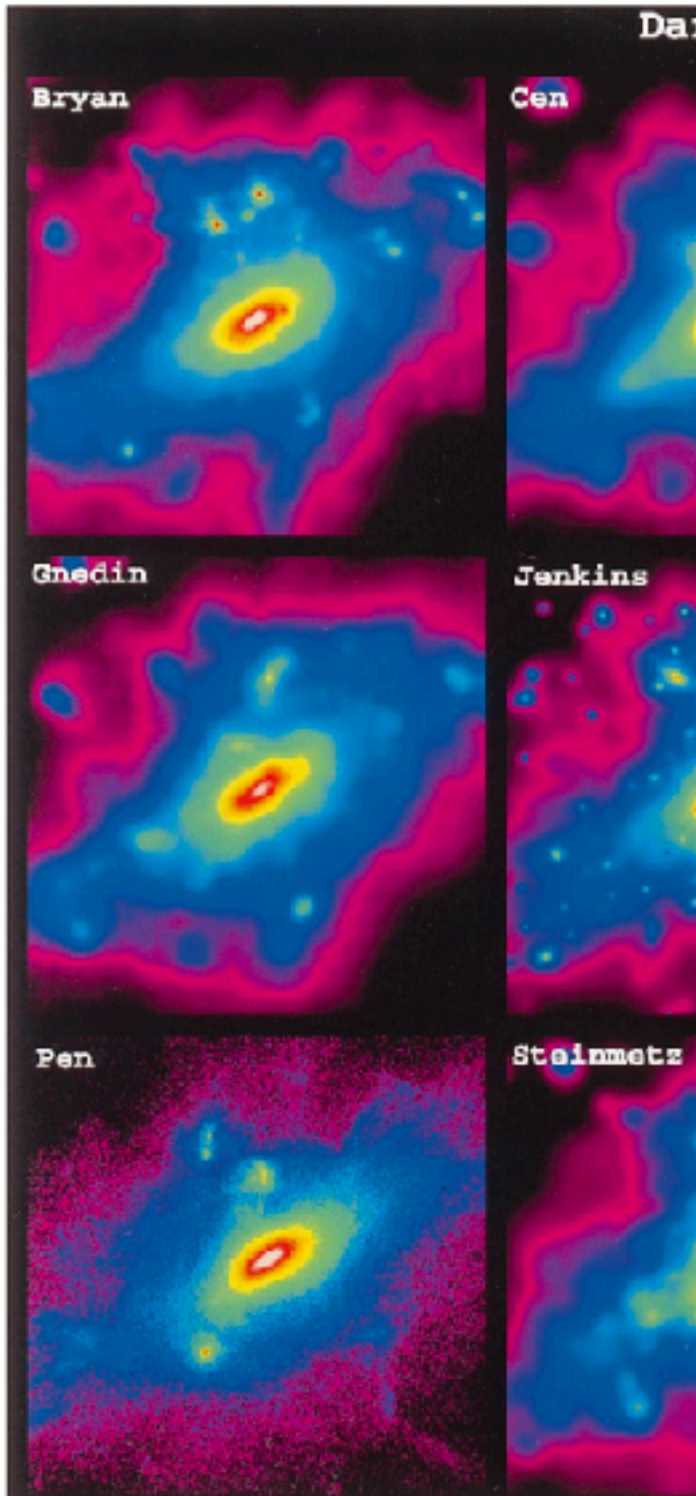
Parallels vs oblique shock tube

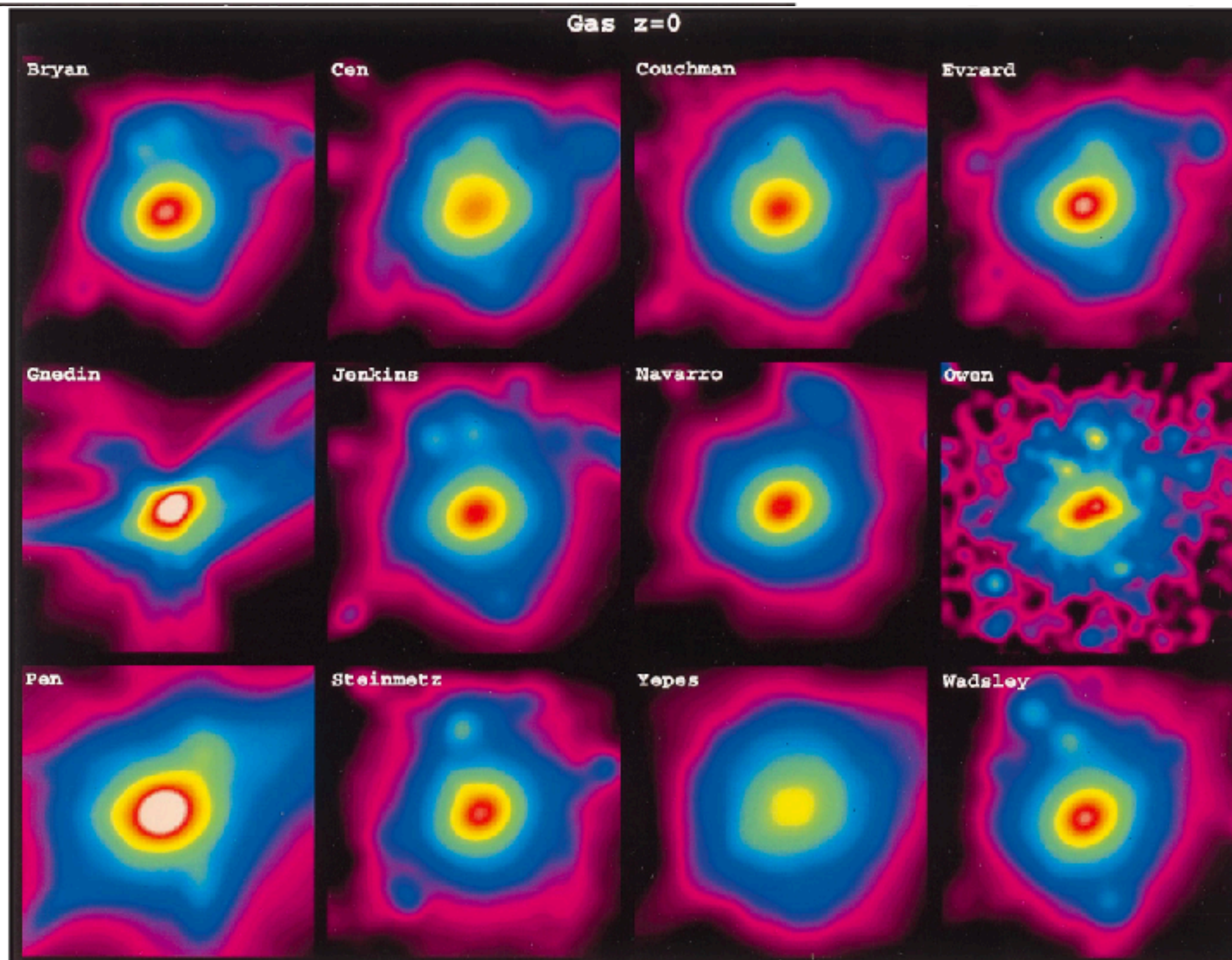
- spurious entropy generation at shock (SPH)

- dependence on shock alignment with axis (Grid)

Comparison between cosmological methods

THE SANTA BARBARA CLUSTER COMPARISON PROJECT: A COMPARISON OF COSMOLOGICAL HYDRODYNAMICS SOLUTIONS

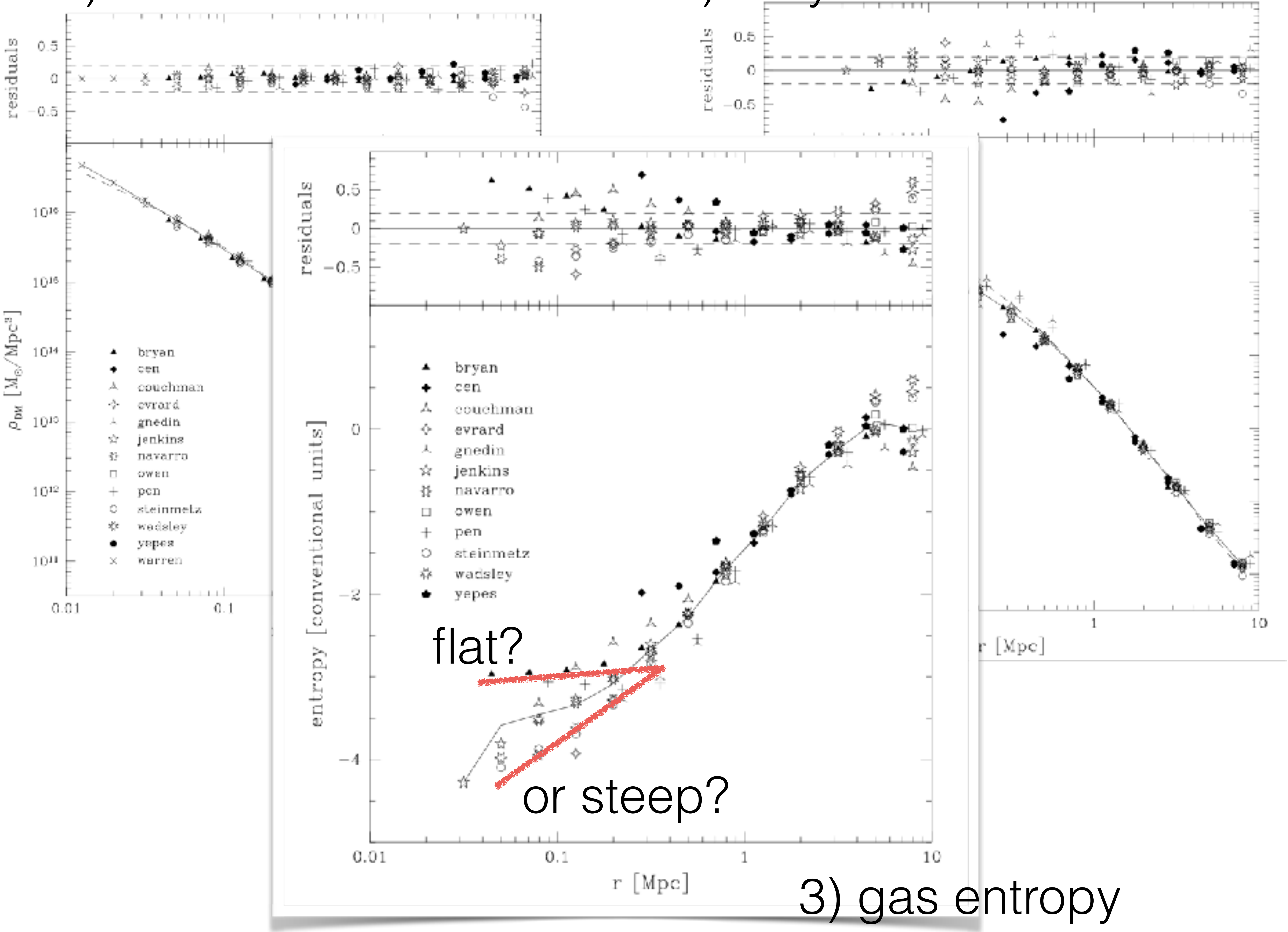




radial profiles of:

1) dark matter

2) baryonic matter



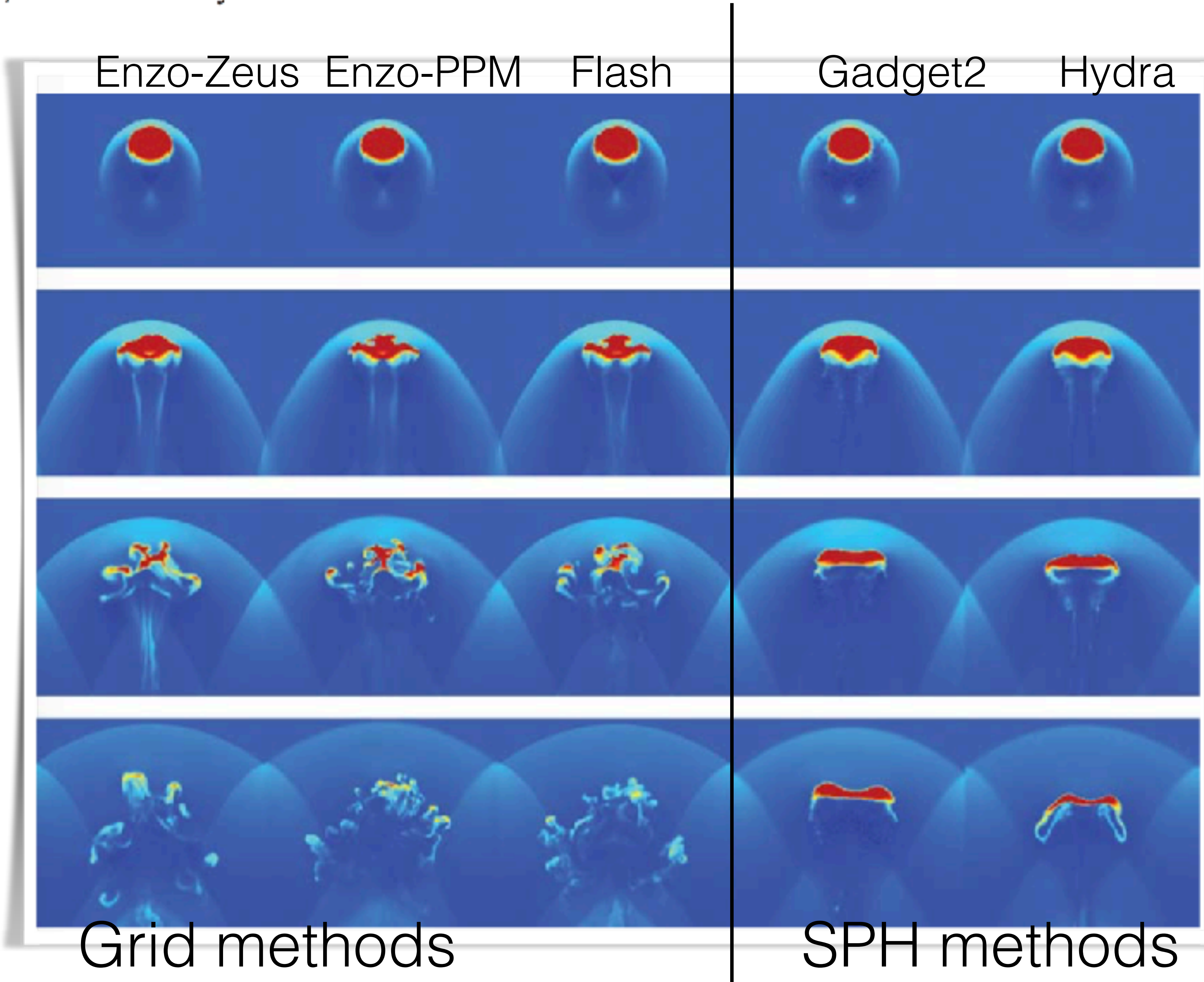
3) gas entropy

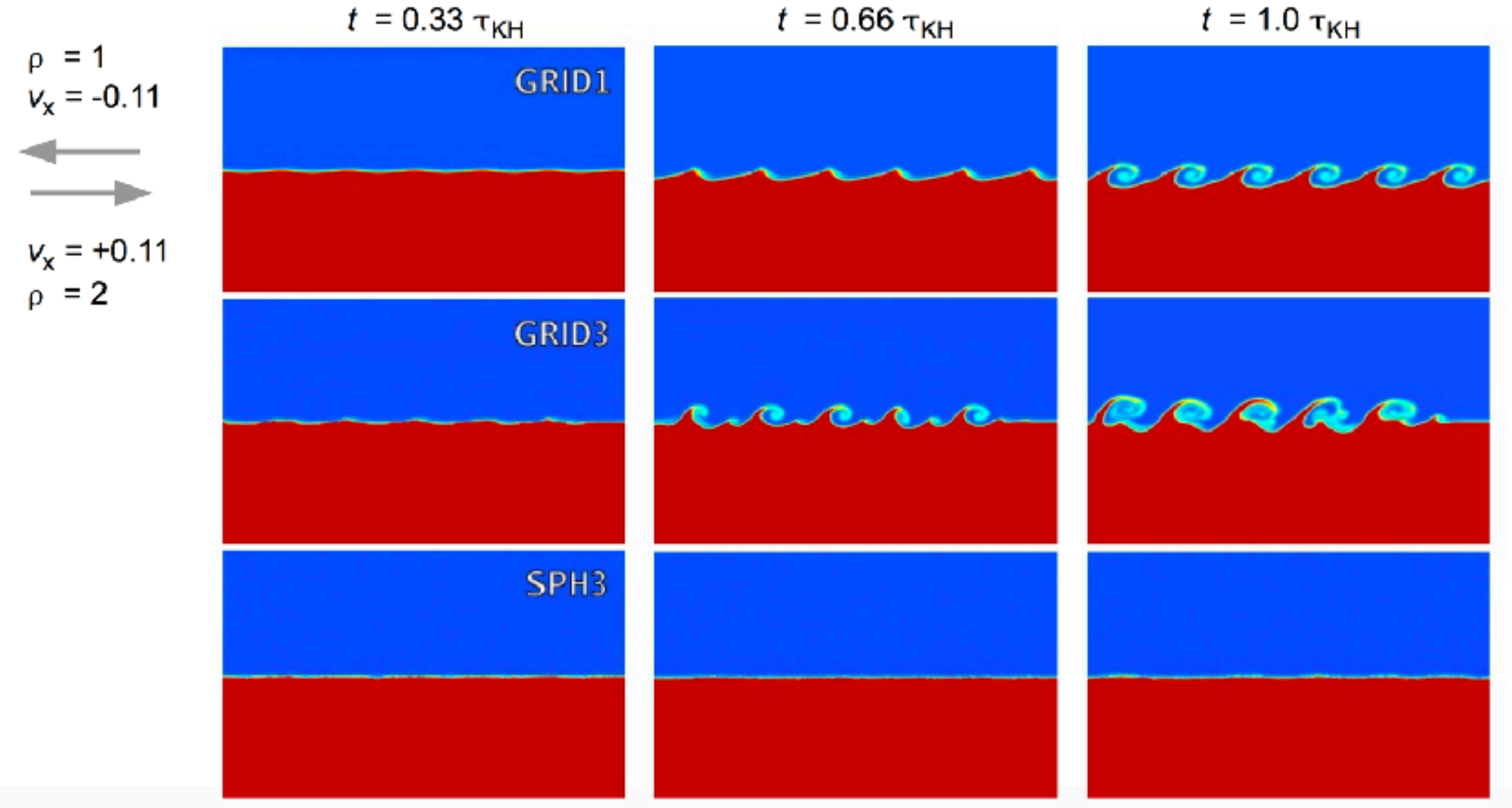
Comparison between cosmological methods

Fundamental differences between SPH and grid methods

Oscar Agertz,[1★] Ben Moore,[1] Joachim Stadel,[1] Doug Potter,[1] Francesco Miniati,[2] Justin Read,[1] Lucio Mayer,[2] Artur Gawryszczak,[3] Andrey Kravtsov,[4] Åke Nordlund,[5] Frazer Pearce,[6] Vicent Quilis,[7] Douglas Rudd,[4] Volker Springel,[8] James Stone,[9] Elizabeth Tasker,[10] Romain Teyssier,[11] James Wadsley[12] and Rolf Walder[13]

Evolution of a supersonic gas cloud in the intracluster medium





Artificial viscosity & mixing!

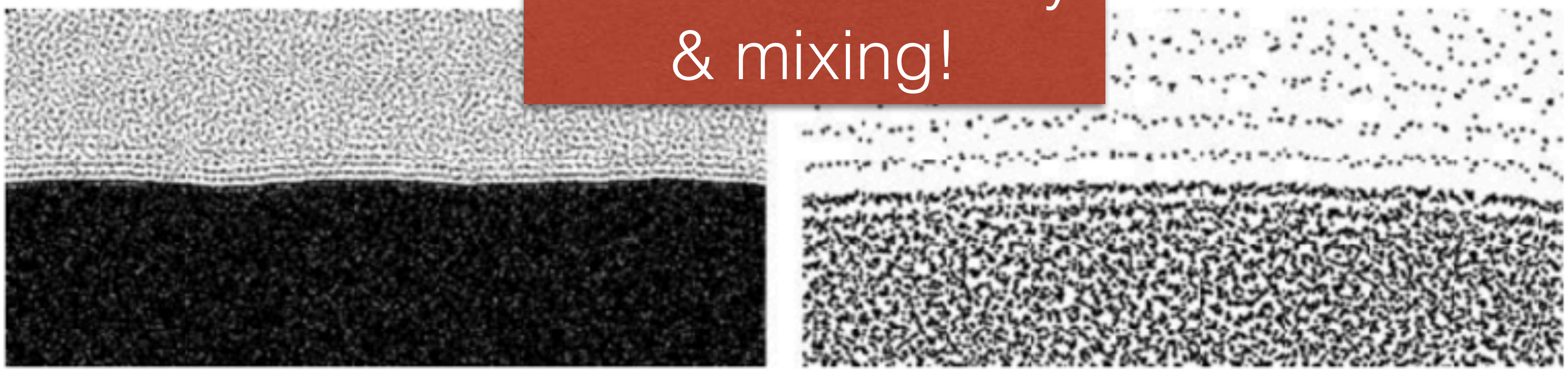
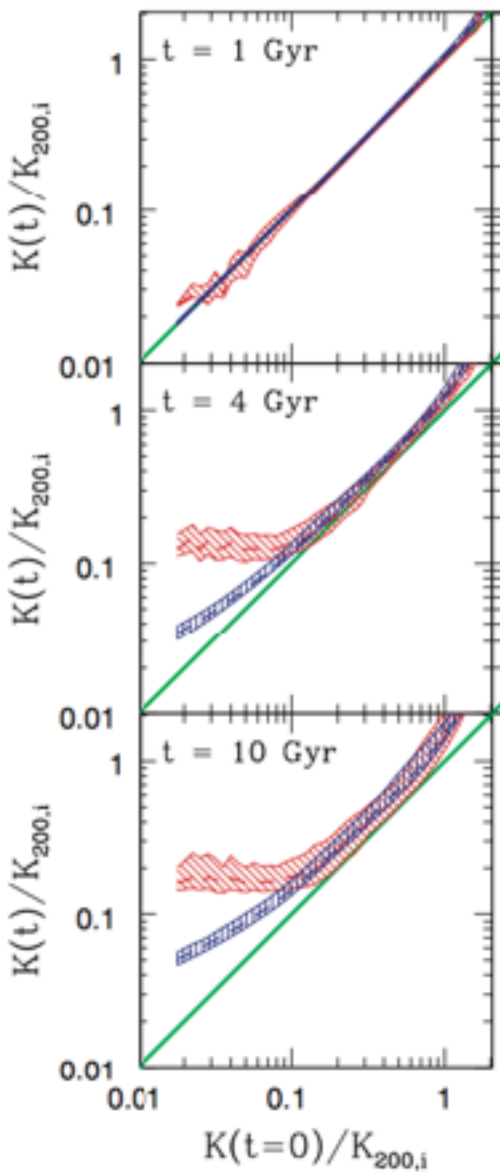
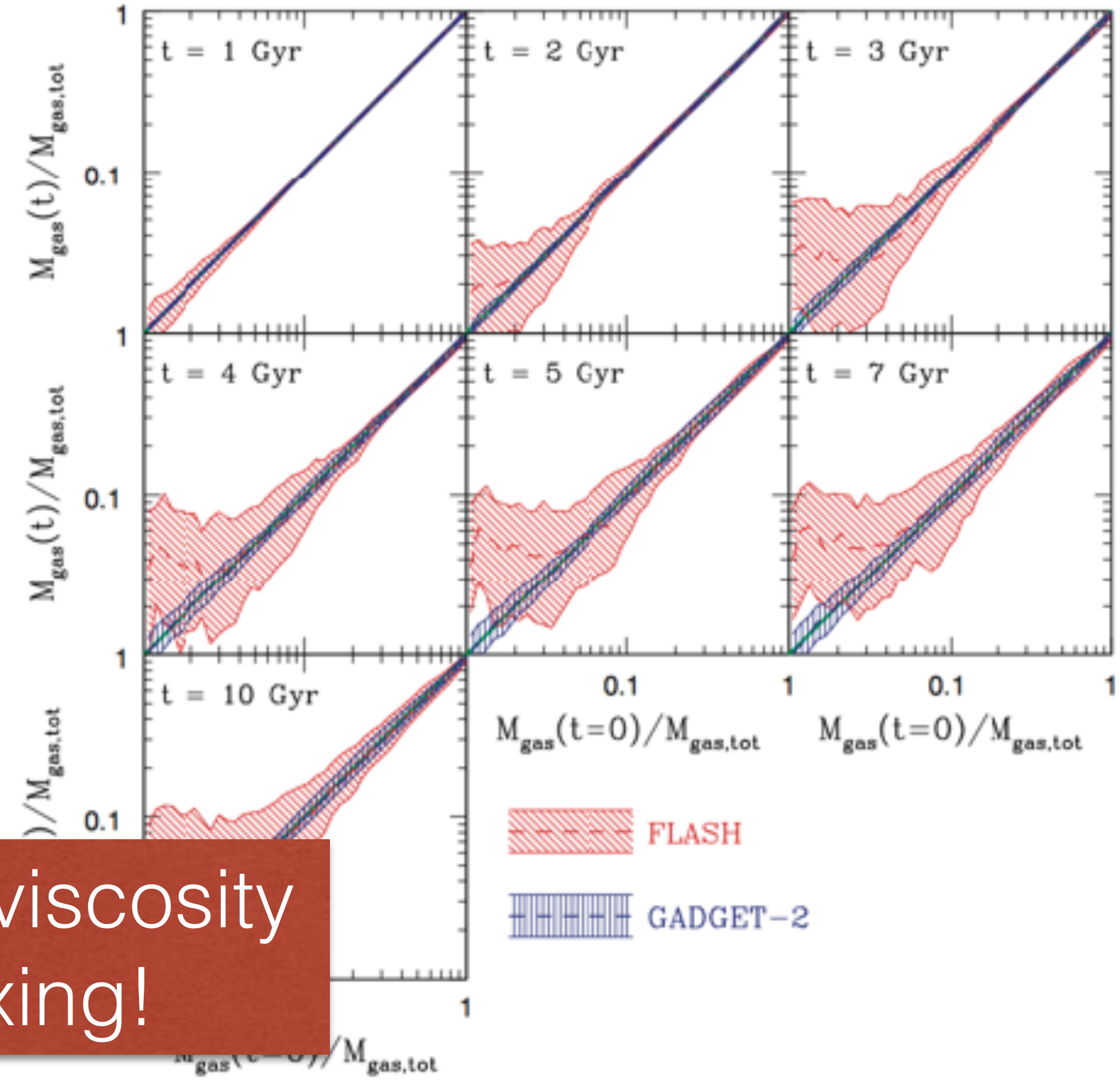


Figure 14. A close up view of the SPH particles at the boundaries between the shearing layers (left) and closer zoom in (right) for SPH3 at τ_{KH}. We can clearly see empty layers formed through erroneous pressure forces due to improper density calculations at density gradients. Even though the two fluids are moving relative to each other, the gap is so large that proper fluid interaction is severely decreased or even absent.

Comparison between cosmological methods





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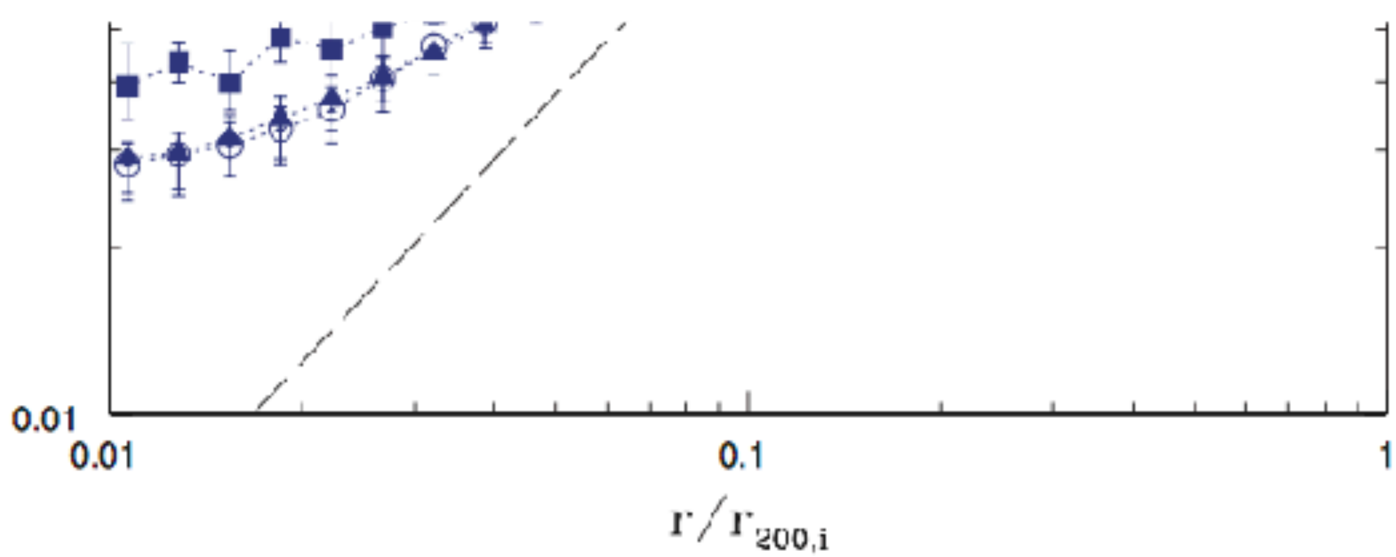


Mitchell et al. 2009 MNRAS

Evolution of binary galaxy cluster collision

Comparison between cosmological methods

A test suite for quantitative comparison of hydrodynamic codes in astrophysics

Elizabeth J. Tasker,[1★] Riccardo Brunino,[2] Nigel L. Mitchell,[3] Dolf Michielsen,[2] Stephen Hopton,[2] Frazer R. Pearce,[2] Greg L. Bryan[4] and Tom Theuns[3,5]

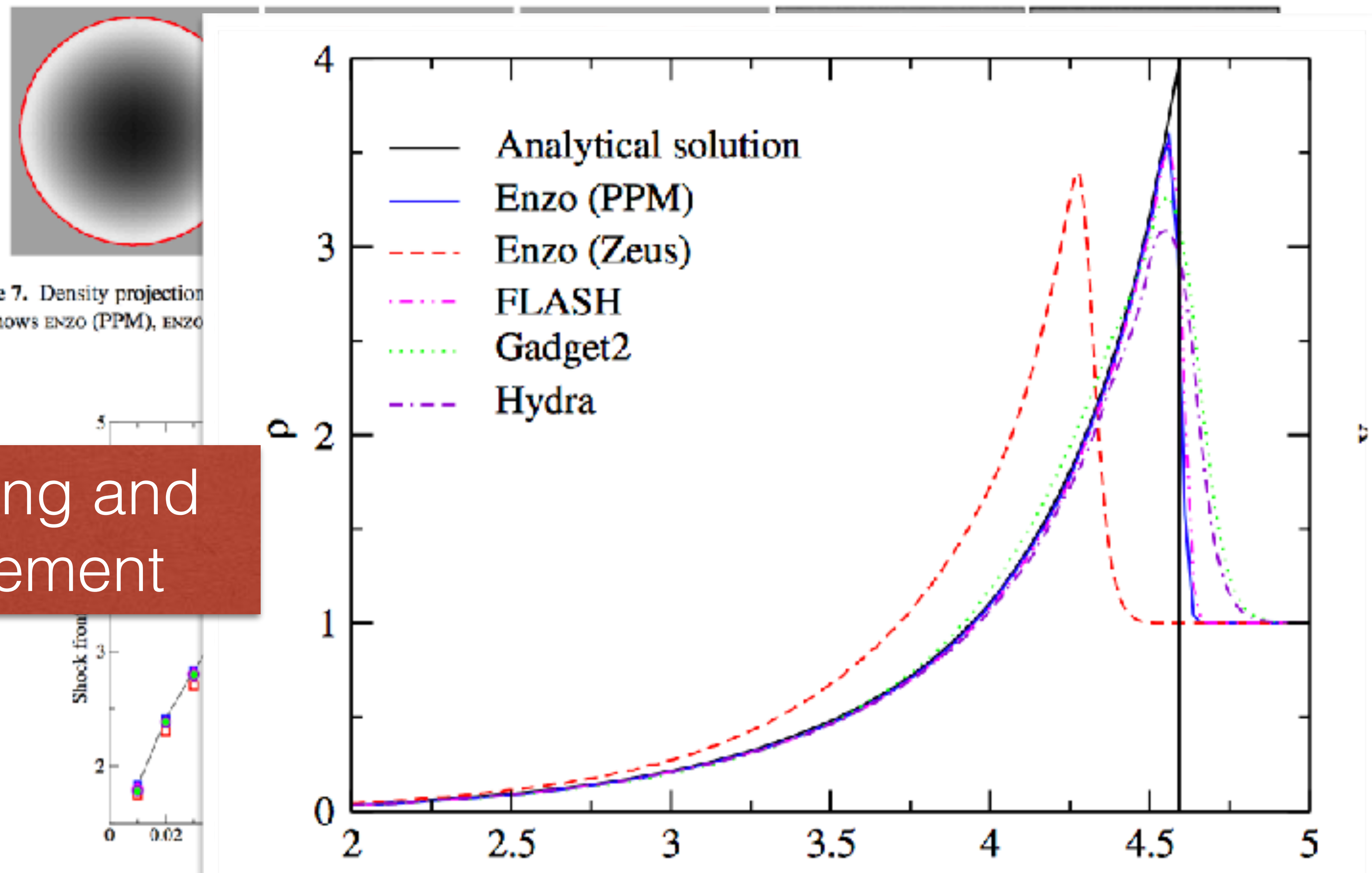


Figure 7. Density projection
side shows ENZO (PPM), ENZO

Timestepping and grid alignement

Figure 8. For the Sedov blast test, from left- to right-hand side, position of shock front over time, the maximum density value and the width of the shock front at half maximum.

Comparison between cosmological methods

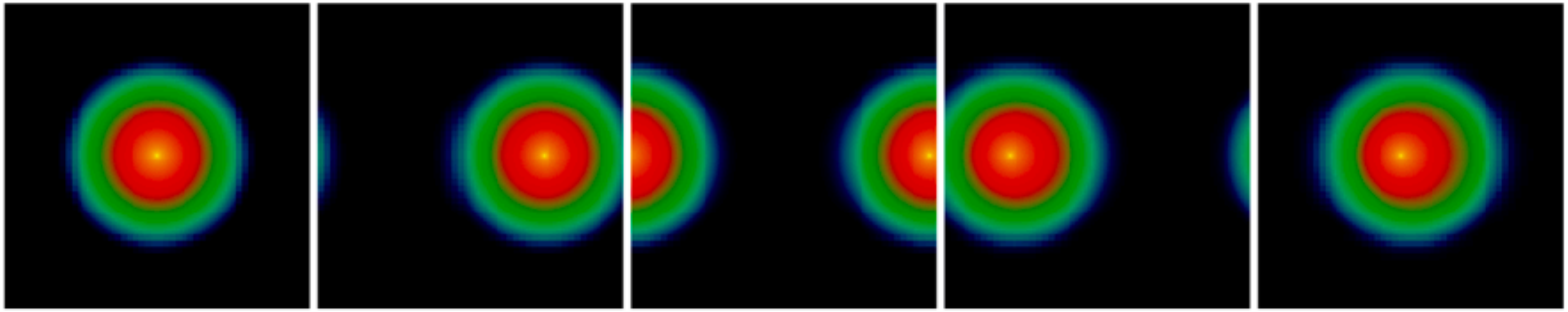


Figure 12. Density projections of the cluster over the course of 1 Gyr in which it moves once around the simulation box. Images taken at 0, 250, 500, 750, 1000 Myr with projected density range $[10^{8.4}, 10^{16.5}]\,M_\odot\,\mathrm{Mpc}^{-2}$. Yellow and red shows higher density regions than green, while black is very low density. [Images produced with ENZO (ZEUS).]

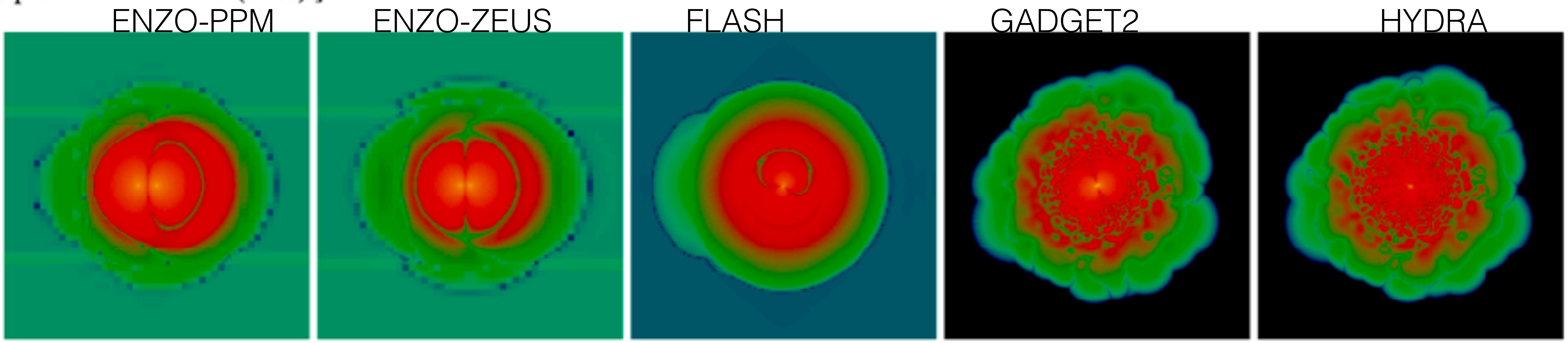
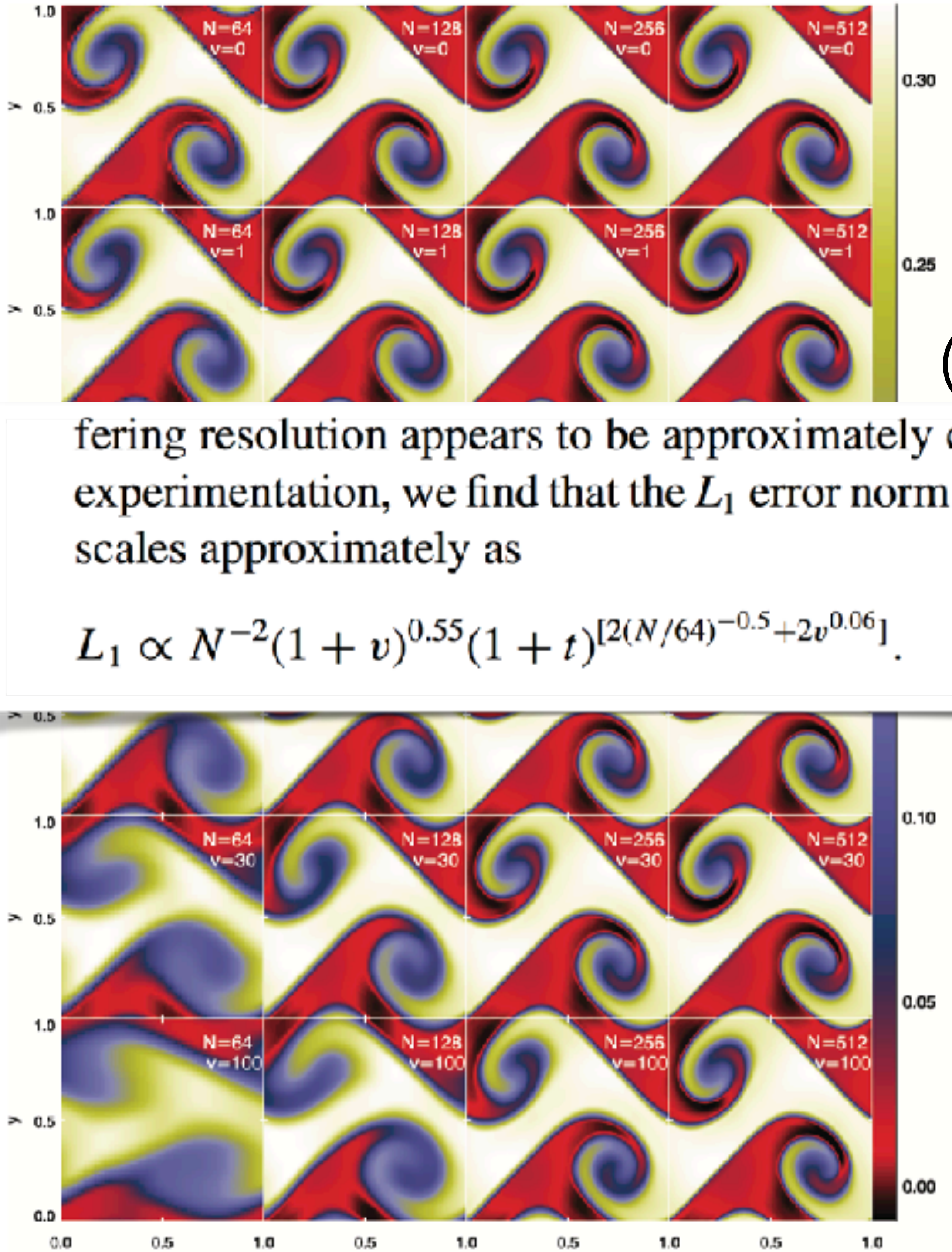


Figure 13. Image subtractions of the density projections at the start and end of the translating cluster test. From left- to right-hand side shows ENZO (PPM), ENZO (ZEUS), FLASH, GADGET2 and HYDRA. The projected density range is $[10^4, 10^{18.6}]\,M_\odot\,\mathrm{Mpc}^{-2}$.

Galileian invariance & advection errors

Comparison between cosmological methods

Kelvin-Helmoltz instabilities



fering resolution appears to be approximately constant. With some experimentation, we find that the L_1 error norm of these simulations scales approximately as

$$L_1 \propto N^{-2}(1+v)^{0.55}(1+t)^{[2(N/64)^{-0.5}+2v^{0.06}]}. \quad (11)$$

Robertson et al. 2010

Ideally, we want to solve:

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} = 0,$$

yet in practice we deal with (~1st order approximation) :

$$= \alpha \frac{\partial^2 \rho}{\partial x^2}.$$

n:

$$\alpha = \frac{1}{2} v \Delta x (1 - |c|),$$

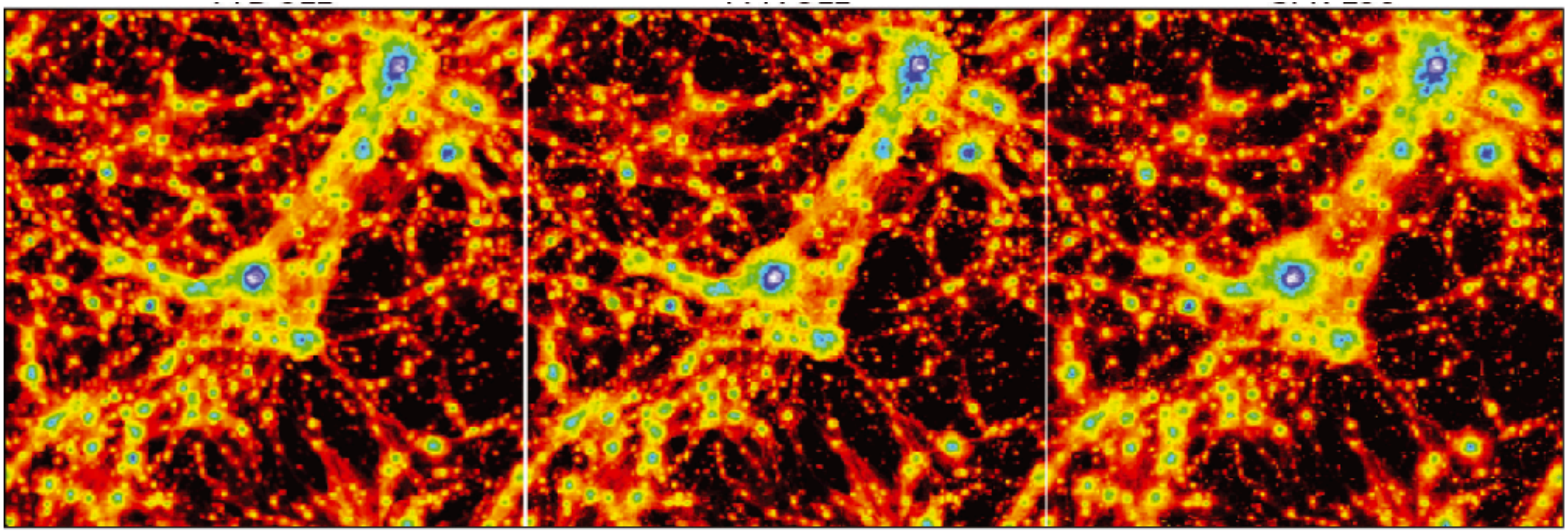
To have advection error under control:
-> increase no. of cells N
-> reduce timestep Δτ

Comparison between cosmological methods

A comparison of cosmological codes: properties of thermal gas and shock waves in large-scale structures

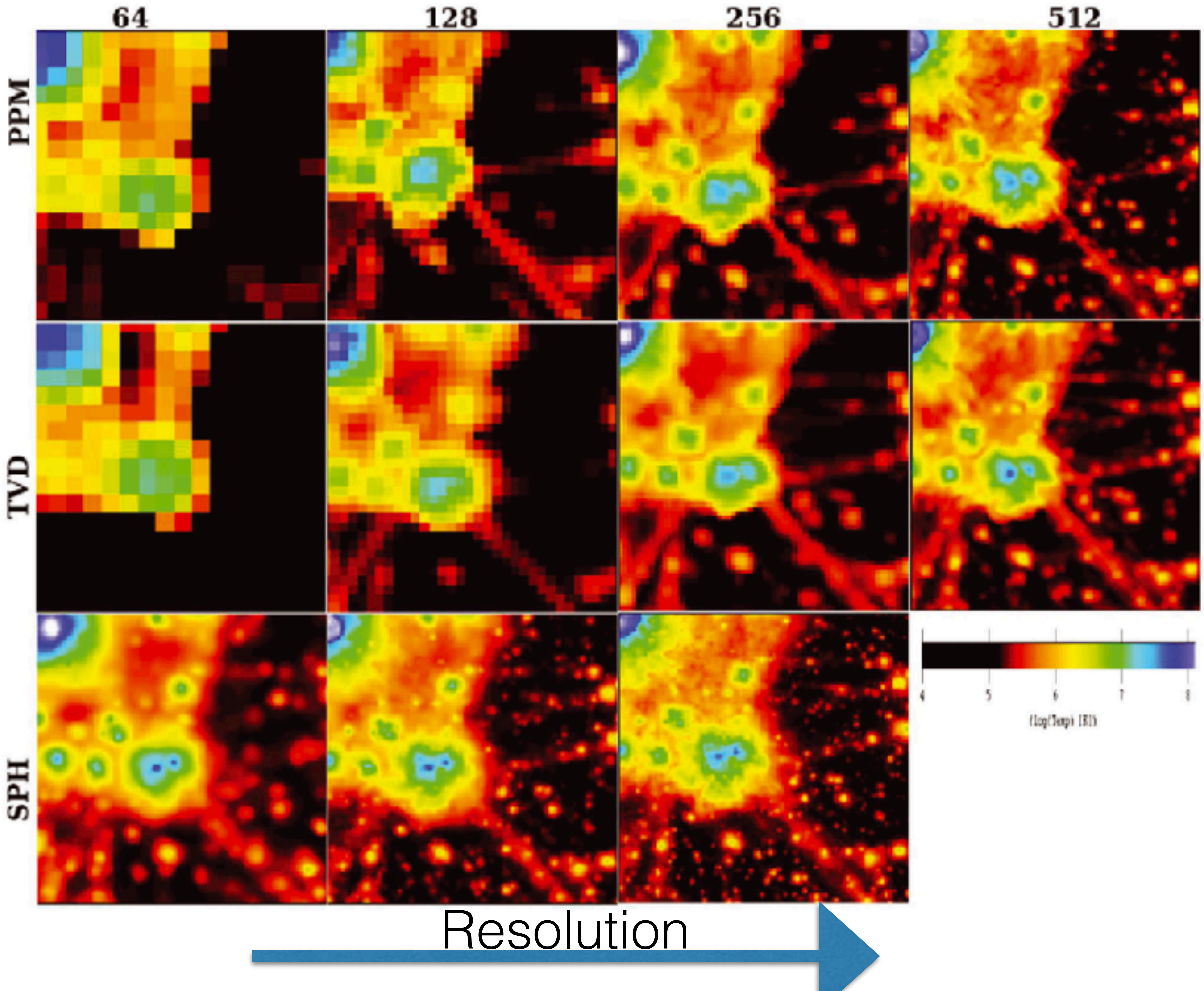
F. Vazza,[1,2★] K. Dolag,[3,4] D. Ryu,[5] G. Brunetti,[2] C. Gheller,[6] H. Kang[7] and C. Pfrommer[8]

TVD PPM SPH

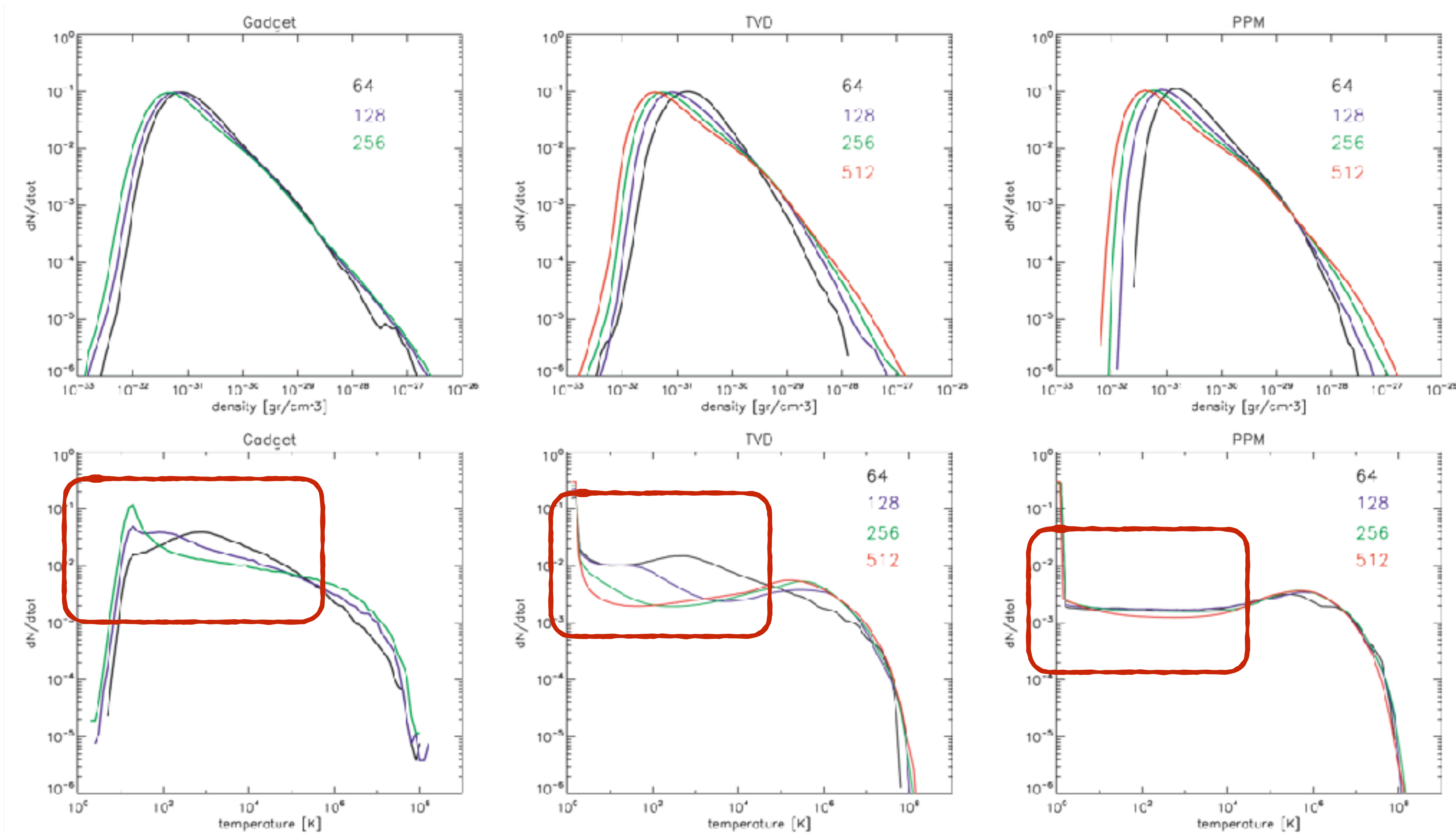


*TVD = Total Variation Diminishing method, **ES-TVD** code by Ryu et al.*
*PPM= Parabolic Piecewise Method, **Enzo** code by Bryan et al.*
*SPH= Smoothed Particle Hydrodynamics, **Gadget** code by Springel et al.*

Comparison between cosmological methods

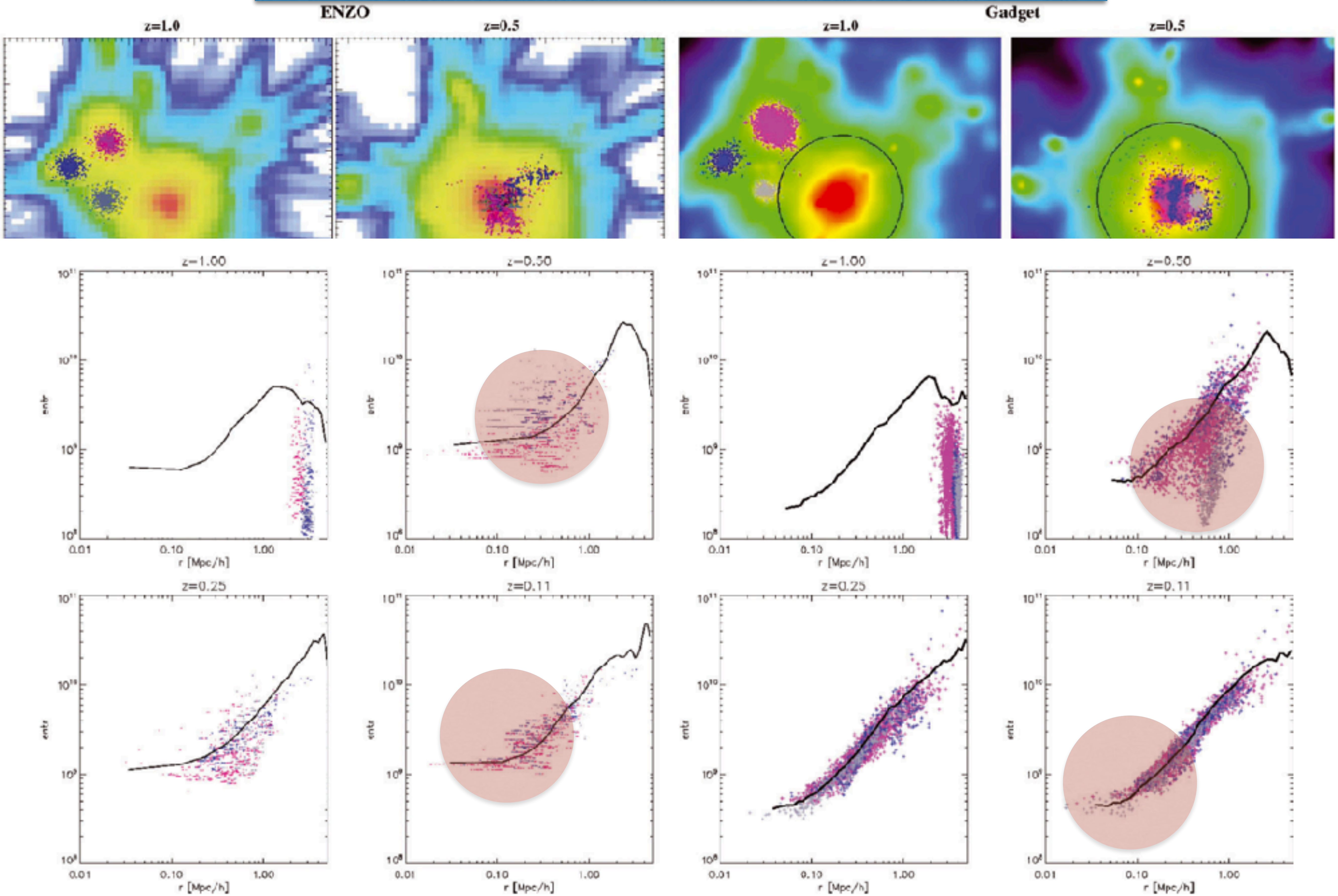


Comparison between cosmological methods

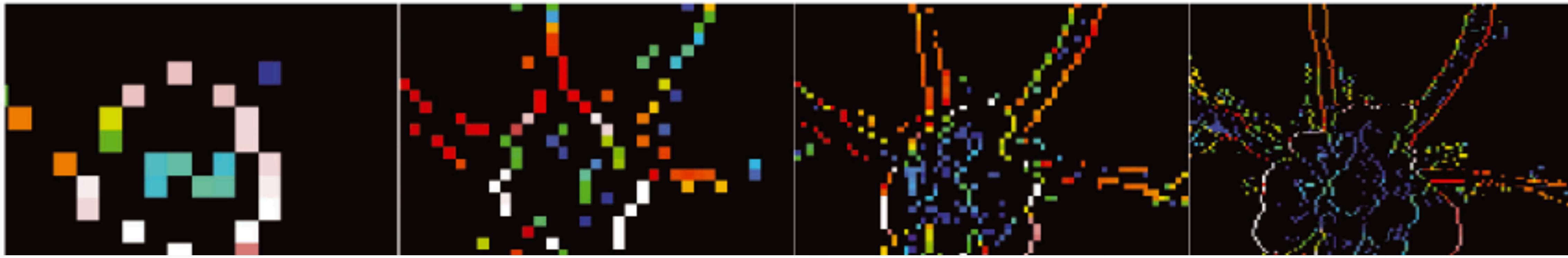


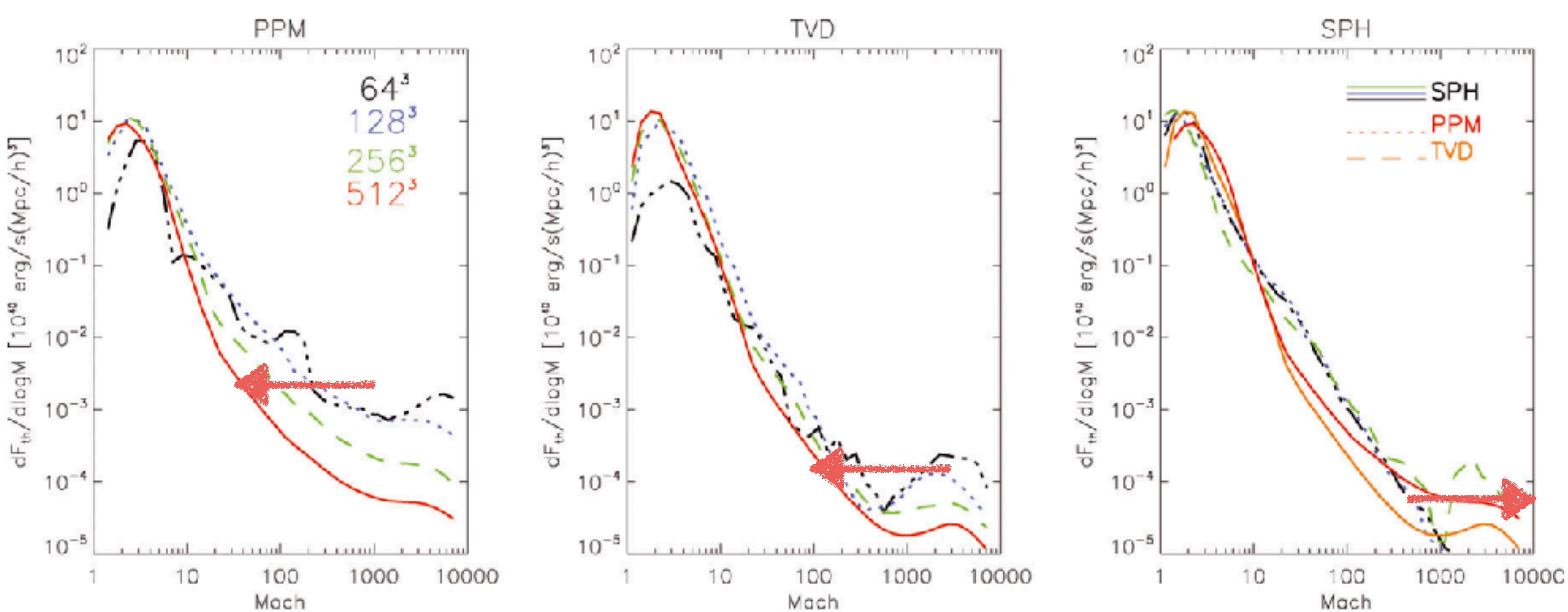
Distribution functions of gas density/temperature

Comparison between cosmological methods



Comparison between cosmological methods





Shock waves in different codes (using different methods)

Comparison between cosmological methods

- good agreement density/temp distribution on >100 kpc
- larger differences in

  a) peripheral regions of clusters (-> shocks)

  b) cluster cores (-> mixing)

SUMMARY OF KNOWN TRENDS IN COSMOLOGICAL HYDRODYNAMICS:

GRID METHODS:

PROBLEMS:

- advection errors
- overmixing
- dependence on grid alignment

SOLUTIONS:

➡ increase resolution (AMR)
➡ increase res. / subgrid model
➡ unsplit methods

SPH METHODS:

PROBLEMS:

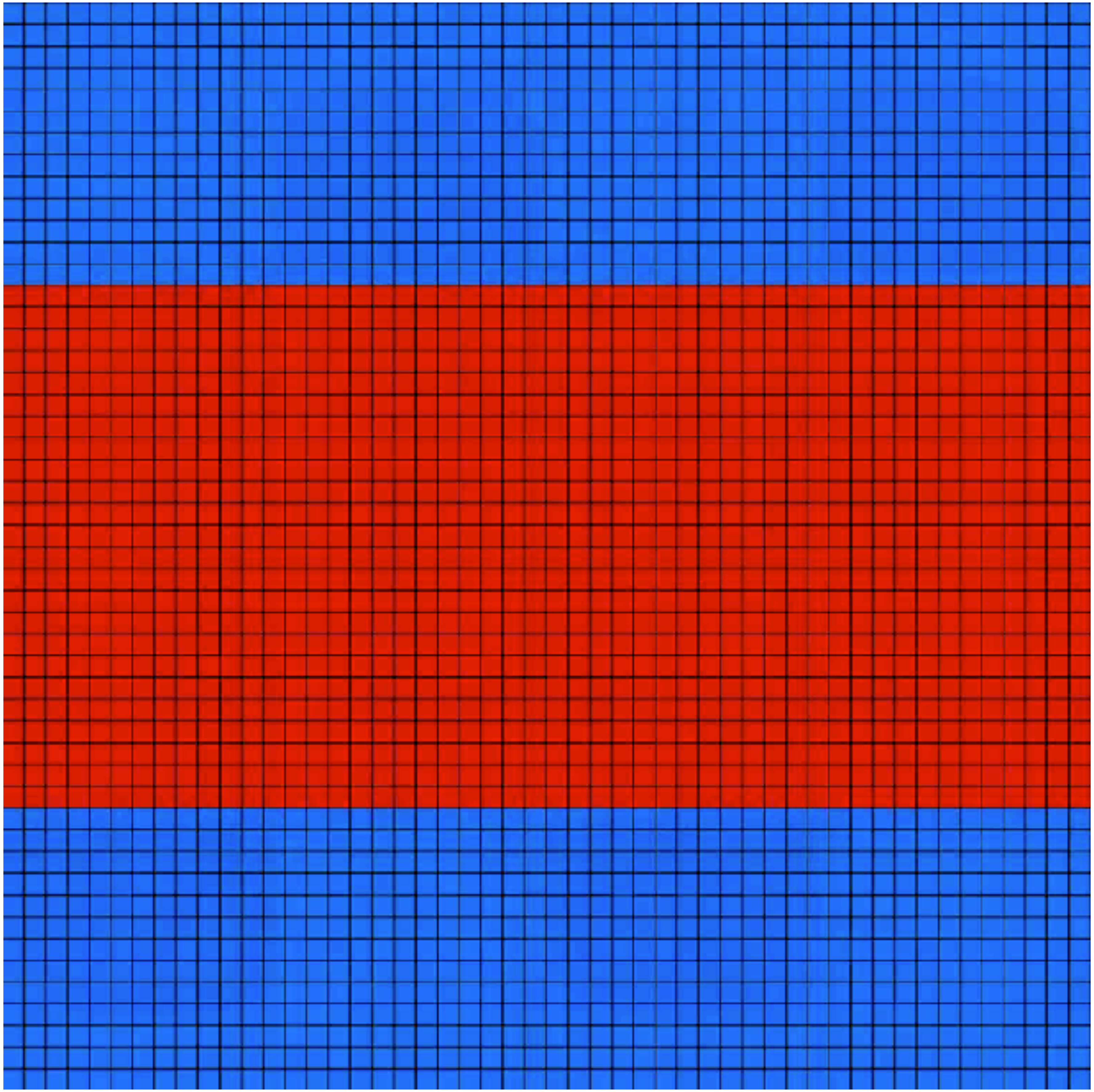
- spurious entr. generation
- absence of mixing
- velocity noise

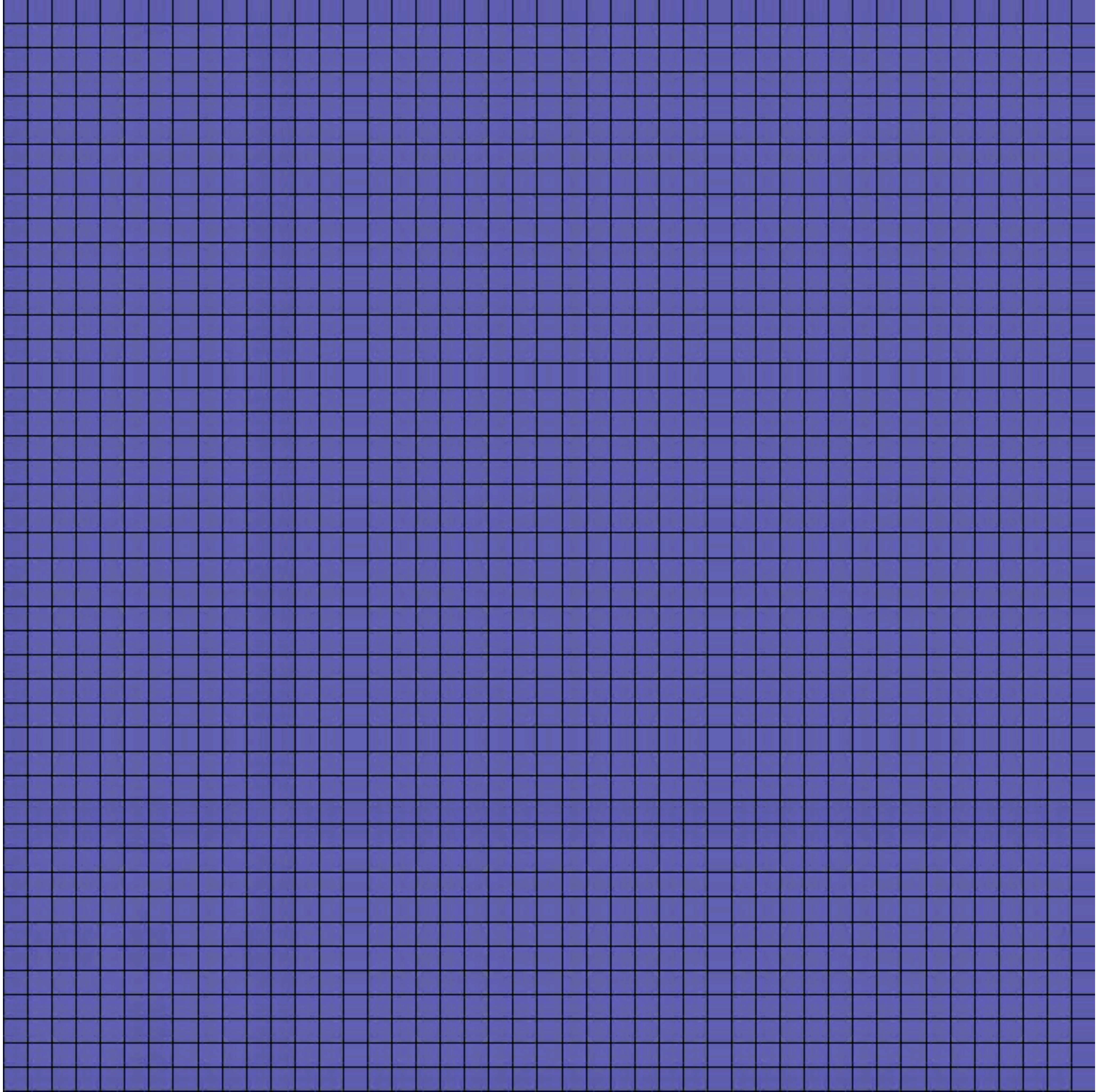
SOLUTIONS:

➡ new artificial viscosity
➡ improve density estimate
➡ new artificial viscosity

one method to rule them all? -> MOVING MESH METHODS

A moving Voronoi-Mesh code: AREPO (Springel 2010)





MHD methods

Momentum conservation $$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} - \frac{1}{4\pi}\mathbf{B}\mathbf{B}) + \nabla P_{tot} = 0$$

Total energy conservation $$\partial_t E + \nabla \cdot \left[(E + P_{tot})\mathbf{u} - \frac{1}{4\pi}\mathbf{B}(\mathbf{B} \cdot \mathbf{u}) \right] = 0$$

Magnetic flux conservation $$\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times \mathbf{u}) = 0$$

No magnetic monopoles $$\nabla \cdot \mathbf{B} = 0$$

from R. Teyssier 2010

-> See Mignone's Talk

No magnetic monopoles $\nabla \cdot \mathbf{B} = 0$

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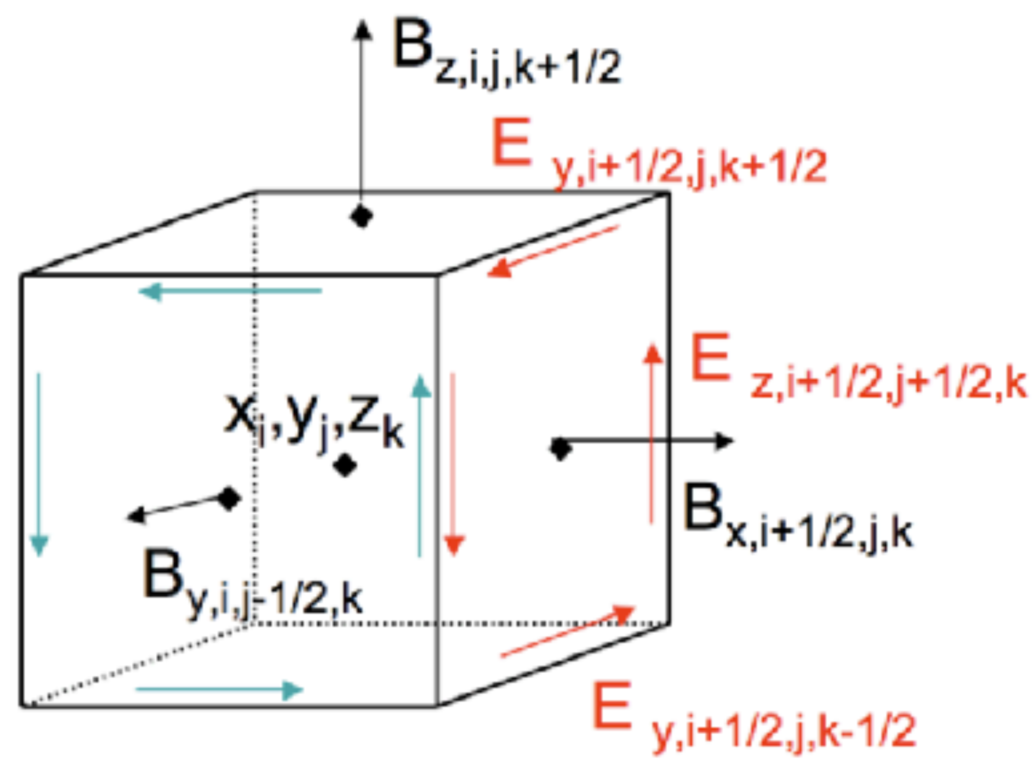
Godunov method with Constrained Transport

The induction equation in integral form suggests a surface-average form:

$$\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times \mathbf{u}) = 0 \qquad \text{(Stokes theorem)} \quad \partial_t \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{s} + \int_L (\mathbf{B} \times \mathbf{u}) \cdot \mathrm{d}\mathbf{l} = 0$$

The magnetic field is face-centred while Euler-type variables are cell-centred (staggered mesh approach).

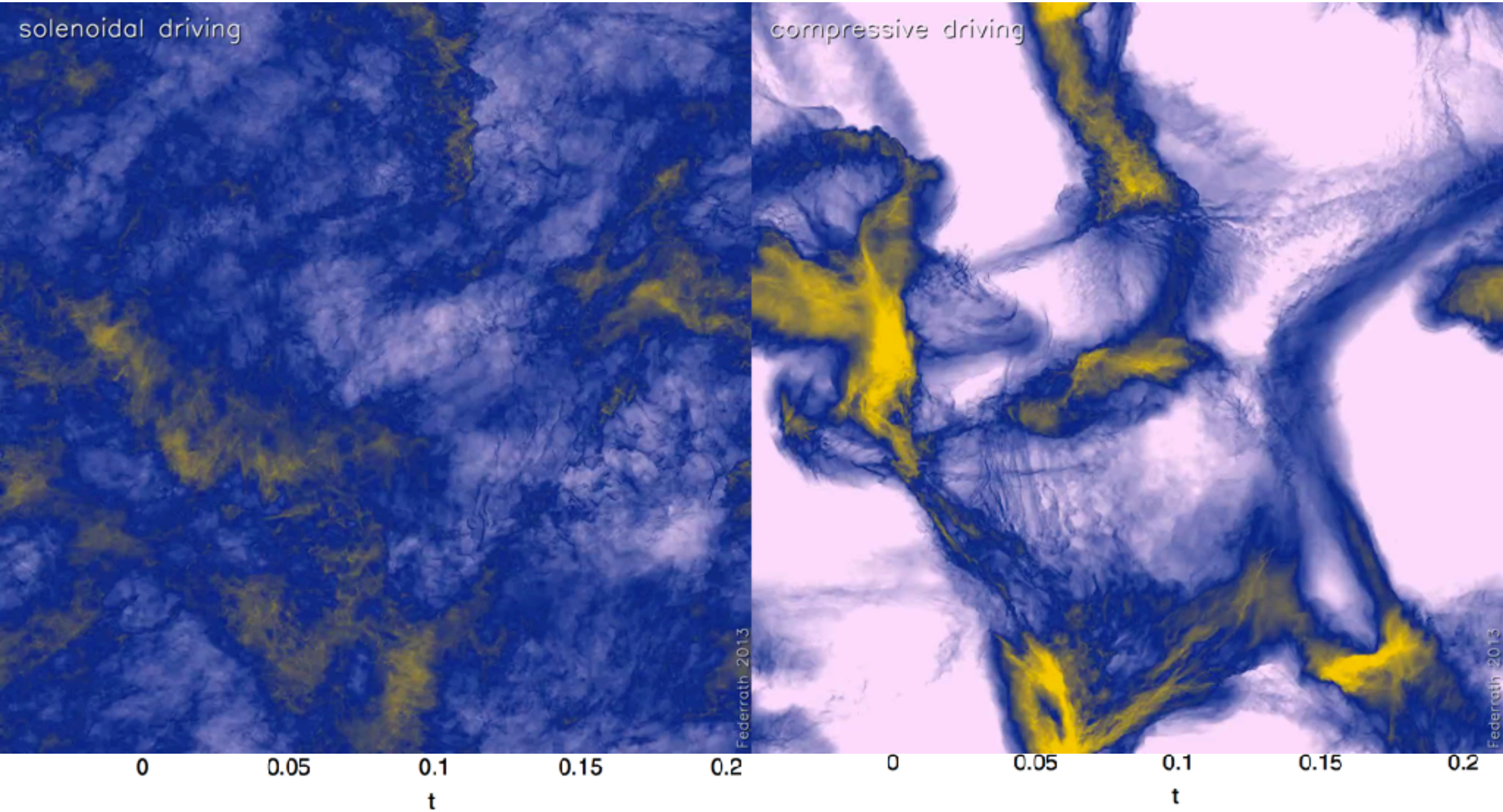
$$(B_x)_{i+1/2,jk} = \frac{1}{S} \int_S B_x(y,z)\,\mathrm{d}y\mathrm{d}z \qquad S = [y_{i-1/2}, y_{i+1/2}] \times [z_{i-1/2}, z_{i+1/2}]$$



Similar to potential vector methods (Yee 1966; Dorfi 1986; Evans & Hawley 1988).

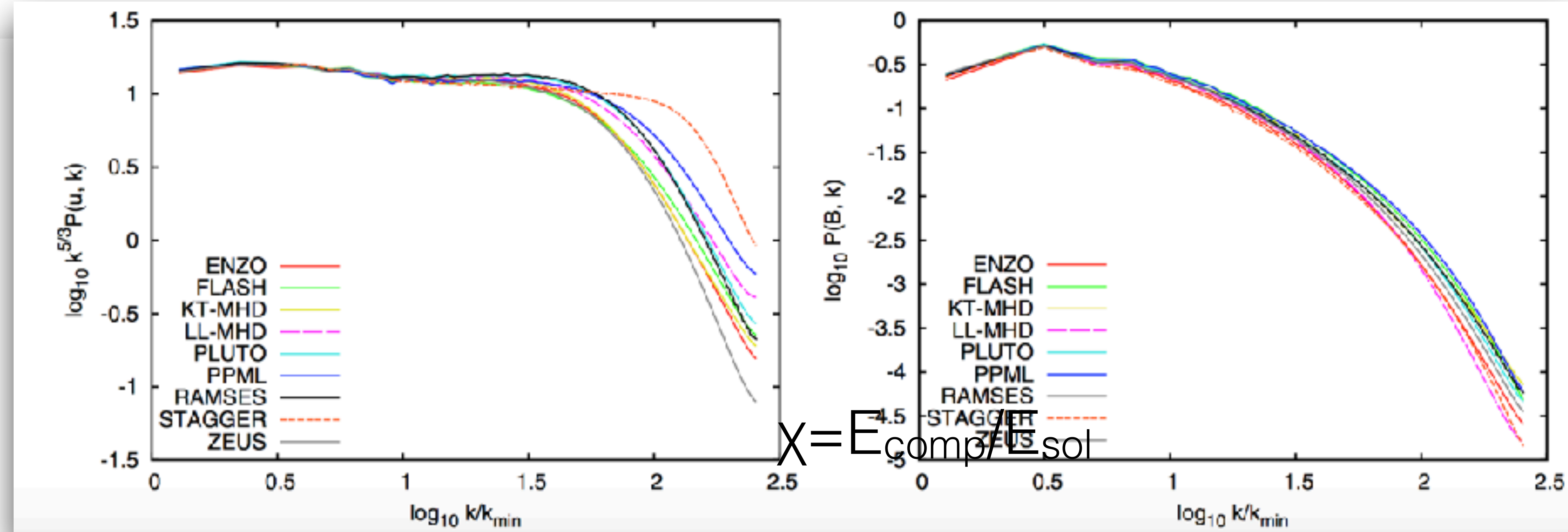
from R. Teyssier 2010 & Vides 2013

Testing MHD methods



Federrath+2011 ApJ

Performance of MHD methods (isothermal turbulence)



$X=E_{comp}/E_{sol}$

Table 2
Solver Design Specifications for the Eulerian Methods[a]

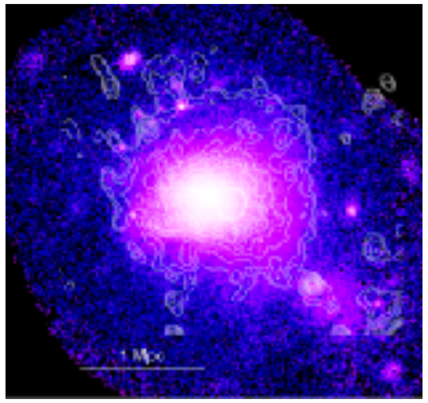
Name	Base Scheme[b]	Spatial Order[c]	Source Terms[d]	MHD[e]	Time Integration[f]	Directional Splitting[g]
ENZO	FV, HLL	Second	Dedner	Dedner	Second-order RK	Direct
FLASH	FV, HLLD	Second	ıı Derivative	Third-order CT	Forward Euler	⊥ Reconstruction
KT-MHD	FD, CWENO	Third	KT	Third-order CT	Fourth-order RK	Direct
LL-MHD	FV, HLLD	Second	None	Athena CT	Forward Euler	Split
PLUTO	FV, HLLD	Third	Powell	Powell	Fourth-order RK	Direct
PPML	FV, HLLD	Third	None	Athena CT	Forward Euler	⊥ Reconstruction
RAMSES	FV, HLLD	Second	None	2D HLLD CT	Forward Euler	⊥ Reconstruction
STAGGER	FD, Stagger	Sixth	Tensor	Staggered CT	Third-order Hyman	Direct
ZEUS	FD, van Leer	Second	von Neumann	MOC-CT	Forward Euler	Split

Kritsuk+2013 ApJ

How to design a large cosmological simulation?

(#1 the hydro case)

- Suppose we want to study
  cosmic rays in massive galaxy clusters



Final radius: ~ 3 Mpc (for a ~10^{15} M_{sol})

They form from fluctuations at least ~4-5 times larger (in diameter), so Volume~30^3 Mpc^3 at least.

However, this is a statistical process.
With given cosmological parameters, we need ~100^3 Mpc^3 for a ~100% chance of forming one big a cluster.

Requirement #1: Volume >~100^3 Mpc^3

How to design a large cosmological simulation?

- Which process do we want to study?

Mass distribution?	*$\Delta x \sim 300$ kpc to sample the profile with ~10 radial bins.*
Shocks/cosmic rays?	*$\Delta x < 200$ kpc to resolve shocks energetic*
Cooling radius?	*$\Delta x \sim 100$ kpc because $t_{cool} << t_{Universe}$ only there*
Turbulence?	*$\Delta x < 50$ kpc for observed density fluctuations*
Galaxy formation?	*$\Delta x < 1$ kpc*

Requirement #2: max. resolution ~100kpc…

Requirement #1: Volume >~100^3 Mpc^3

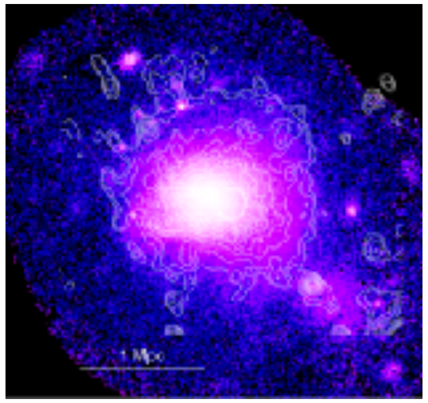
Requirement #2: max. resolution ~100 kpc

2048^3 cells/DM particles on 200^3 Mpc^3
~1,200 000 core hours on Curie/Piz-Daint
(FV,Gheller,Bruggen 2014,2016)

How to design a large cosmological simulation?

(#2 the MHD case)

- Suppose we want to study **magnetic fields in massive galaxy clusters**



Final radius: ~ 3 Mpc (for a ~10^{15} M_{sol})

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However, this is a statistical process.
With given cosmological parameters, we need ~100^3 Mpc^3 for a ~100% chance of forming one big a cluster.

Requirement #1: Volume >~100^3 Mpc^3

How to design a large cosmological simulation?

(the hydro-MHD case)

- Which process do we want to study?

Mass distribution? *$\Delta x \sim 300$ kpc to sample the profile with ~10 radial bins.*

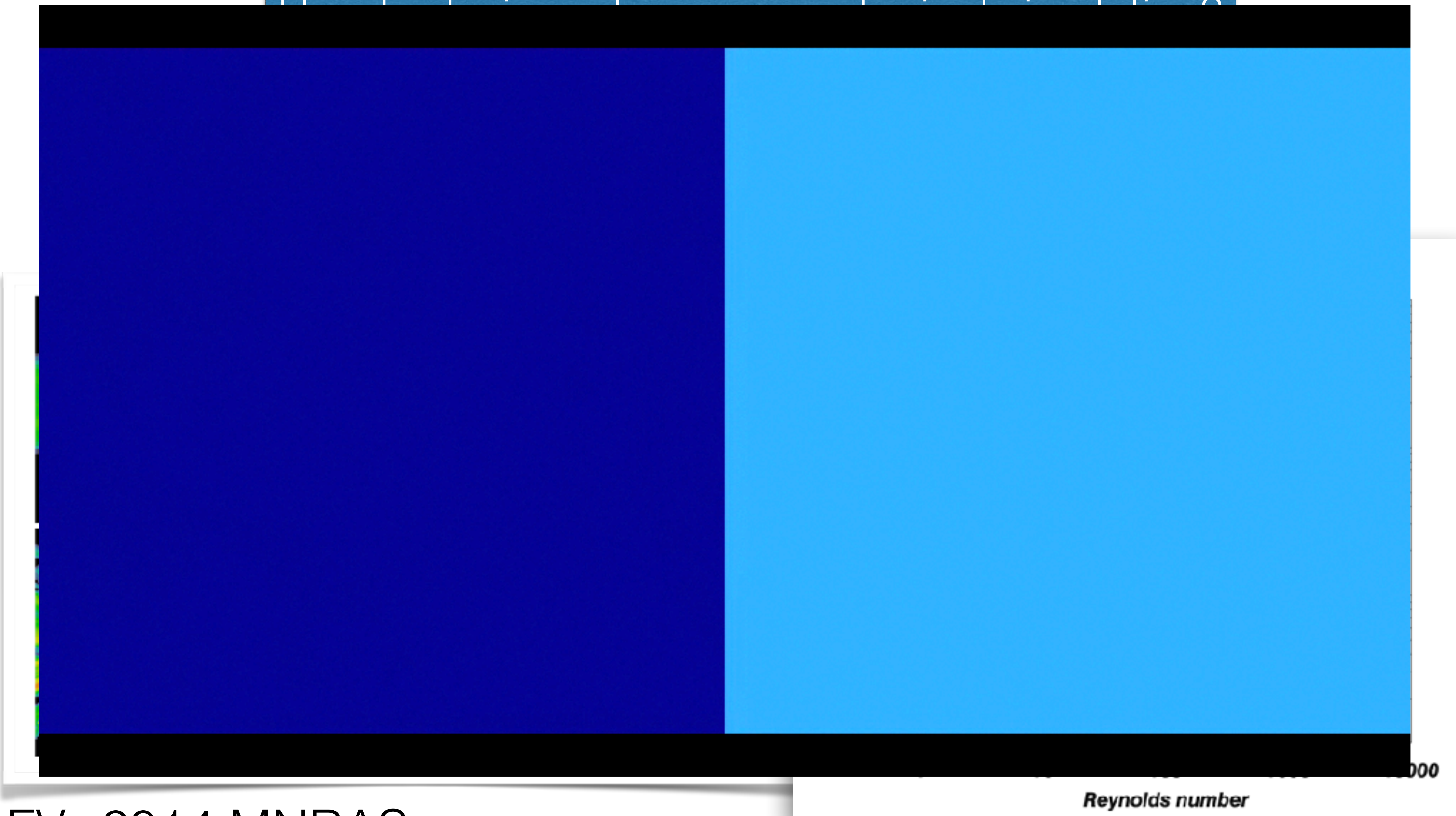
Shocks/cosmic rays? *$\Delta x \sim 100$ kpc to resolve shocks energetic*

Cooling radius? *$\Delta x < 100$ kpc because $t_{cool} < t_{Universe}$ only there*

Turbulence? *$\Delta x < 50$ kpc for observed density fluctuations*

Galaxy formation? *$\Delta x < 1$ kpc*

Requirement #2: max. resolution < 50kpc…

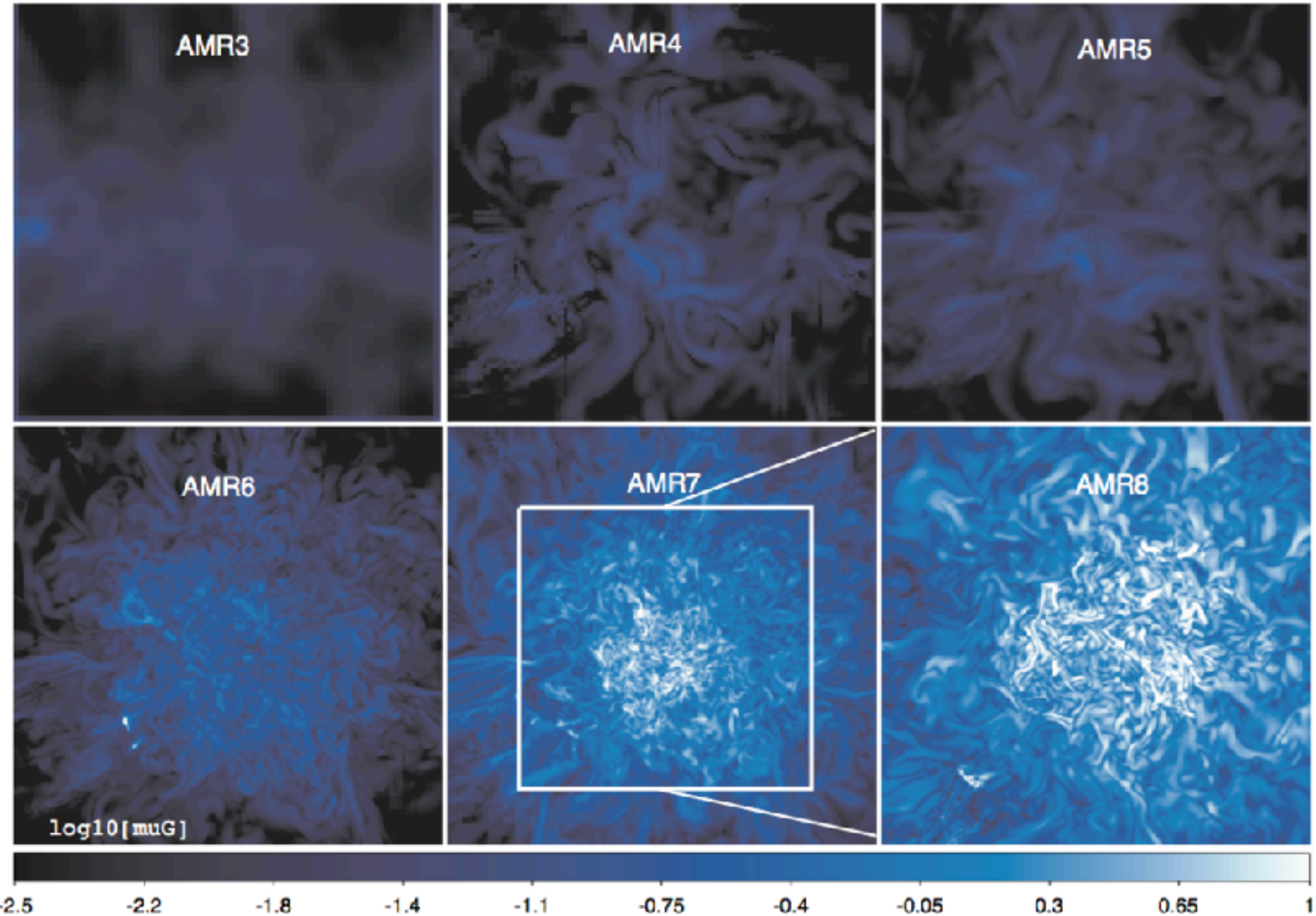


FV+2014 MNRAS

Requirement #3: >200^3 res.elements to *start* a dynamo

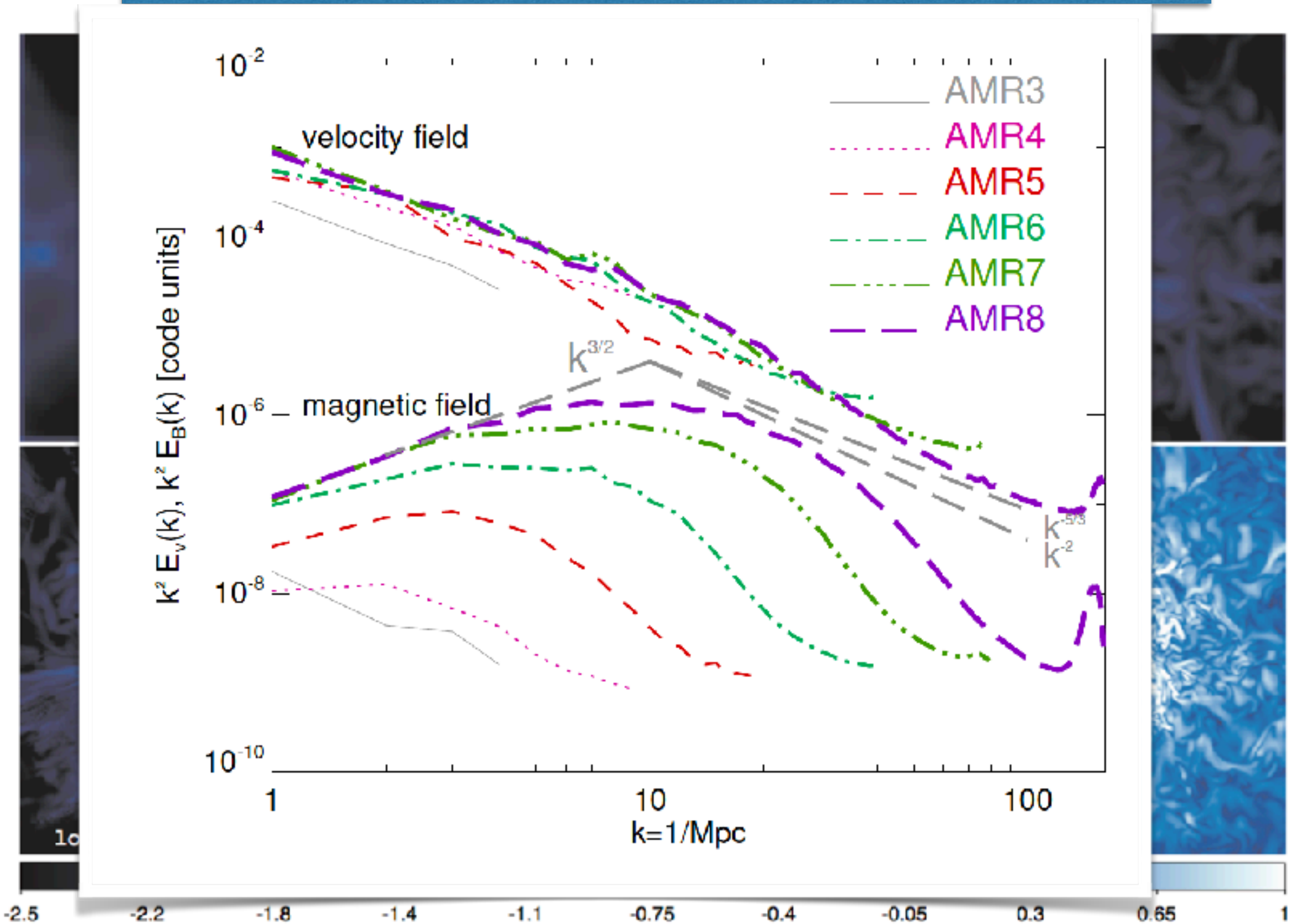
How to design a large cosmological simulation?

FV+2017 MNRAS



Requirement #4: >1000^3 res.el. for a saturated dynamo

How to design a large cosmological simulation?



FV+2017 MNRAS

Requirement #4: >1000^3 res.el. for a saturated dynamo

Requirement #1: Volume $>\sim 100^3$ Mpc^3

Requirement #2: max. resolution ~300 to 1kpc…

Requirement #3: >200^3 res.elements to *start* a dynamo

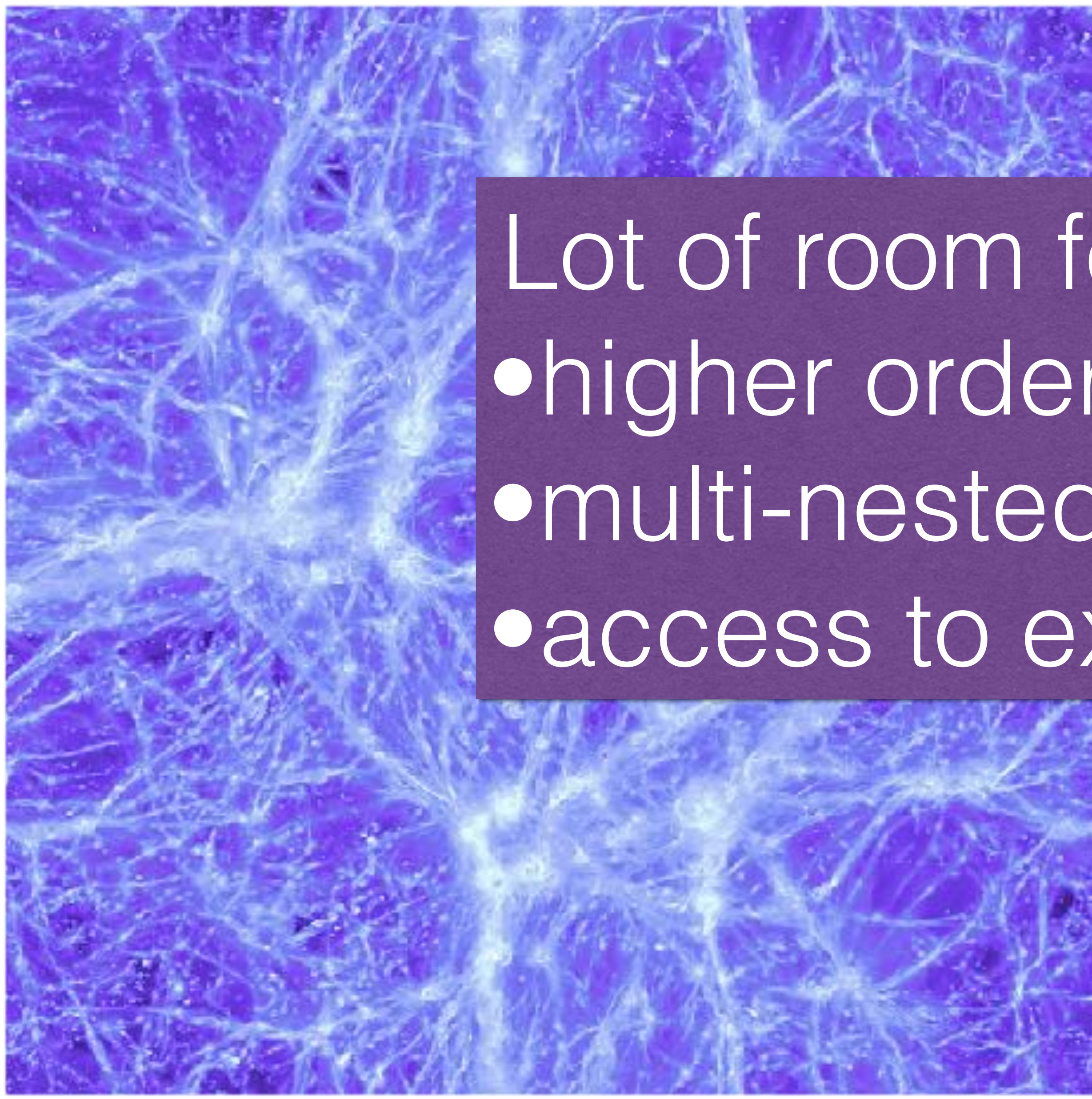
Requirement #4: >1000^3 res.el. for a saturated dynamo

How can we have such a run?

We just cannot (yet).

Fixed resolution *=20kpc*
Size=50 Mpc
2400^3 cells
~10 clusters, filaments…

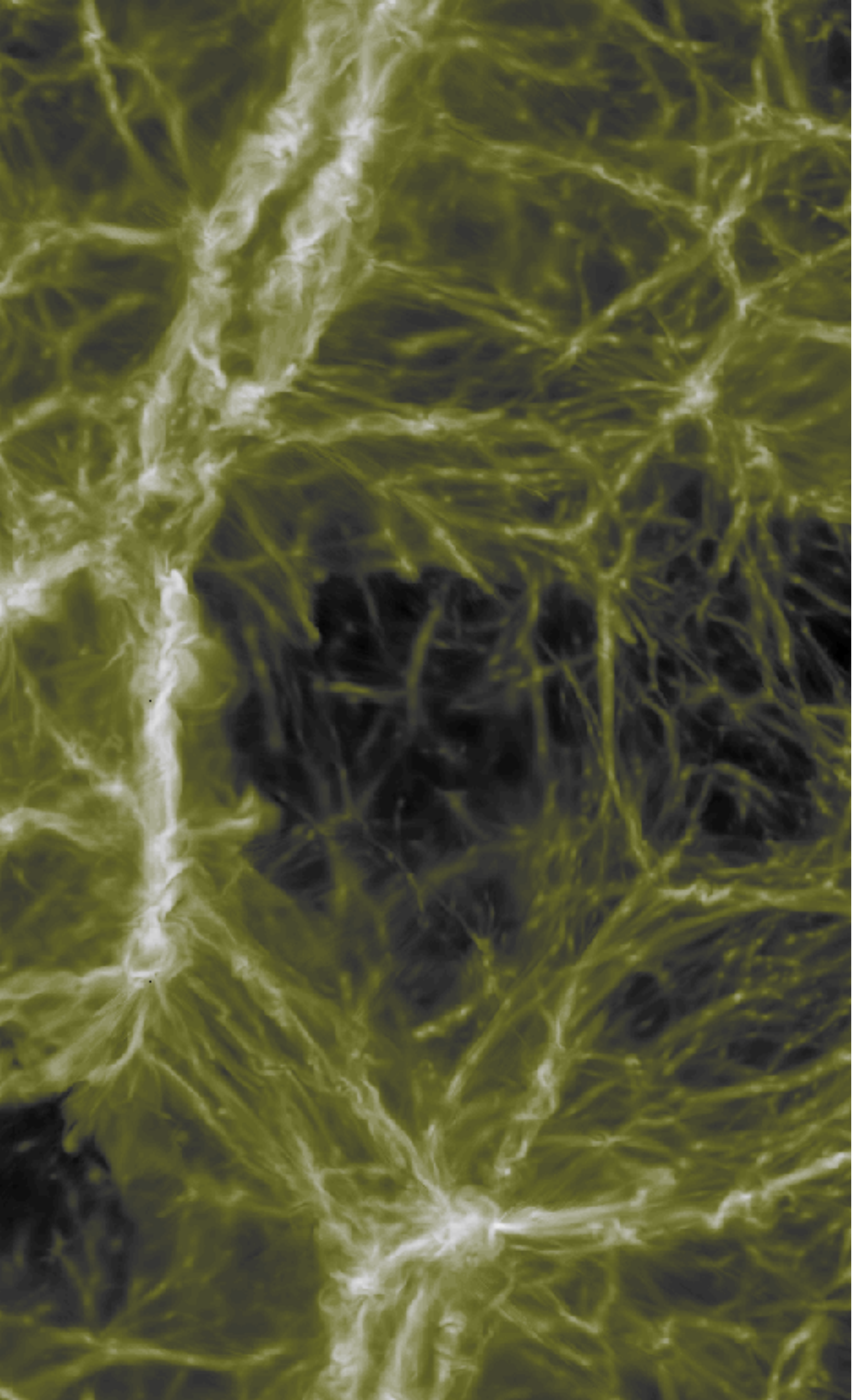
Adaptive mesh resolution = *3.9kpc*
L_{root}=200 Mpc, L_{AMR}=25 Mpc
8 levels of AMR
1 cluster, shocks, turbulence



Lot of room for improvement:
- higher order MHD scheme
- multi-nested setup
- access to exascale….

~1.5 million core hours on 2400 nodes
(GPU accelerated @ PizDaint)

~200k core hours on 512 nodes
(Jureca @ Jülich FZC)



Conclusions(?)

Main challenges in cosmological MHD:

- large density contrasts/dynamical range
- high velocity flows
- mixing of multi-phase gases

Main (known) issues in methods:

- SPH: entropy generation, little mixing
- Grid: galileian invariance, overmixing

The future:

- More complex schemes to increase the dyn.range (higher order / subgrid)
- Moving mesh / Mesh-less techniques
- Major porting of codes to exascale architectures

Thanks, questions?