Energy conserving schemes in OpenFOAM

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Introduction

Objectives

- Introduce the concept of **"discrete energy consistency"**
- Capability of different **schemes/solver** to satisfy **the kinetic energy conservation** property of **Navier-Stokes** equations
- Comparative study: **OpenFOAM** vs. **other CFD** solvers
- **Test Cases**:
  1. Taylor-Green flow
  2. Laminar flow past circular cylinder
  3. Turbulent flow past circular cylinder (URANS and LES)
  4. DNS of supersonic channel flows
  5. Flows with shock waves

Numerical solvers

OpenFOAM

- Open Source(GPL)
- Unstructured,
  Finite Volume(FV)
- **dnsFoam**,
  incompressible DNS
  PISO algorithm
- **rhoCentralFoam**,
  compressible,
  Kurganov and
  Tadmor (TVD).
- **rhoEnergyFoam**,
  novel compressible
  solver

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Commercial Solver

- Proprietary
- Unstructured(FV)
- Upwinding, TVD

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Commercial Solver

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Finite Differences in-house solver

- Compressible energy-conserving
- 3D, Cartesian
- Arbitrary order of accuracy

Kinetic energy

$$\frac{d}{dt}\int\limits_V \left(\rho\,\frac{u_i\,u_i}{2}\right)\,\mathrm{d}V = -\int\limits_S \left(\rho\,\frac{u_i\,u_i}{2} + p\right) u_i\,n_i\,\mathrm{d}S + \int\limits_V \left(p\,\frac{\partial u_i}{\partial x_i}\right)\,\mathrm{d}V + \int\limits_V \frac{\partial}{\partial x_i}\left(\sigma_{ik}\,u_k\right)\,\mathrm{d}V$$

Conservation principles

Kinetic Energy

$$\frac{d}{dt}\int_V \left(\rho\,\frac{u_i\,u_i}{2}\right)\mathrm{d}V = -\int_S \left(\rho\,\frac{u_i\,u_i}{2}+p\right)u_i\,n_i\,\mathrm{d}S + \int_V \left(p\,\frac{\partial u_i}{\partial x_i}\right)\mathrm{d}V + \int_V \frac{\partial}{\partial x_i}\left(\sigma_{ik}\,u_k\right)\mathrm{d}V$$

It is important to satisfy the conservation of the kinetic energy in the **inviscid** and **incompressible** limit, in an **unbounded domain**

Conservation principles

Kinetic Energy

$$\frac{d}{dt}\int\limits_V \left(\rho\,\frac{u_i\,u_i}{2}\right)\mathrm{d}V = \cancel{-\int\limits_S \left(\rho\,\frac{u_i\,u_i}{2} + p\right) u_i\,n_i\,\mathrm{d}S} + \cancel{\int\limits_V \left(p\,\frac{\partial u_i}{\partial x_i}\right)\mathrm{d}V} + \cancel{\int\limits_V \frac{\partial}{\partial x_i}\left(\sigma_{ik}\,u_k\right)\mathrm{d}V}$$

It is important to satisfy the conservation of the kinetic energy in the **inviscid** and **incompressible** limit, in an **unbounded domain**

Discrete energy conservation:

the **nonlinear** terms in the Navier-Stokes equations:

- do **not** contribute to the **net** variation of kinetic energy
- do not **dissipate** or **inject** spurious energy

Is OpenFOAM energy conserving?

- OpenFOAM is not provided with any strictly **energy conserving** solver, neither compressible or incompressible.

Low-dissipative schemes

- Vuorinen, V., et al. **"A low-dissipative, scale-selective discretization scheme for the Navier-Stokes equations."** Computers & Fluids 70 (2012): 195-205.
- Vuorinen, V., et al. **"On the implementation of low-dissipative Runge-Kutta projection methods for time dependent flows using OpenFOAM."** Computers & Fluids 93 (2014): 153-163.

rhoEnergyFoam: Shock Capturing Capabilities

Kinetic energy and shock waves

The **kinetic energy** is **not conserved** in the presence of **shock waves**, neither in the inviscid limit:

- energy conserving scheme near the shock is **conceptually wrong**
- **shock waves** are **detected** with an appropriate **shock sensor**
- **some** amount of **numerical dissipation** is added near the shock

Hybrid Numerical Flux

$$\widehat{f}_{j+1/2} = \widehat{f}^{ec}_{j+1/2} + \begin{cases} 0 & if\ ishock = 0 \\ \widehat{f}^{AUSM}_{j+1/2} & if\ ishock = 1 \end{cases}$$

energy conserving + dissipative part of AUSM flux

Implementation into OpenFOAM:**rhoEnergyFoam**

Space discretization

Energy consistent **numerical fluxes** implemented
in the OpenFOAM library:

- **stability** without **numerical dissipation**
- **shock capturing** capabilities with hybrid **energy conserving/AUSM**

Time integration

Explicit fourth-order **Runge-Kutta** time integration:

- **low-storage** implementation, suitable for LES and DNS.

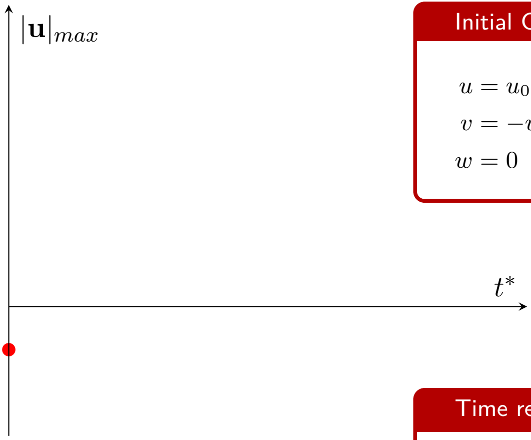
Implementation into OpenFOAM:**rhoEnergyFoam**

rhoEnergyFoam:

- compressible unsteady solver (subsonic and supersonic)
- exact kinetic energy conservation in the inviscid and incompressible limit
- 2nd order accurate in space, 4th order in time
- Integration with the OpenFOAM **thermodynamic** and **turbulence** libraries
- Suitable for DNS and LES of compressible flows.

Taylor-Green flow

Test case proposed by **Duponcheel et al. (2008)**



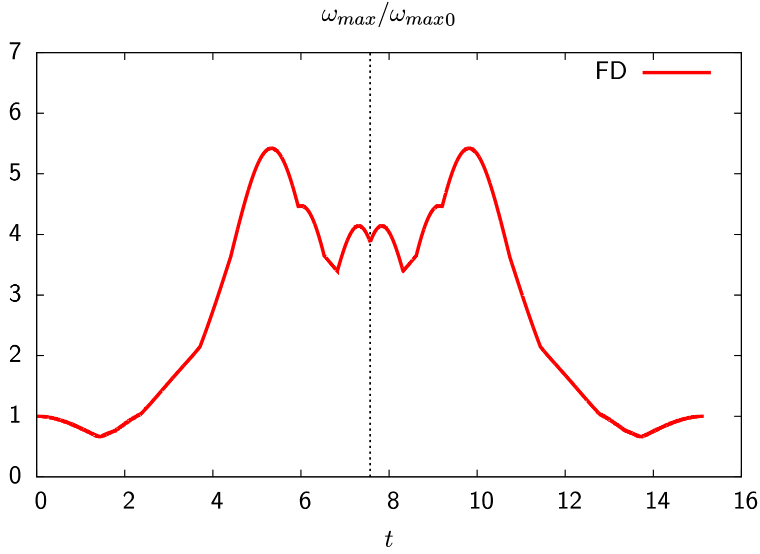
Initial Conditions

$$u = u_0 \sin(k_0\, x)\, \cos(k_0\, y)\, \cos(k_0\, z)$$
$$v = -u_0 \cos(k_0\, x)\, \sin(k_0\, y)\, \cos(k_0\, z)$$
$$w = 0$$

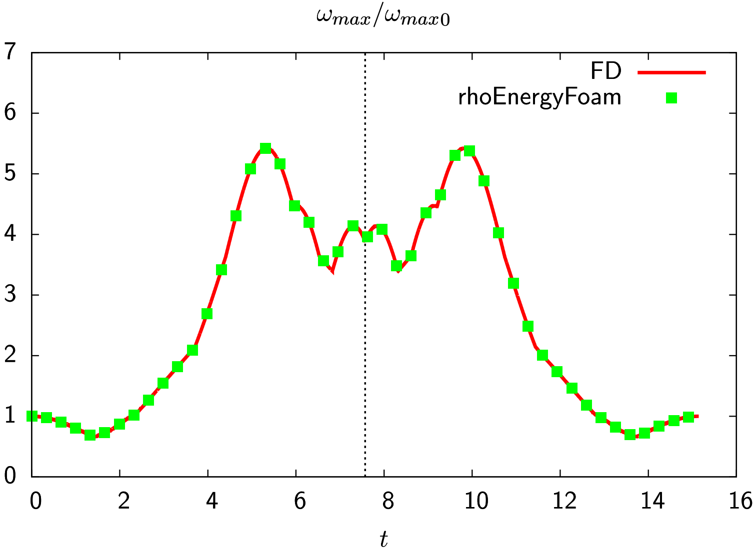
Time reversibility

Euler equations are **time reversible**, that is:
$\mathbf{u}(t, \mathbf{x}) \Rightarrow -\mathbf{u}(-t, \mathbf{x})$

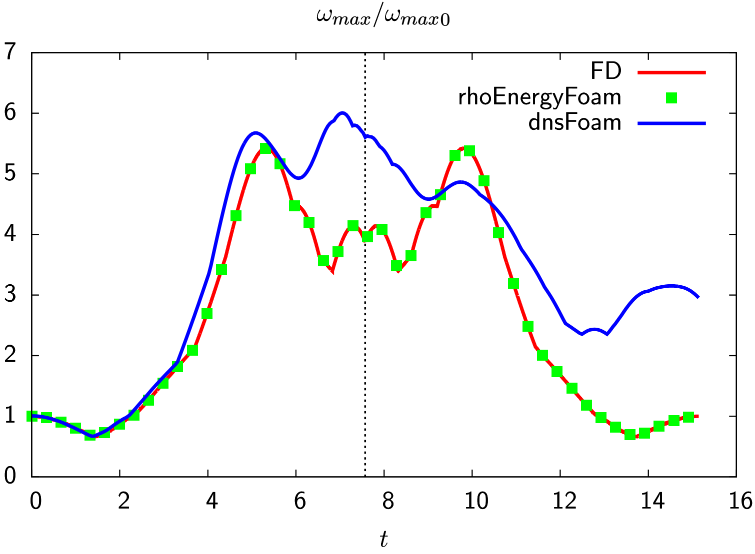
Taylor-Green flow



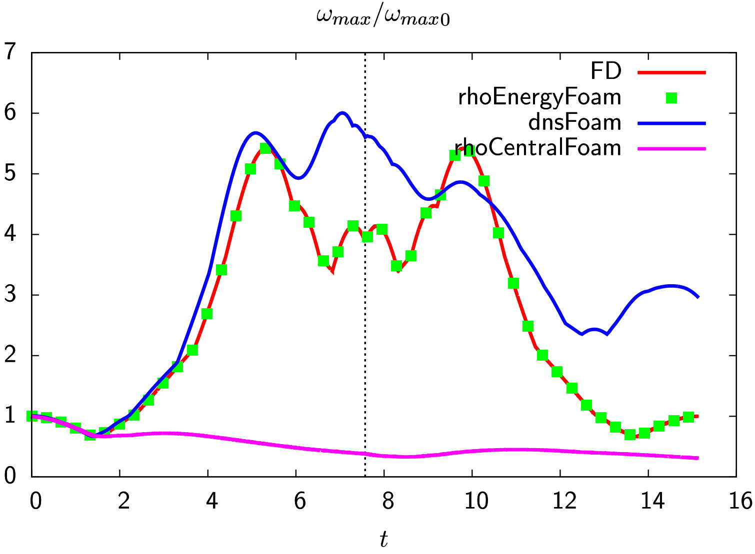
Taylor-Green flow



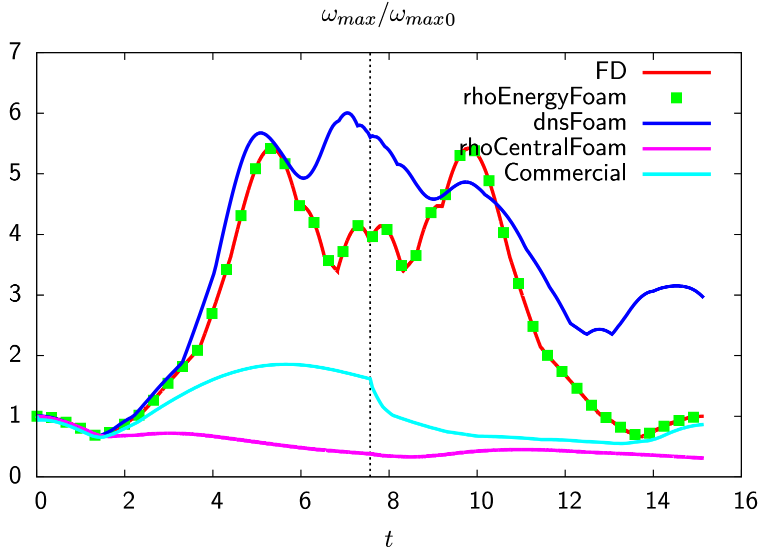
Taylor-Green flow



Taylor-Green flow

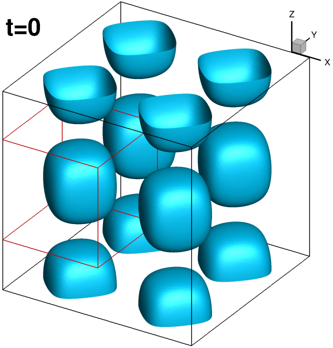


Taylor-Green flow

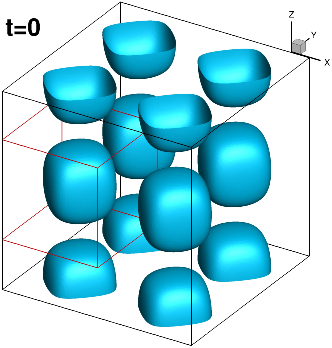


Taylor-Green flow

dnsFoam



rhoEnergyFoam



Circular Cylinder at low Reynolds number

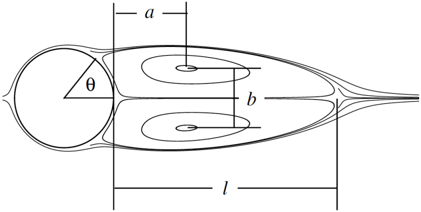
Regimes

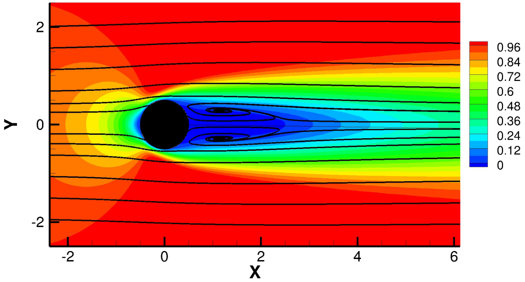
Williamson (1996)

- $Re < 49$ stationary laminar
- $49 < Re < 190$ laminar, vortex shedding
- $190 < Re < 260$ transitional wake

Re=40, Mesh=64x64

	l/d	C_D
rhoEnergyFoam	2.40	**1.54**
dnsFOAM	2.36	**1.53**
rhoCentralFoam	1.19	**1.90**
Commercial	1.58	**1.63**
Taira et al. (2007)	2.30	1.56
Linnick et al. (2005)	2.28	1.54
Coutanceau et al. (1977)*	2.13	1.59





Circular Cylinder at low Reynolds number

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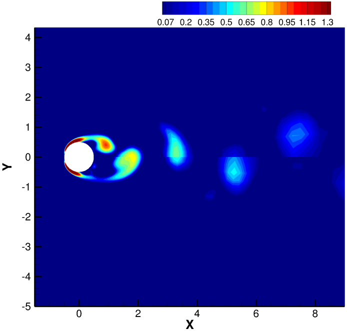
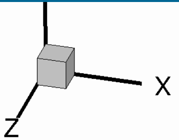


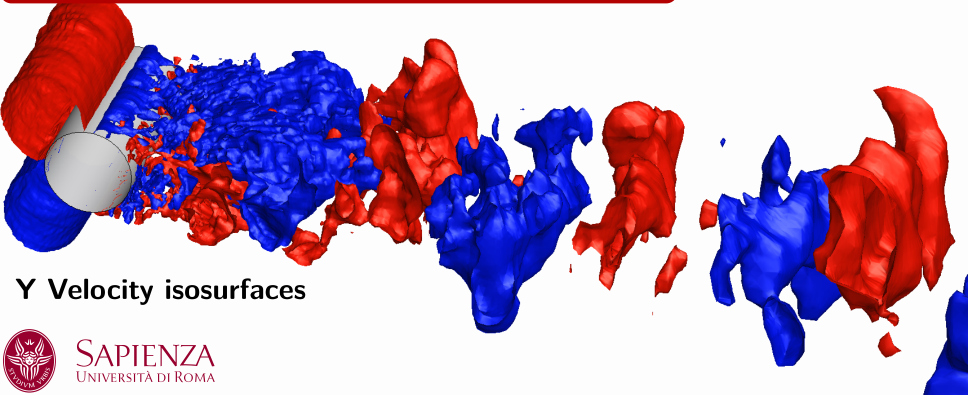
Re=200, Mesh=128x128

	C_D	C_L
rhoEnergyFoam	$\mathbf{1.33} \pm 0.050$	± 0.66
dnsFoam	$\mathbf{1.33} \pm 0.045$	± 0.69
Commercial	$\mathbf{1.26} \pm ?$	± 0.57
rhoCentralFoam	$\mathbf{1.30} \pm ?$	± 0.40
Taira et al.	1.35 ± 0.044	± 0.69
Linnick et al.*	1.34 ± 0.044	± 0.69



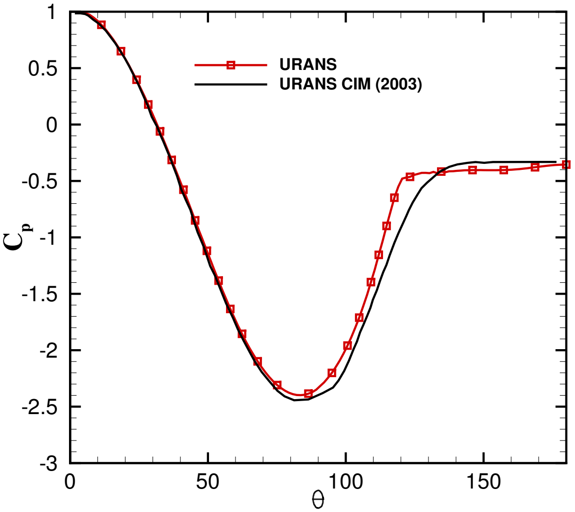
Flow Parameters

- $Re_D = 10^6 \; M_\infty = 0.1$
- URANS (2D) and LES (3D)
- Mesh $256 \times 256 \times 48 \; L_z = 2D$
- Ref. Catalano Iaccarino Moin (CIM) (2003)

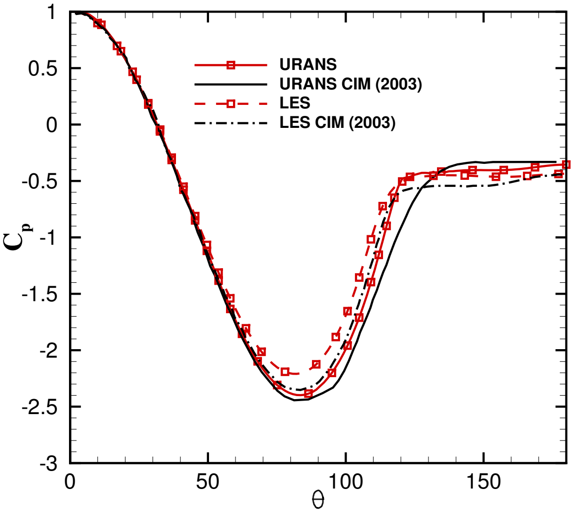


Y Velocity isosurfaces

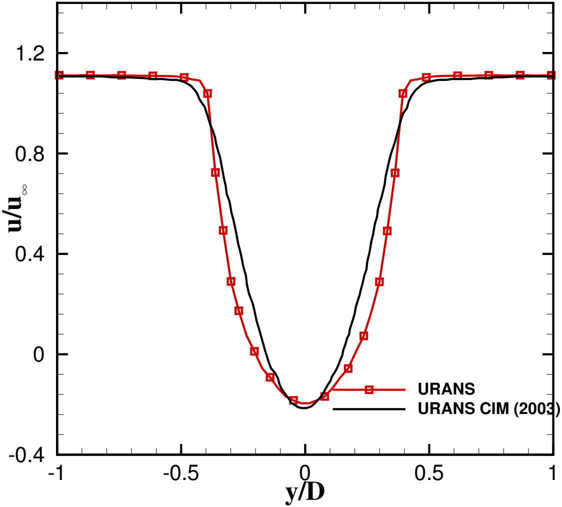
Pressure Coefficient



Pressure Coefficient



Streamwise velocity at $x = 0.75$



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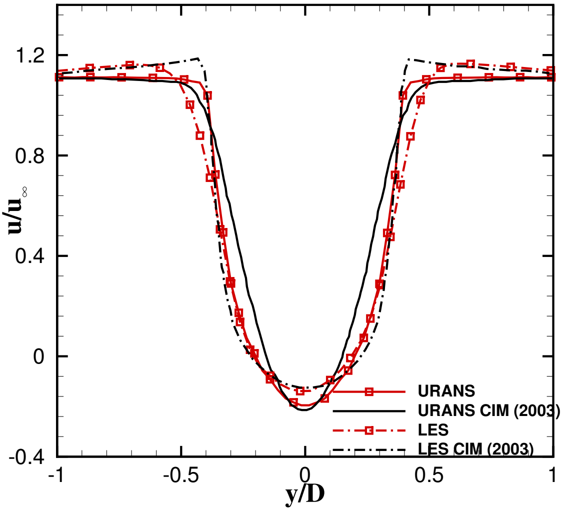

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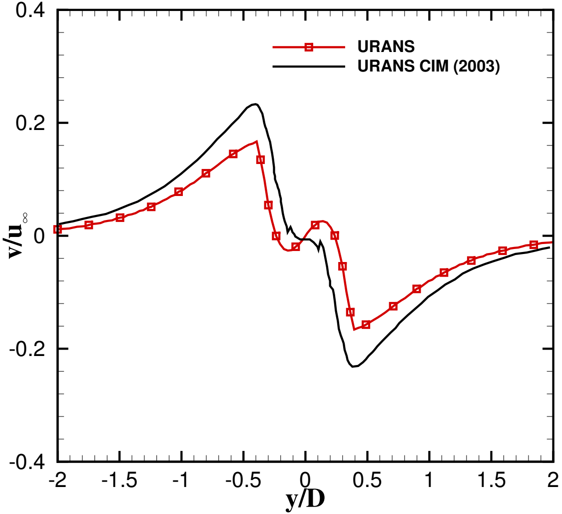

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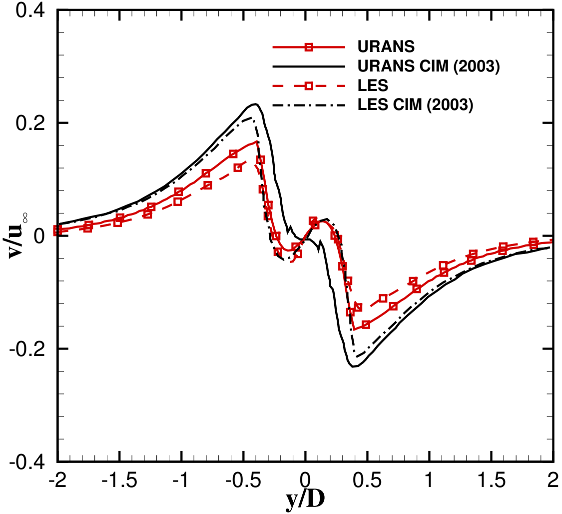

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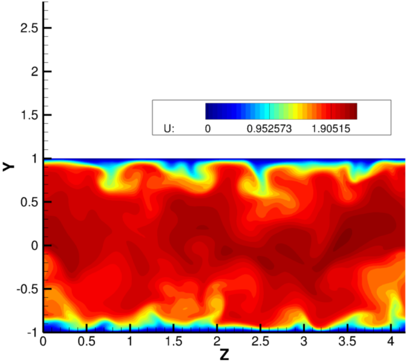
Compressible channel flow: DNS

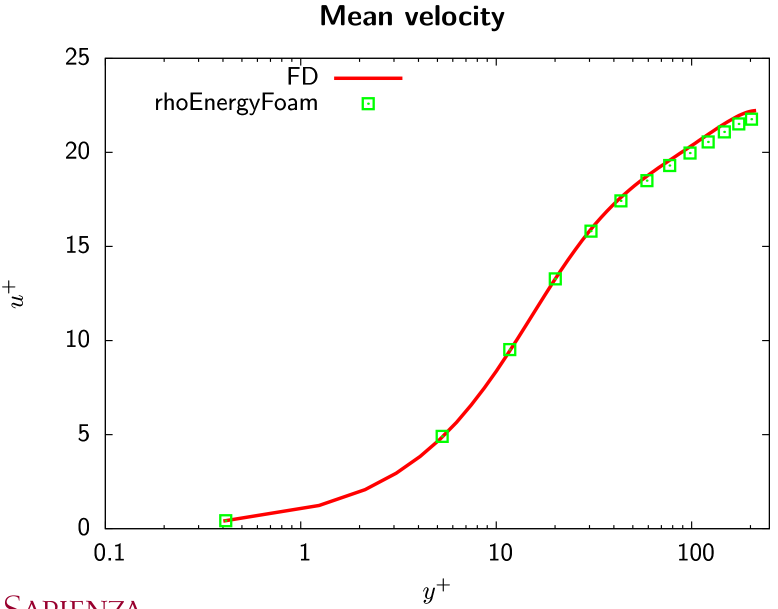
rhoEnergyFoam

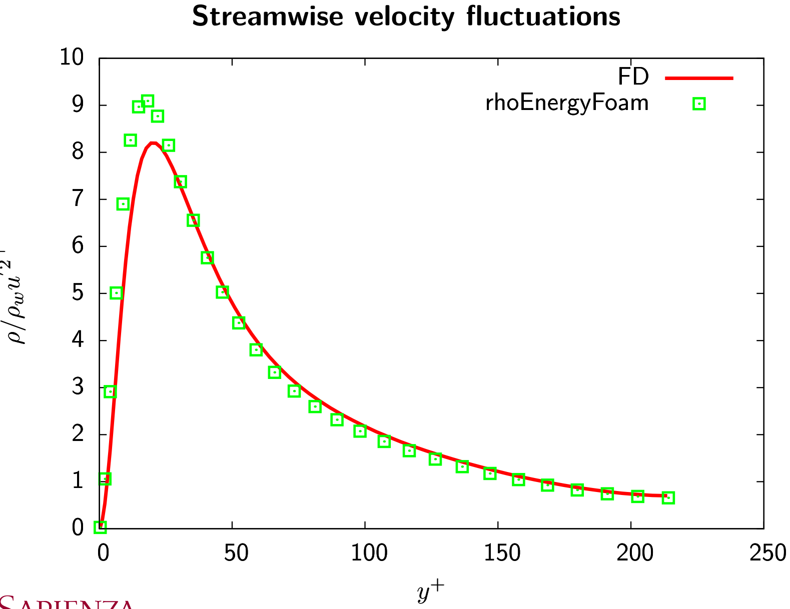


Computational set-up

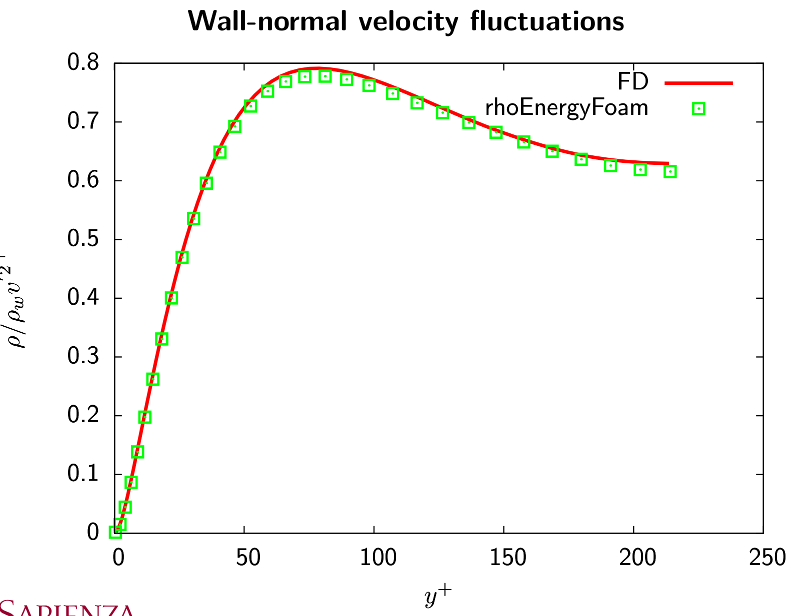
- **Parameters:** $Re_\tau (= u_\tau h/\nu) = 220$, $M_b (= u_b/a_w) = 1.5$
- **Forcing: momentum** and **total energy** equation, to maintain constant mass flow rate.
- **Box dimensions:** $L_x \times L_y \times L_z = 4\pi h \times 2h \times 4/3\pi h$
- **Mesh resolution:** $\Delta x^+ \approx 10$, $\Delta y^+ < 4$, $\Delta z^+ \approx 5$ **About 9 milions cells**

Compressible channel flow: DNS

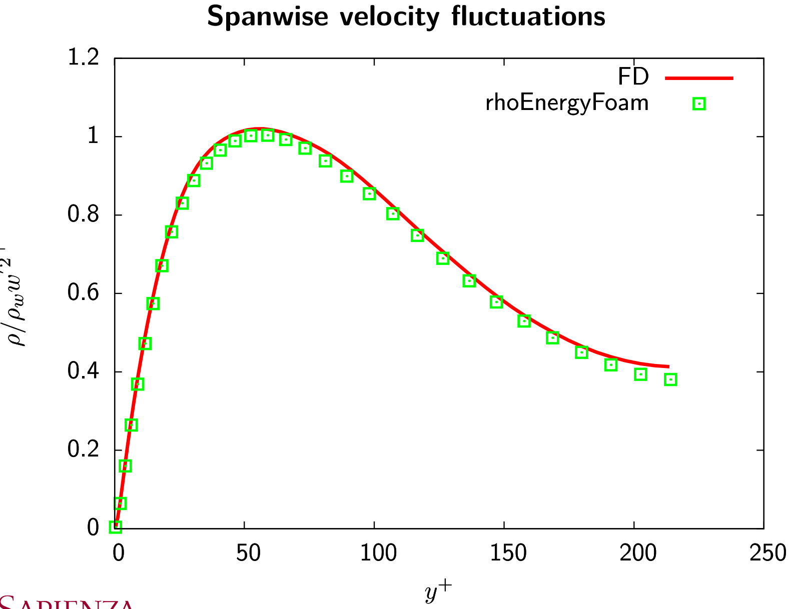



Streamwise velocity fluctuations
FD
rhoEnergyFoam
$\rho/\rho_w u'^{2+}$
y^+
0
1
2
3
4
5
6
7
8
9
10
0
50
100
150
200
250

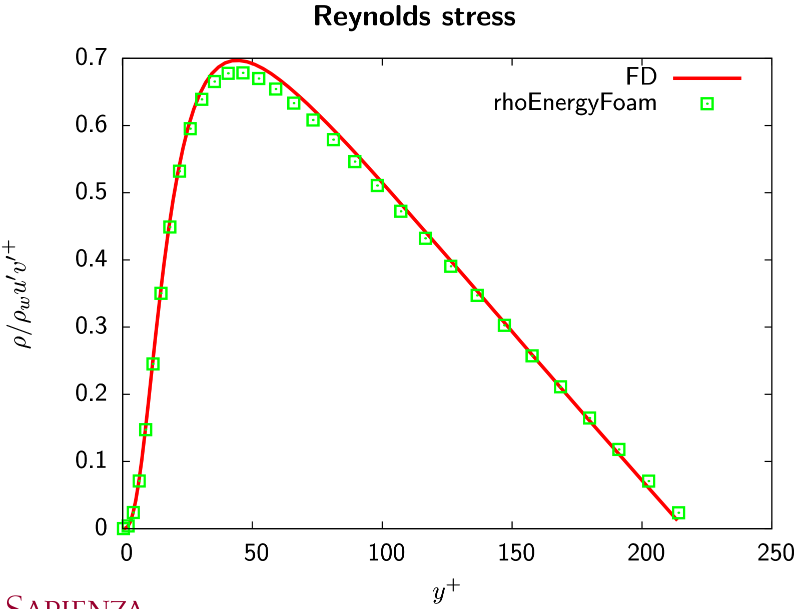
Compressible channel flow: DNS



Compressible channel flow: DNS



Compressible channel flow: DNS

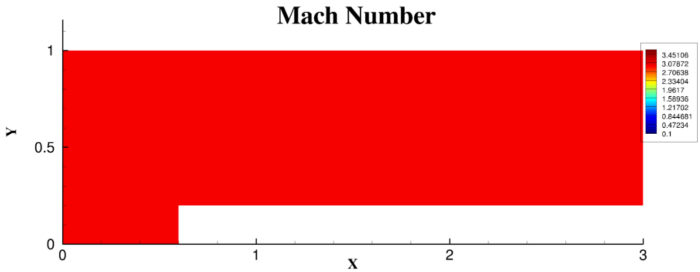


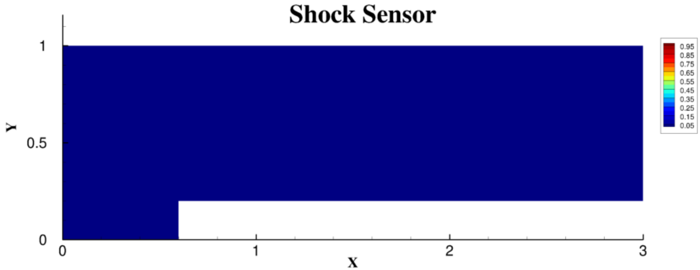
Flows with shocks

Inviscid test cases

- Flow past backward facing step, $M_\infty = 3$ (Woodward&Colella (1984))
- Flow past Onera M6 wing, $M_\infty = 0.84$ $\alpha = 3.04$ (AGARD Report 138)

Flows with shocks: backward facing step





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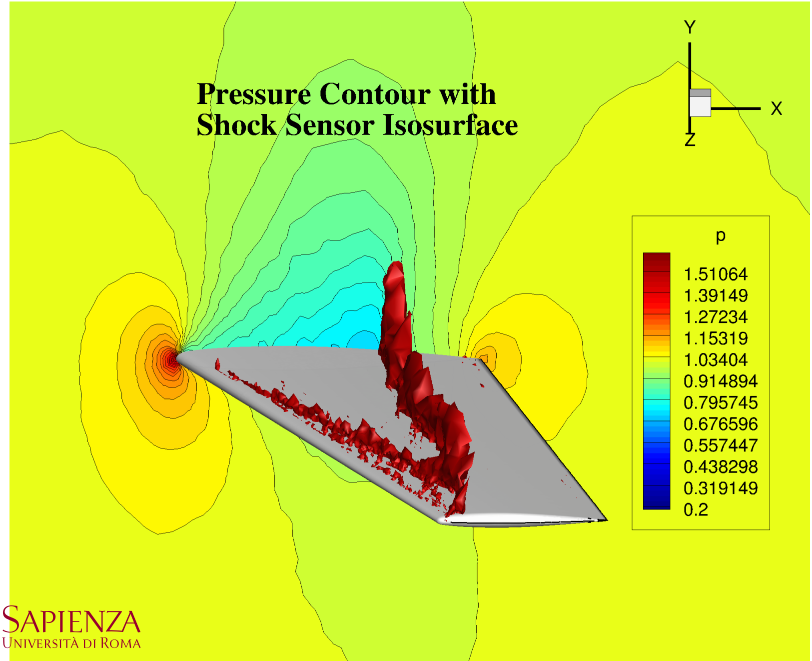
Flows with shocks: M6 wing



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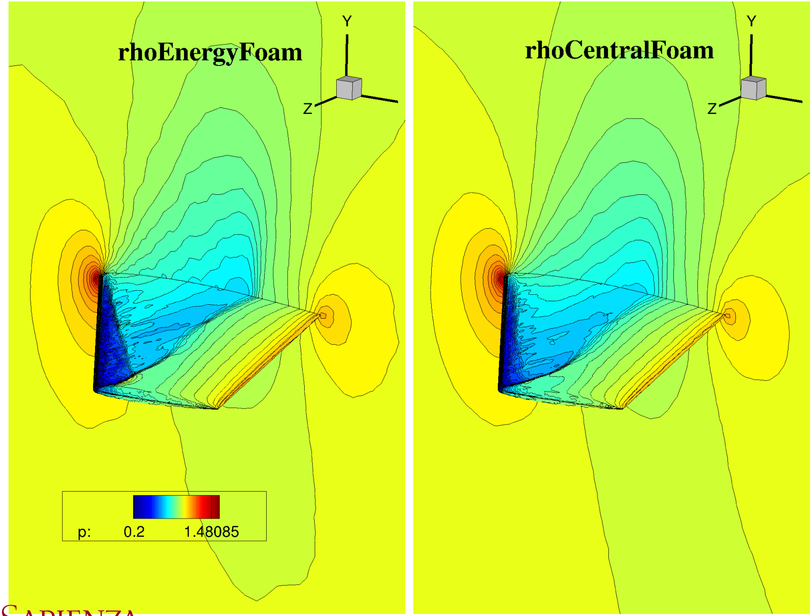
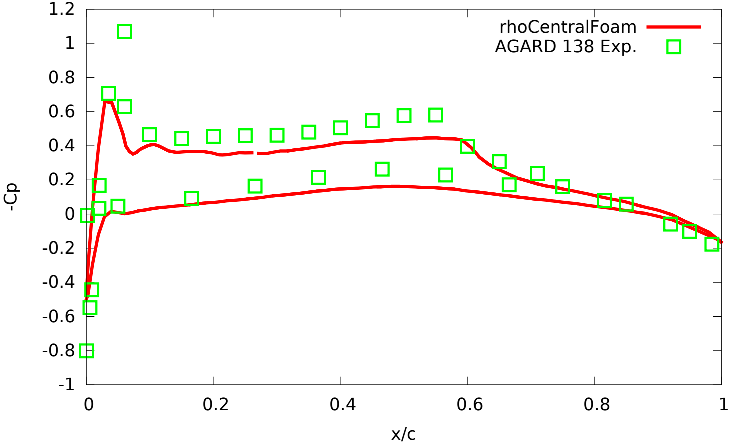


Flows with shocks: M6 wing

Spanwise station $y/c = 0.2$



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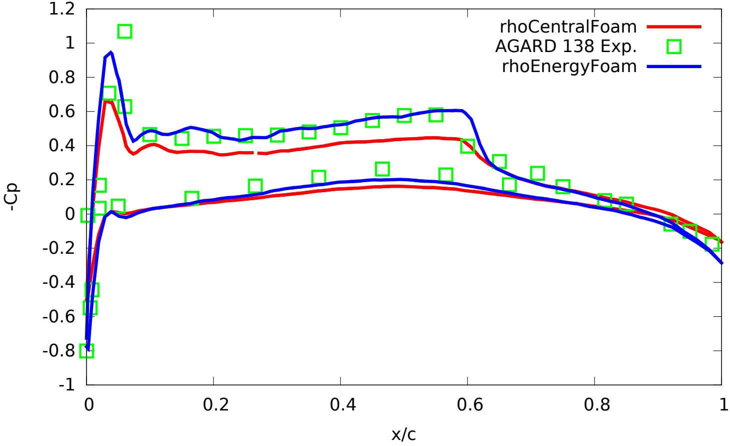
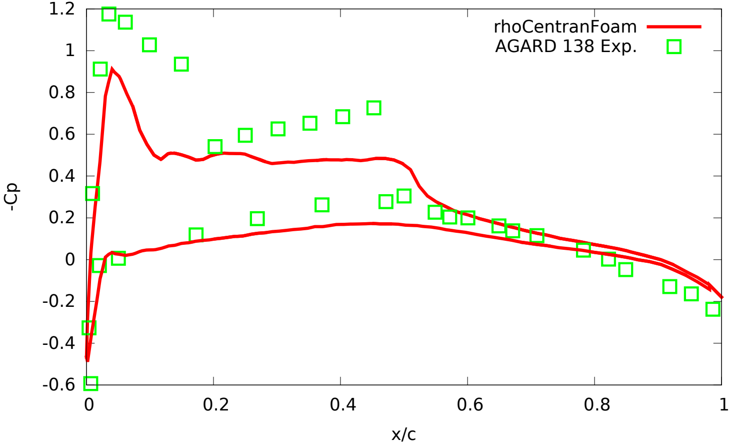
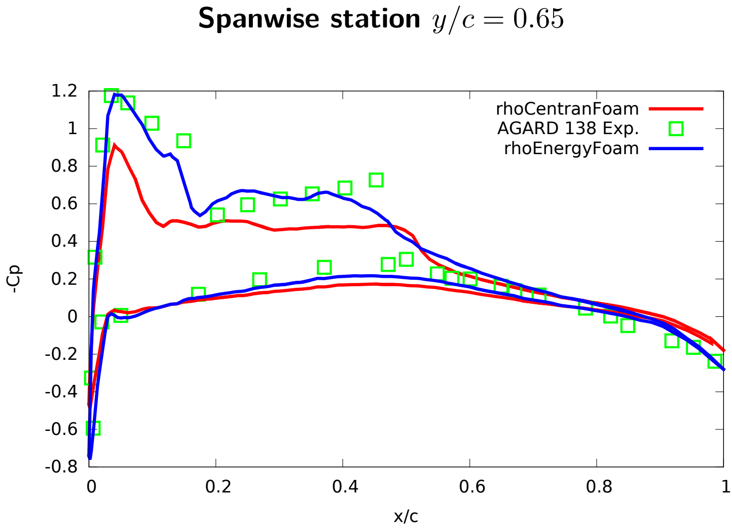


Flows with shocks: M6 wing

Spanwise station $y/c = 0.65$





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Computational Efficiency

What about the **computational efficiency** ?

Computational times

FD	rhoEnergyFoam	rhoCentralFoam	dnsFoam
1.0	5.9	5.9	5.1

Scalability

Linear Scalability

- **FD:** up to 64K CPUs and more
- **standard OpenFOAM solvers:** up to 1K CPUs.
  Bottlenecks: I/O and linear solvers
- **rhoEnergyFoam: ?**

Conclusions

OpenFOAM/Commercial are **not energy-conserving**
The **novel** solver **rhoEnergyFoam** guarantees:

- **stability** without addition of artificial **dissipation**
- shock capturing with **hybrid** energy conserving/AUSM
- a greater **fidelity** to the physics

Future work

- Scalability analysis of rhoEnergyFoam
- RANS and LES simulations of flows with shocks

Conclusions

Conclusions