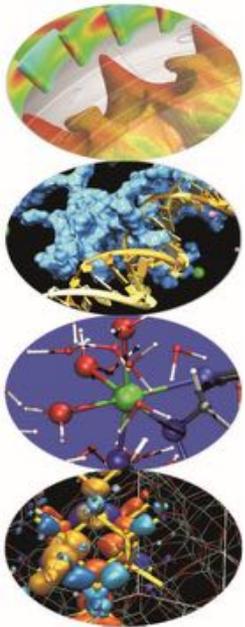


FFD, mesh morphing & reduced order models: Enablers for efficient aerodynamic shape optimization

F Salmoiraghi, G Rozza
A Scardigli, **H Telib**

SISSA mathlab
OPTIMAD engineering srl



Outline



1. Practical problems in shape optimization
2. Enabling of large scale aerodynamic shape optimization
 - Shape and mesh morphing
 - Efficient sampling strategies
 - ROMs based on POD
3. Questions & hopefully answers

Motivation

1. Difficult to set-up (Integration)

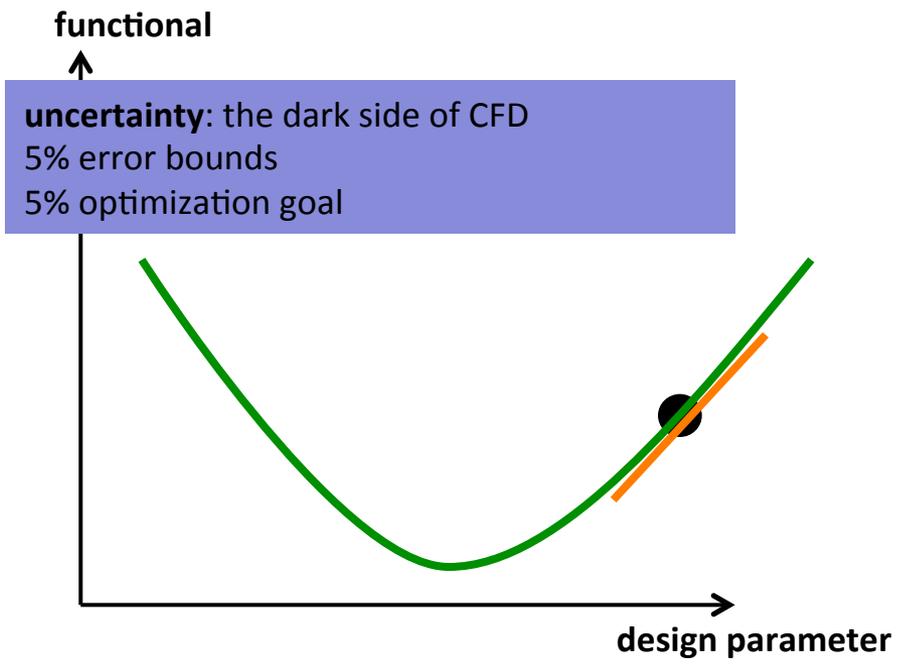
- identification of parameters, parameterization itself etc
- totally automatized (geometry creation, pre-processing)
- especially critical if at advanced design

2. Expensive (Availability)

- computing resources sized for analysis
- licenses CAD, CFD

Information provided by solver

- **Evaluate functional (Comparative)**
– Industry standard
- **Evaluate gradient (convergence, direction)**
– Advanced capabilities
- **Evaluate Hessian (step, local topology)**
– Cutting edge



Time & Costs

- cost of real life RANS approx. $C_{\text{HFM}} = 2000 \text{ cpuh}$
- # of design variables $O(10)$
- cost of computing $0.1\text{€}/\text{cpuh}$
- cost of licenses 0

2 Level multi-fidelity approach using response surface (neglectable cost) + HFM

- global optimization run $O(100) - O(1000)$ $2.e5 - 2.e6 \text{ cpuh}$
- computing resources $O(10) - O(1000)$ $2. e2 - 2.e4 \text{ h}$

1week – 2years

:stop we you have to

20K€ - 200K€

:convince your management

3 Level multi-fidelity approach using response surface (neglectable cost) + ROM + HFM

- cost of Reduced Order Model $C_{\text{ROM}} = \sigma C_{\text{HFM}}$
- global optimization run $O(1000) \text{ ROM} + O(10) \text{ HFM}$ $2.e6 \sigma + 2.e4 \text{ cpuh}$
- necessary saving factor σ $O(1\text{month}), O(10\text{K€})$ $1 - 1/100$

About optimization

- uncertain
 - i. hope in “systematic errors” or “conservation of trends”
 - ii. what if your new prototype(!) performs worse than original??
 - iii. mastered by empirical knowledge
 - iv. limited basin of validity
- it takes specialized technical staff
 - i. to set all optimization parameters (parameters?, strategy)
 - ii. to build an automatic workflow (geometry??. mesh??. 1month)
 - iii. and to do some preliminary investigations (sensitivity, uncertainty) (1.5 month)
 - iv. which help you to set up the optimization run (0.5 month)
- very costly wrt to analysis
 - i. computing: 10x – 100x
 - ii. staff: 10x – 100x

Automatic shape optimization is used only if strategic

HPC view

- 2 level parallelization
 - concurrent jobs
 - parallel execution of single job
- serial part of workflow $0.01-0.1 * (\text{parallel part})$
 - geometry & mesh processing



Free-Form Deformation & Mesh-Morphing using Level-Sets

Requirements to geometrical engine

Geometry represented as surface triangulation (CAD neutral)

1. parameterization of complex geometries

- Free-Form Deformation developed by Desideri et al @INRIA
- Mesh-Morphing using RBFS

2. constraints handling

- C^0 , C^1 , C^2 conditions on arbitrary boundaries
- no-penetration condition

3. features & curvature based surface mesh adaptation

- if deformed geometry needs finer surface mesh than original geometry

Different approaches

Direct morphing of surface & mesh

- surface constraints are handled by choosing wisely CPs (e.g. inner points of lattice, rbf nodes with given distance to boundaries)
- mesh constraints handled via limitations on bounds of CP
- very cheap, since only evaluations of Deformation Lattice or RBF required

<http://mathlab.sissa.it/pygem>

used in the final examples!!

1. **surface deformation**
2. **mesh morphing**

- surface constraints via topological information (geodesic distances) on surface
- volume constraints via computation of Euclidean distances
- expensive, since constraints are computed explicitly at each deformation
- very expensive, since mesh morphing is formulated as an interpolation (minimization) pb on surface deformation

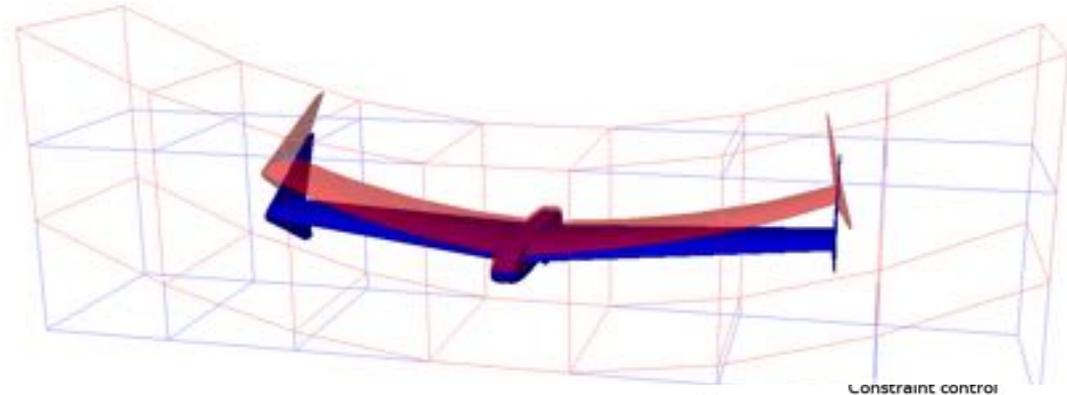
<https://github.com/optimad/MIMMO>

<http://www.optimad.it/products/camilo>

explicit surface constraints

Free-Form Deformation applies a displacement vector $N_i = S_i + D(S_i)$

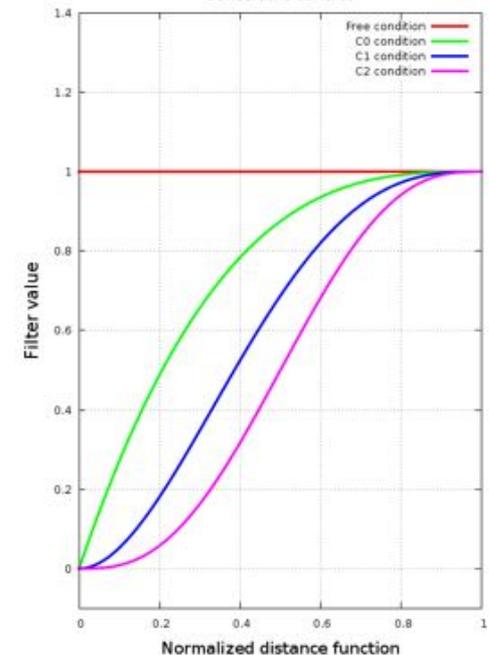
- difficult to impose regularity
- conditions on an arbitrary shaped boundary Γ



- our approach introduces a weight function
- $N_i = S_i + w[\phi(S_i|\Gamma)] D(S_i)$

- with $w(0) = 0$ for C^0 condition
- with $w(0) = 0, w'(0) = 0$ for C^1 condition
- with $w(0) = 0, w'(0) = 0, w''(0) = 0$ for C^2 condition

- $\phi(S_i|\Gamma)$ must provide topological information
- but it requires that $\phi(S_i|\Gamma)$ is C^0, C^1 and C^2 respectively



topological information 1: exact geodesic distances



resulting function is only C^0 , cannot impose higher regularity

source: Crane et al.

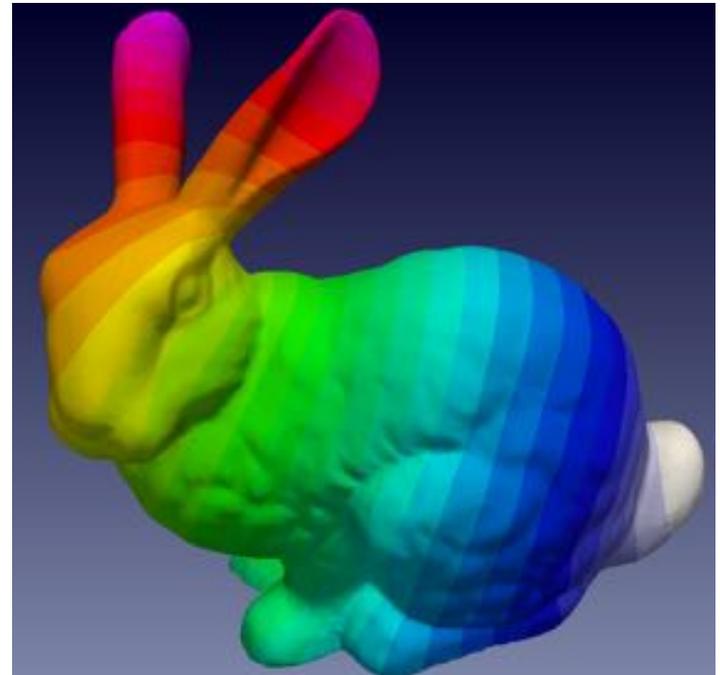
topological information 2: smoothed geodesic distances

we impose the following optimization problem:

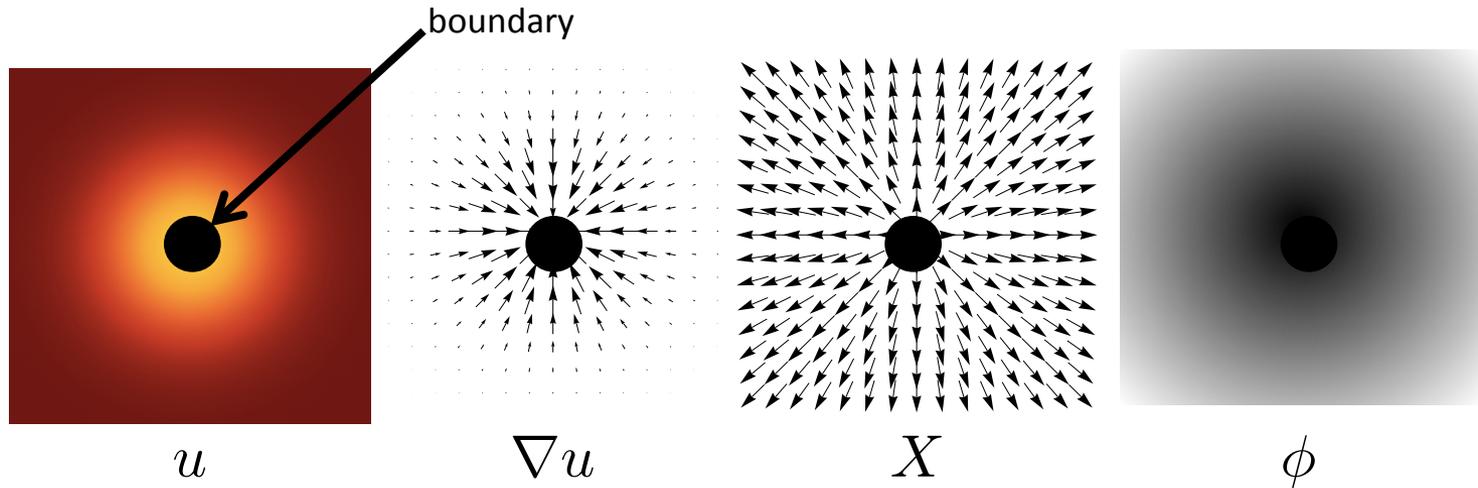
- as close as possible to geodesic LS to keep topology
- constraint on C^2 continuity
- infinite solutions \rightarrow smoothing parameter

this leads to the solution of 1 parabolic and 1 elliptic PDE

- sparse direct solver with reordering is used



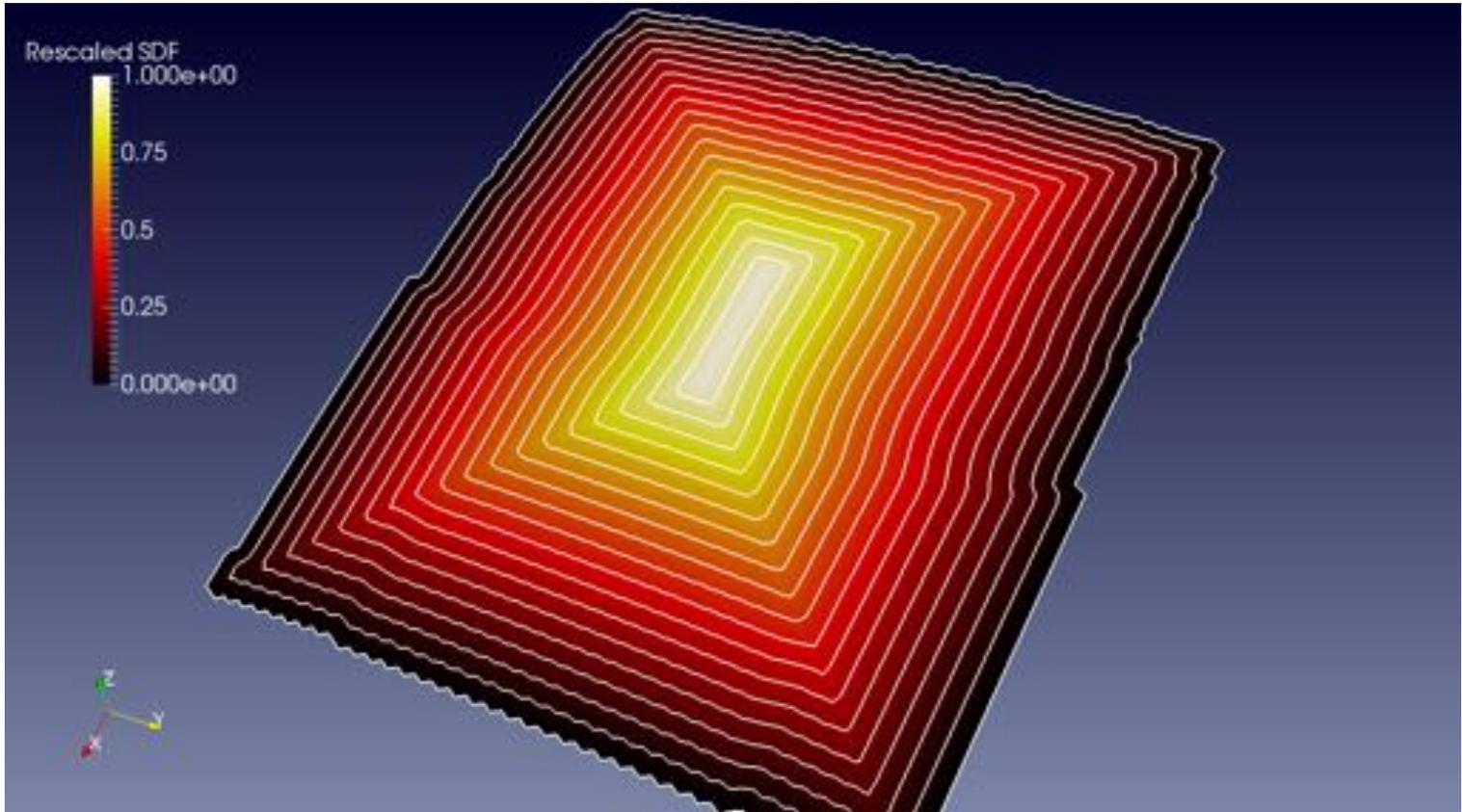
Geodesics based on heat kernel



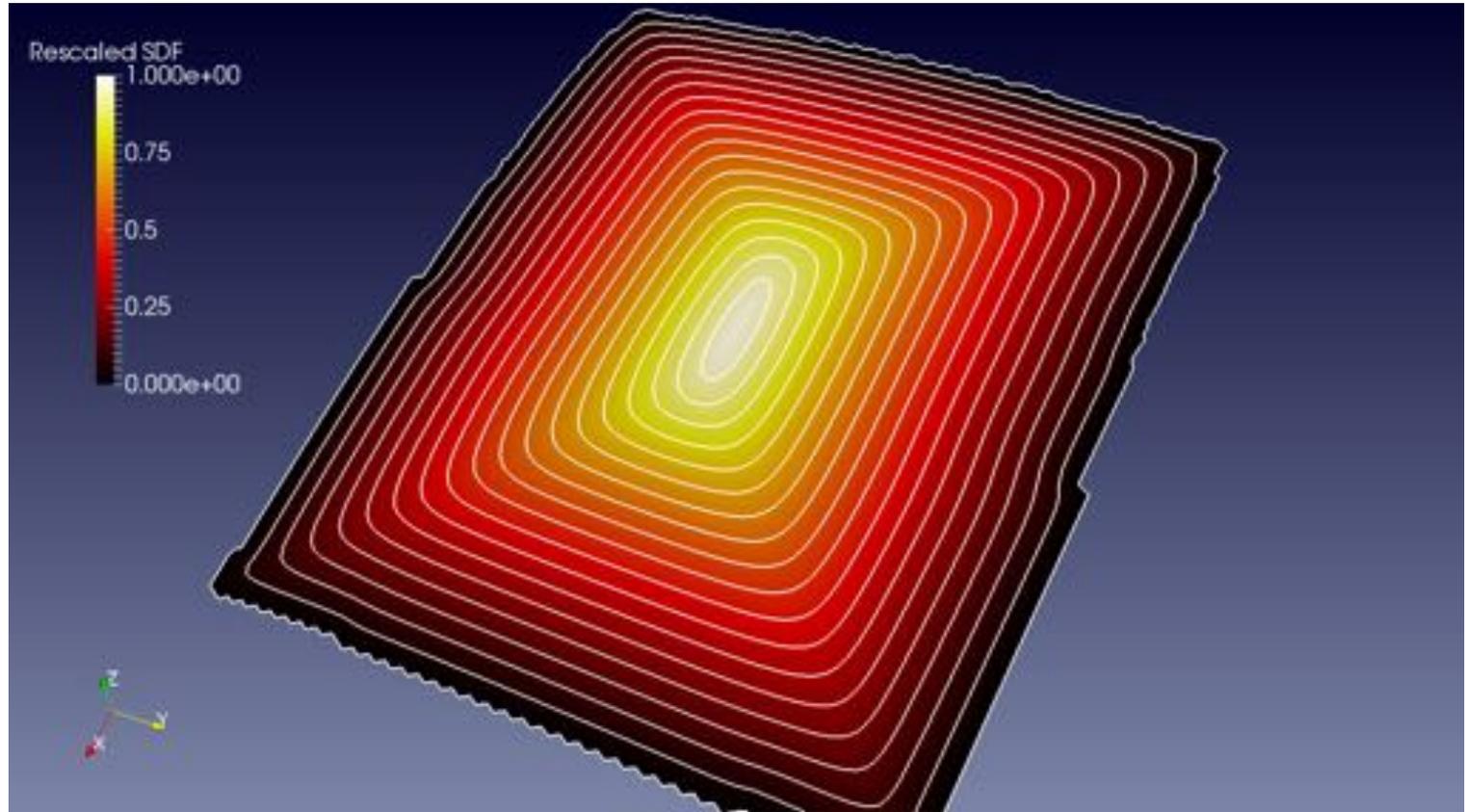
1. resolve heat equation $u_t = -u_{,xx}$ for a given time (parameter for smoothing)
2. calculate $X = -\text{grad } u / |\text{grad } u|$
3. solve $\text{lap } \Phi = \text{div grad } X$

As similar as possible to geodesic distance, but imposes smoothness

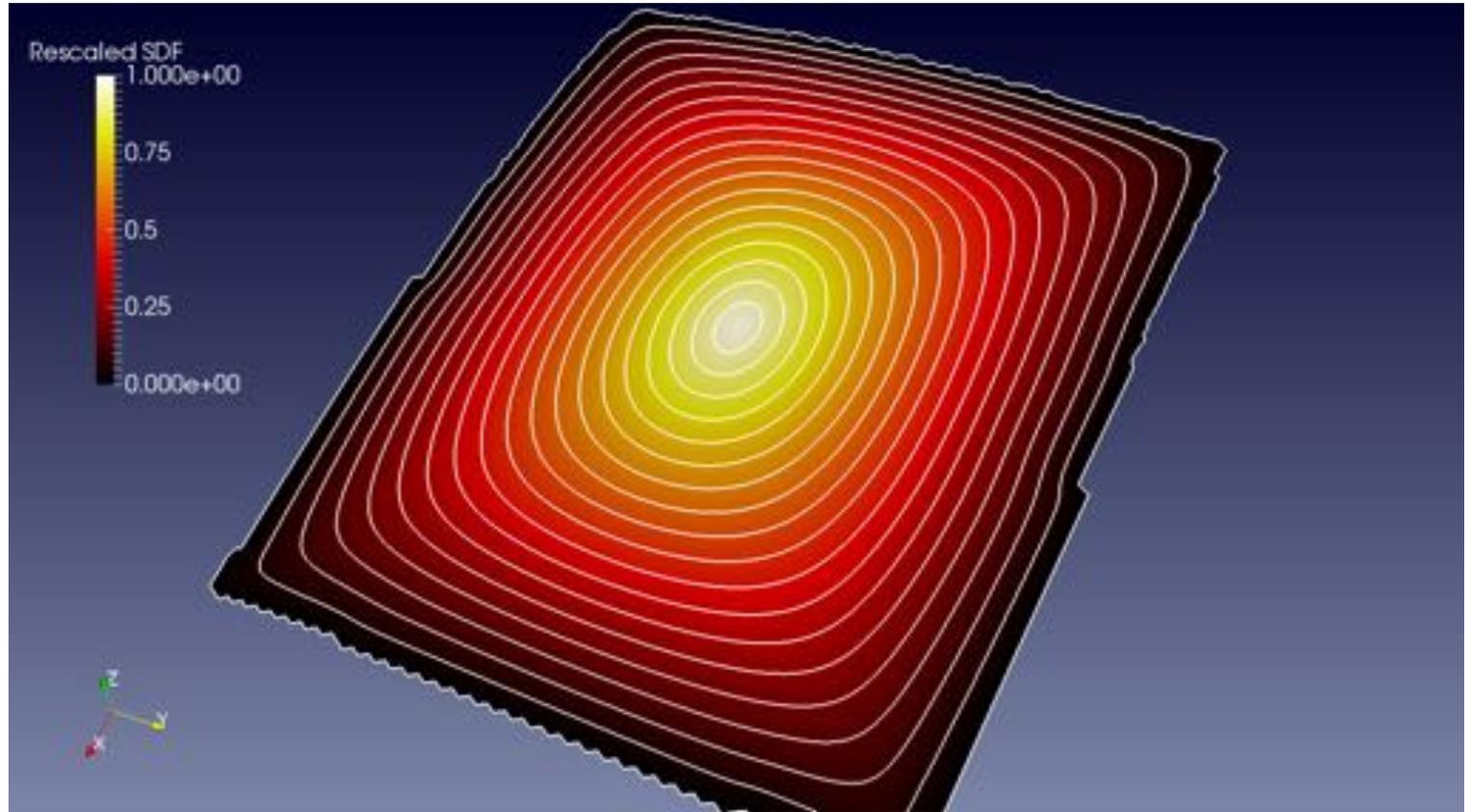
Heat kernel solution: $t=0$



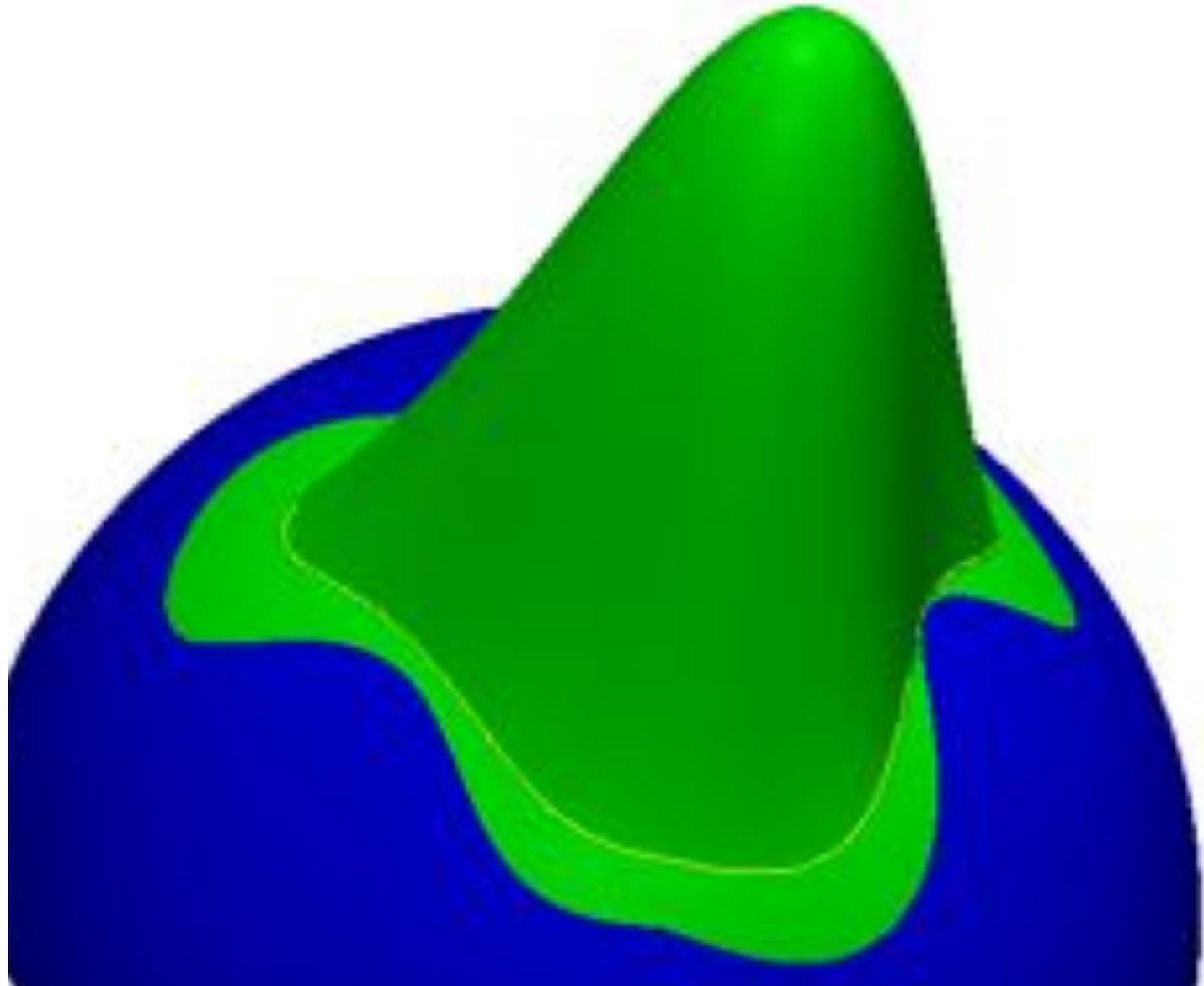
Heat kernel solution: $t=0.1$



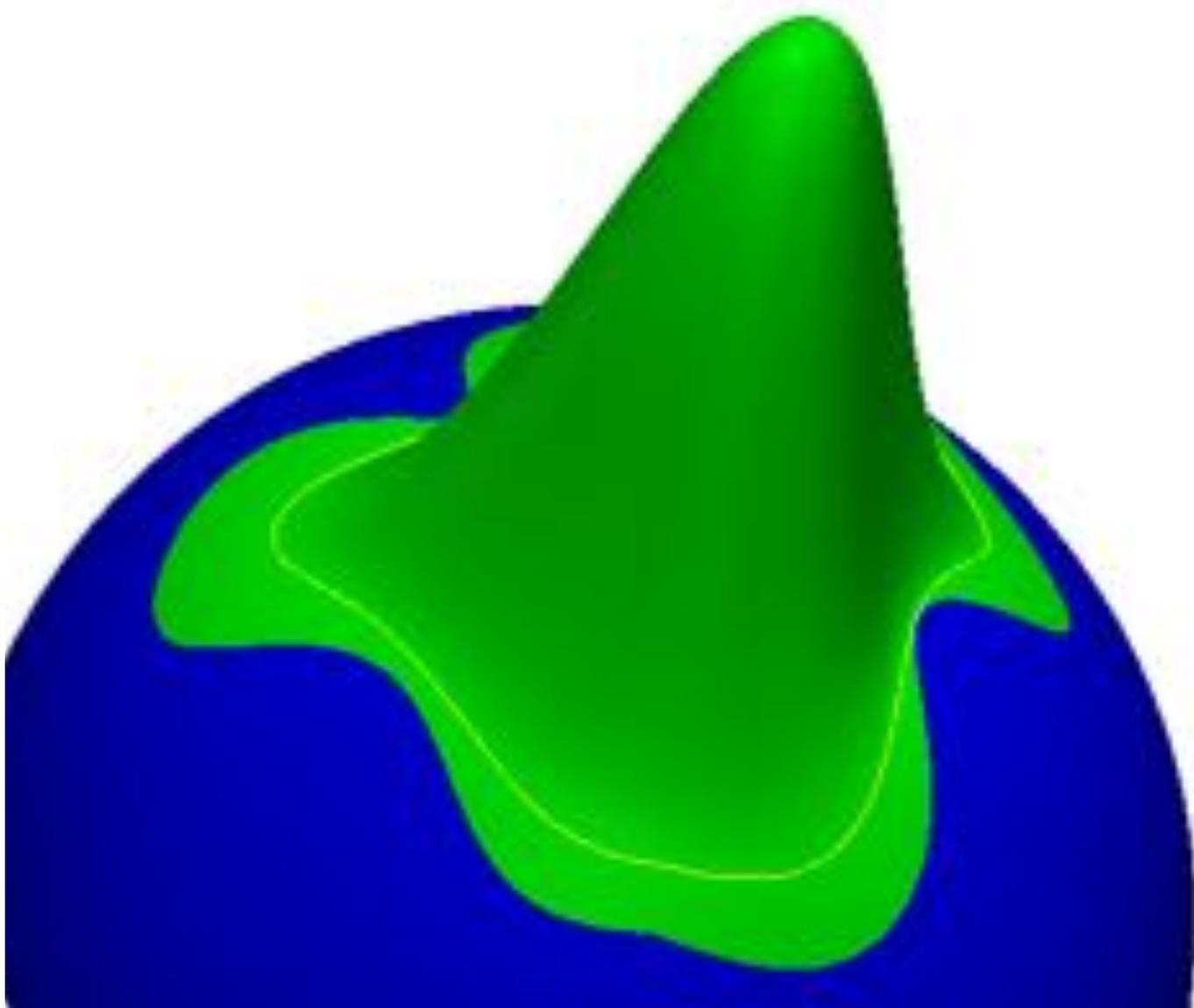
Heat kernel solution: $t=1.0$



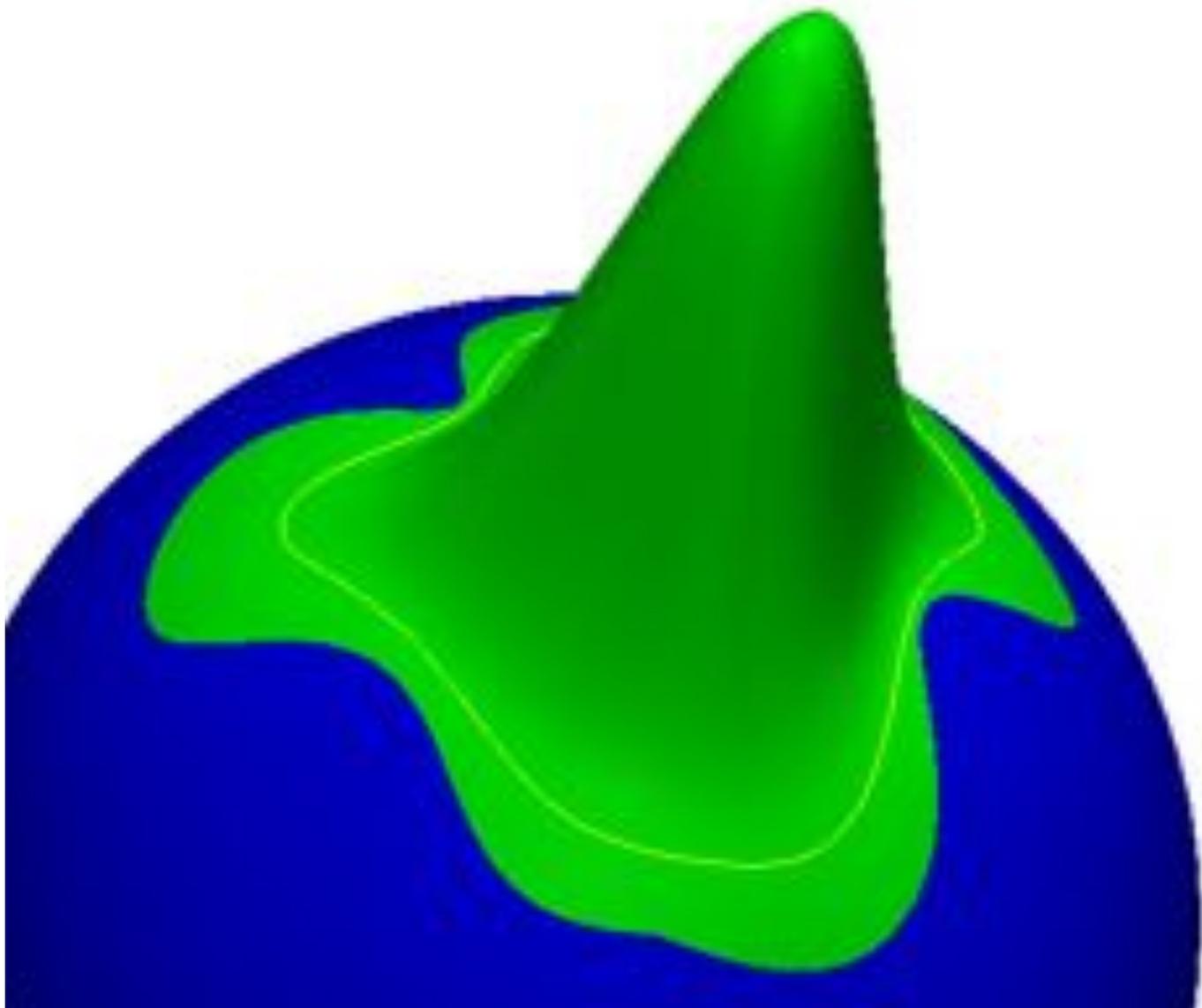
Deformation using C^0 constraint



Deformation using C^1 constraint



Deformation using C^2 constraint



volume constraints

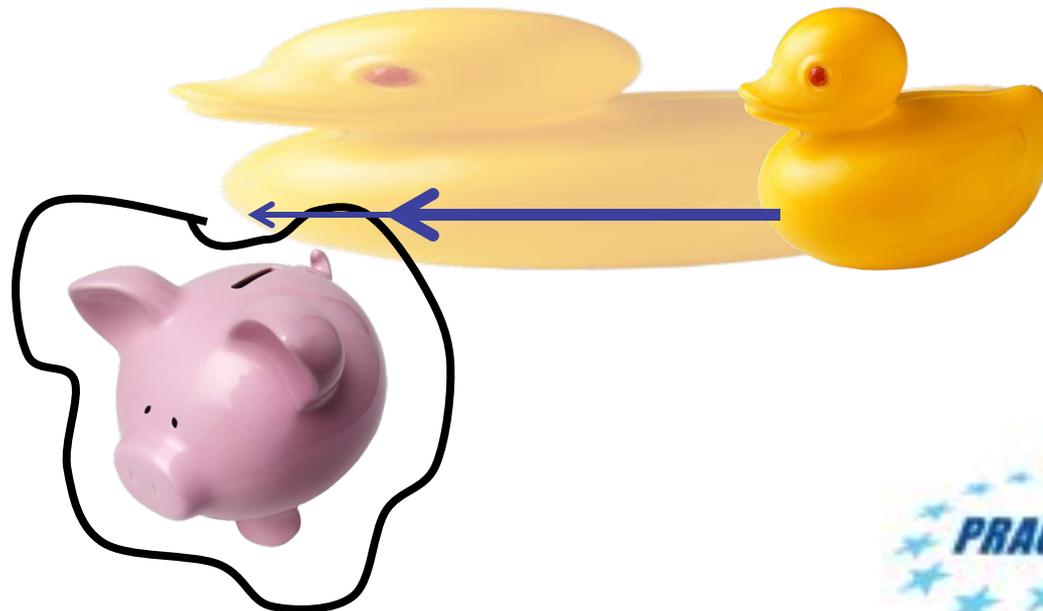
Common constraint: Distance to a given surface should be maintained

- 1. User should indicate only intuitive information
 - surface (open or closed)
 - distance to be maintained
 - bounds on CP non-intuitive and often not efficient
- 2. Combined control algorithm
 - Ray-tracing
 - Level Set
 - Line search
- 3. Two different types of rescaling algorithms available

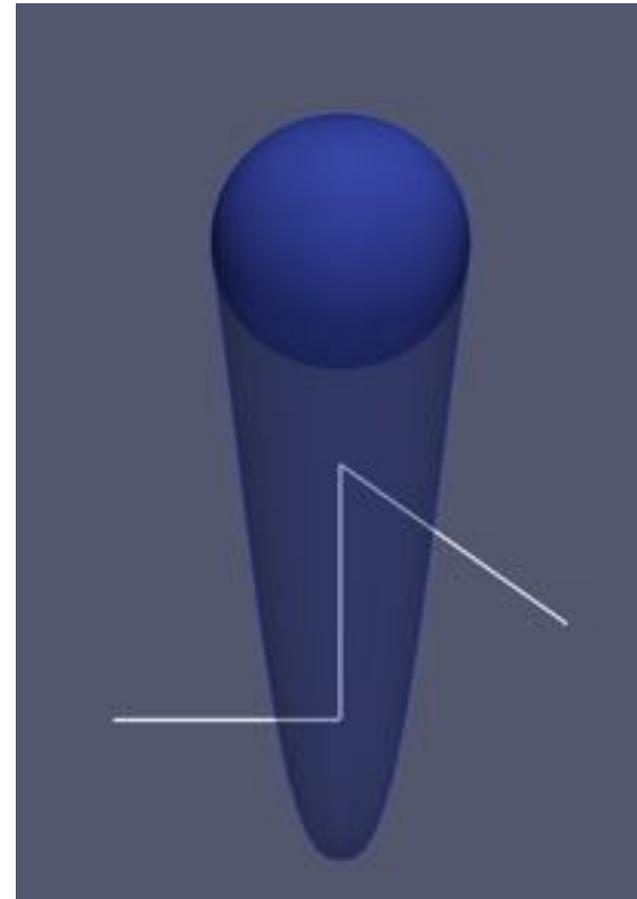
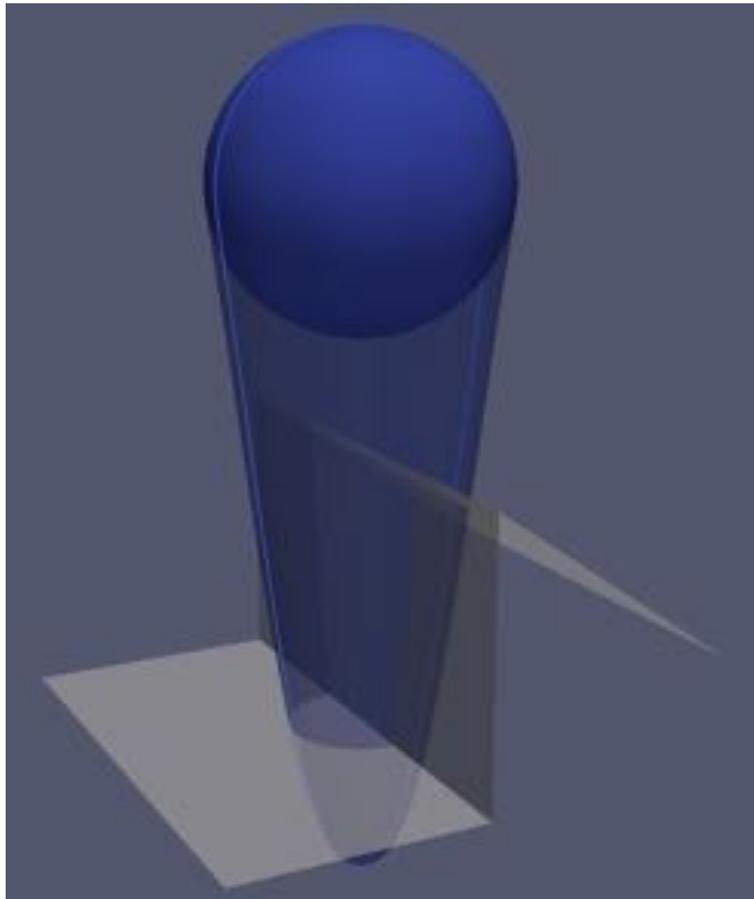
basic algorithm

- Given an unconstrained deformation field and a control surface
- Calculate the Level-Set function (signed distance) of constraint
- Perform a ray-tracing step using deformation field as rays
- Compute for each surface point allowable fraction

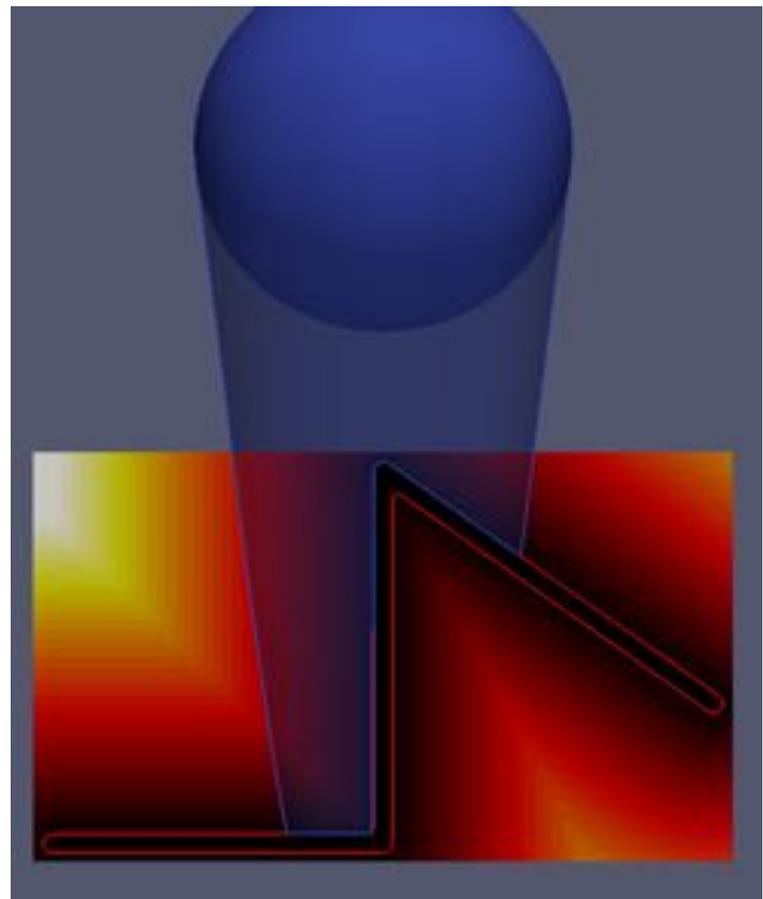
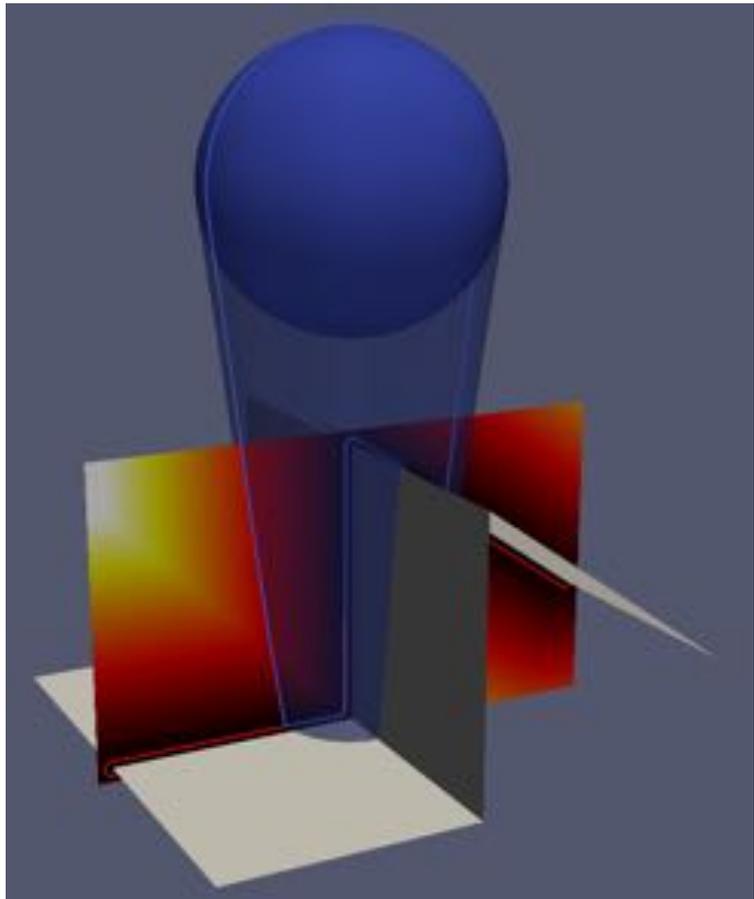
- All steps are extremely scalable



Control off

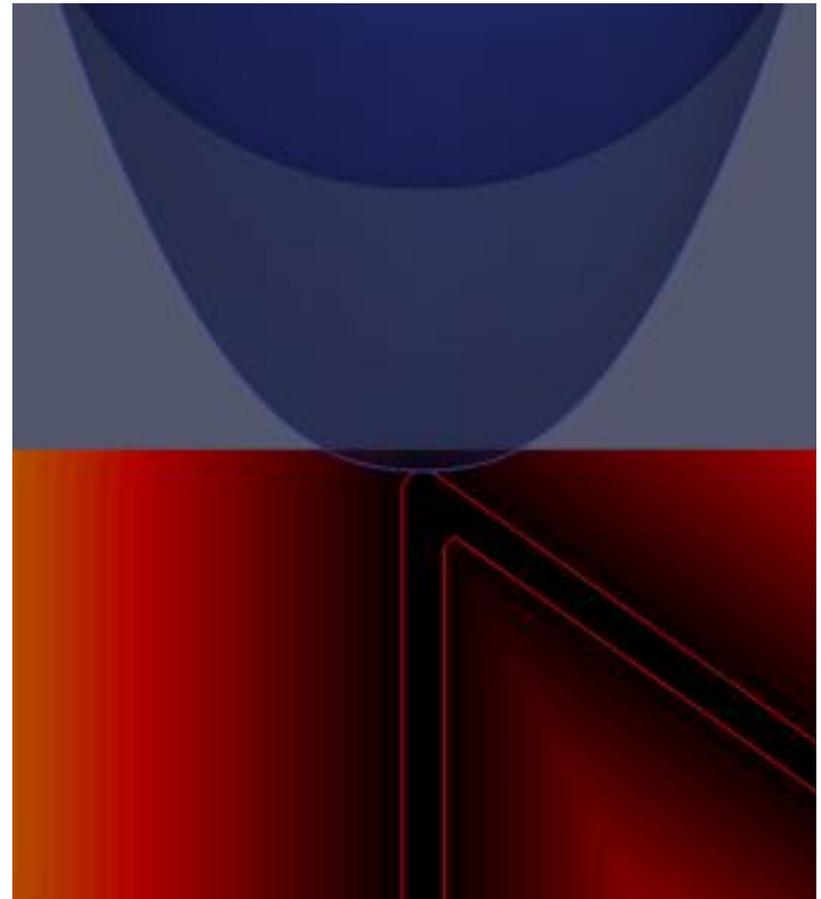
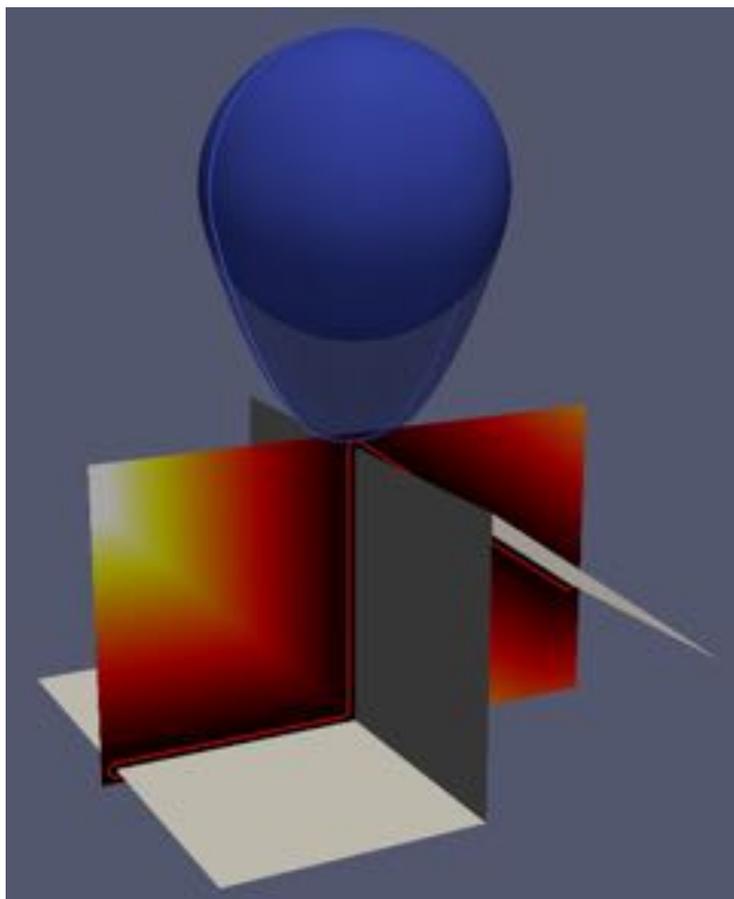


Local rescaling



→ ←
distance wrt tangential projection

Global rescaling



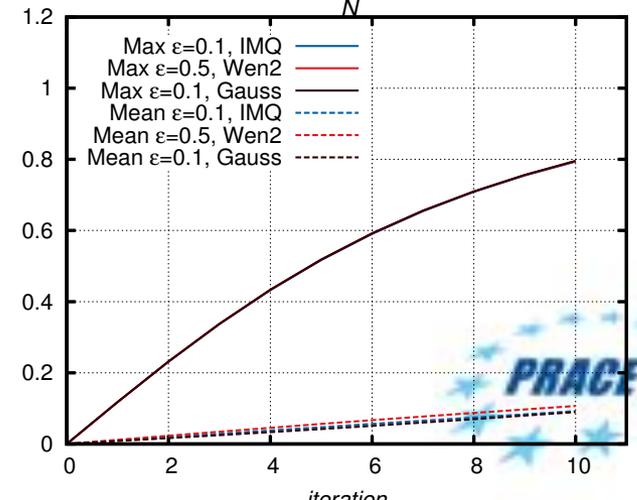
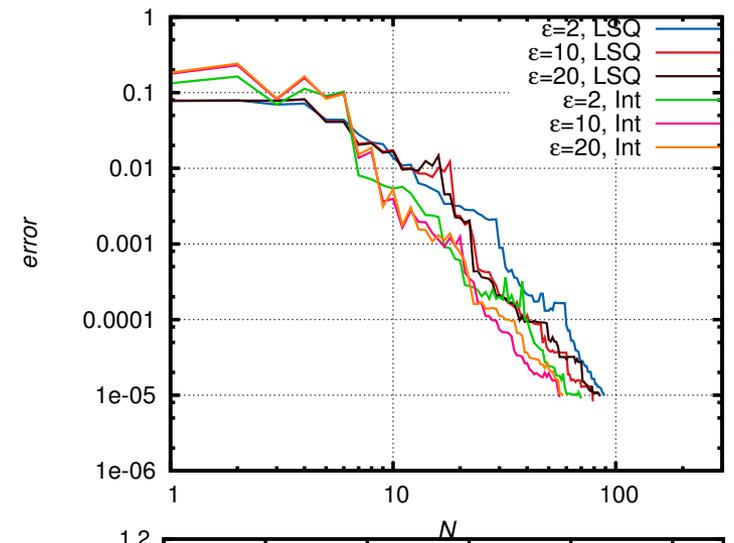
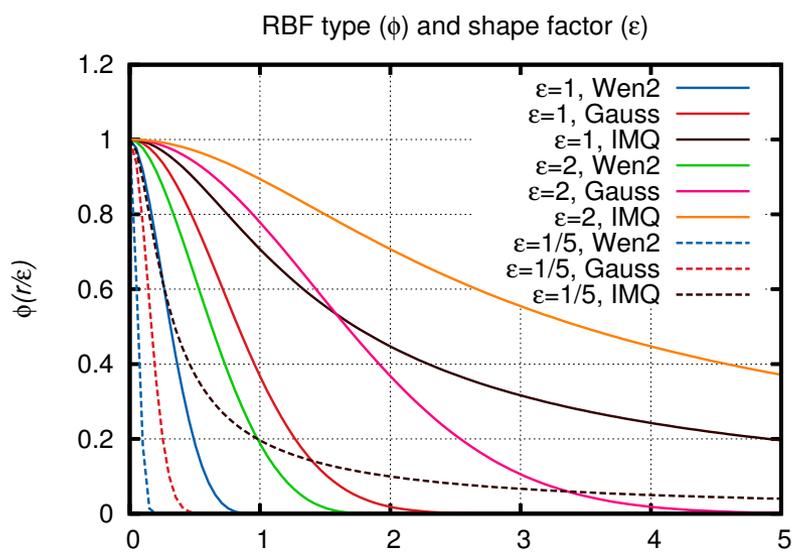
↑
PRACE
CINECA
identification of critical point

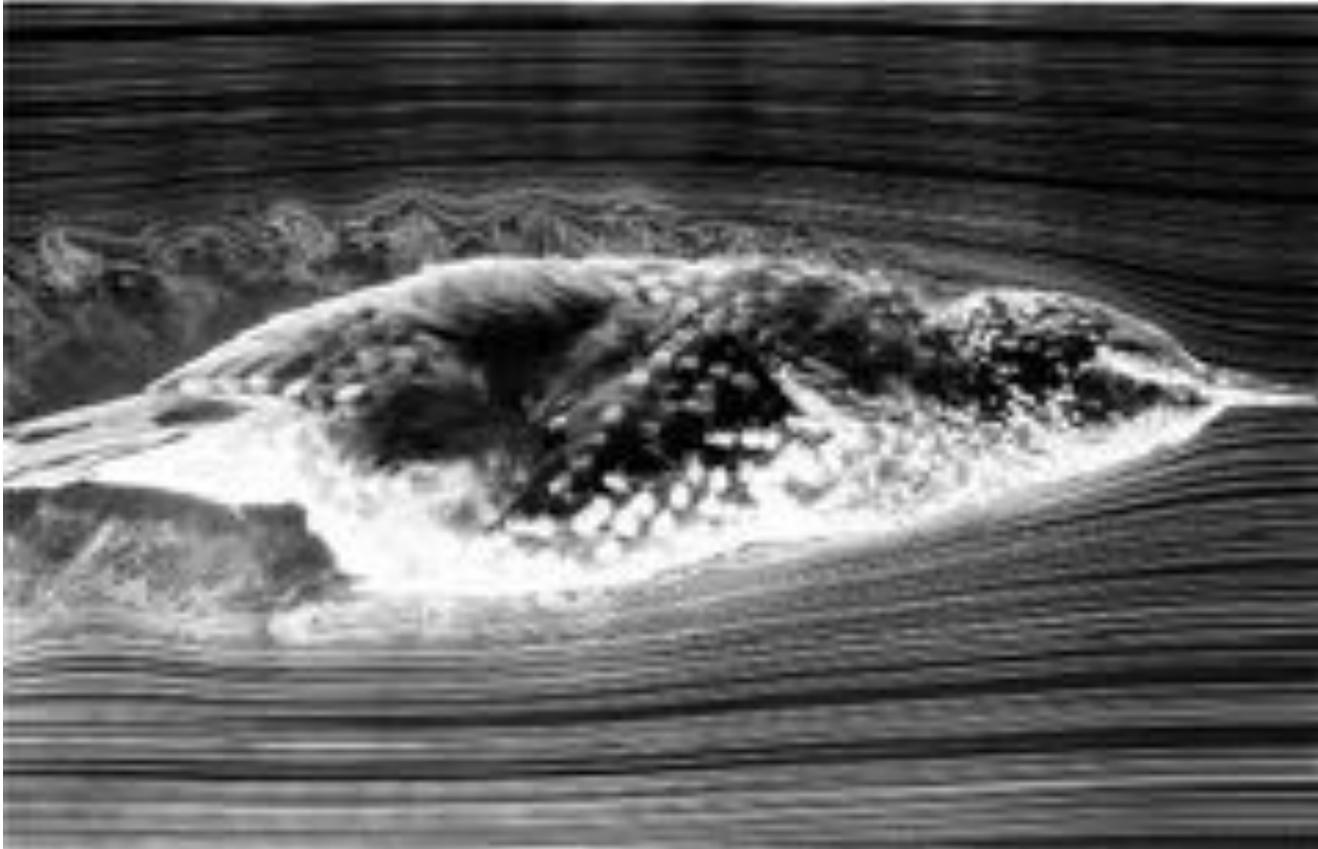
Mesh-Morphing

- propagate surface deformation to mesh
- avoid usage of sHM but creation of one initial high quality grid
- brute force: RBF with one node on each surface vertex
- very costly (N_V volume grid nodes, N_S surface grid nodes):
 - solve phase: dense linear system N_S DoF
 - evaluation phase: $N_V * N_S$ operations (n^5)
- greedy algorithm:
 - initial RBF with one node @ largest surface displacement;
 - calculate initial error;
 - while (maxError > tolerance) do
 - evaluate RBF at each surface node
 - calculate error = ||realDispl-reconDispl||
 - add new node @maxError
 - enddo

Mesh-Morphing

- RBF types, convergence and quality, $N_S = O(10^4)$





podFOAM & ezRB: Reduced Order Models based on POD

HPC based HF + ROMs

- Models tend to saturate HPC resources
 - bigger & more complex (e.g. DES, multi-physics)
 - more reliable & accurate (??)
- Computing time does not decrease as computing power increases
 - big challenge for optimization
- Scenario
 - use high-fidelity simulations to build a knowledge database (few, but which?)
 - recycle your data through semi-empirical Reduced Order Models

Scenario 1

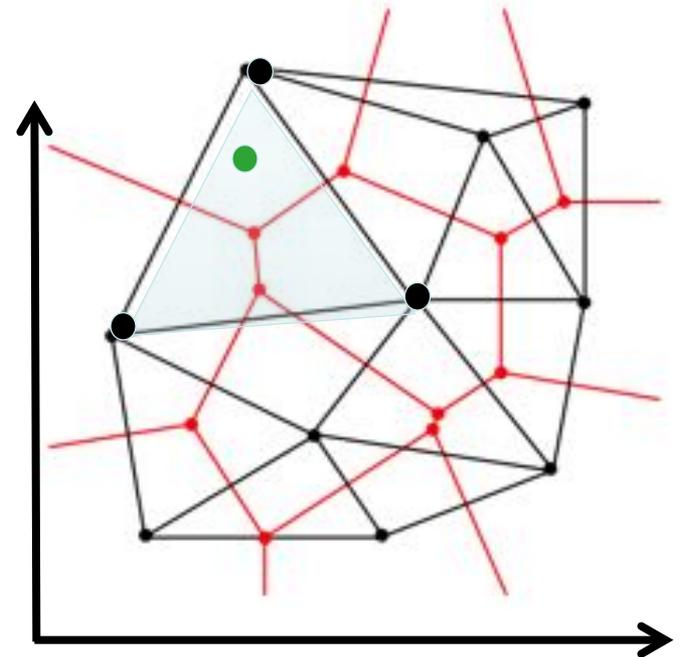
you knew from the very beginning of your project that you would do shape optimization

- parametric geometrical model ($a_0 \dots a_{N-1}$)
- create a database of solutions (DoE)
- associate set of parameters a_i to each solution of DB

- make a Voronoi tessellation of the parameter space
- build a linearized model for each simplex

ezRB

- tessellation of parameter space
 - can be done efficiently in N dimensions
 - small problem size
- requested solution
 - locate right simplex
 - interpolate solution at simplex vertices
- can be performed efficiently via POD



Scenario 2

you didn't know that you would need some shape optimization

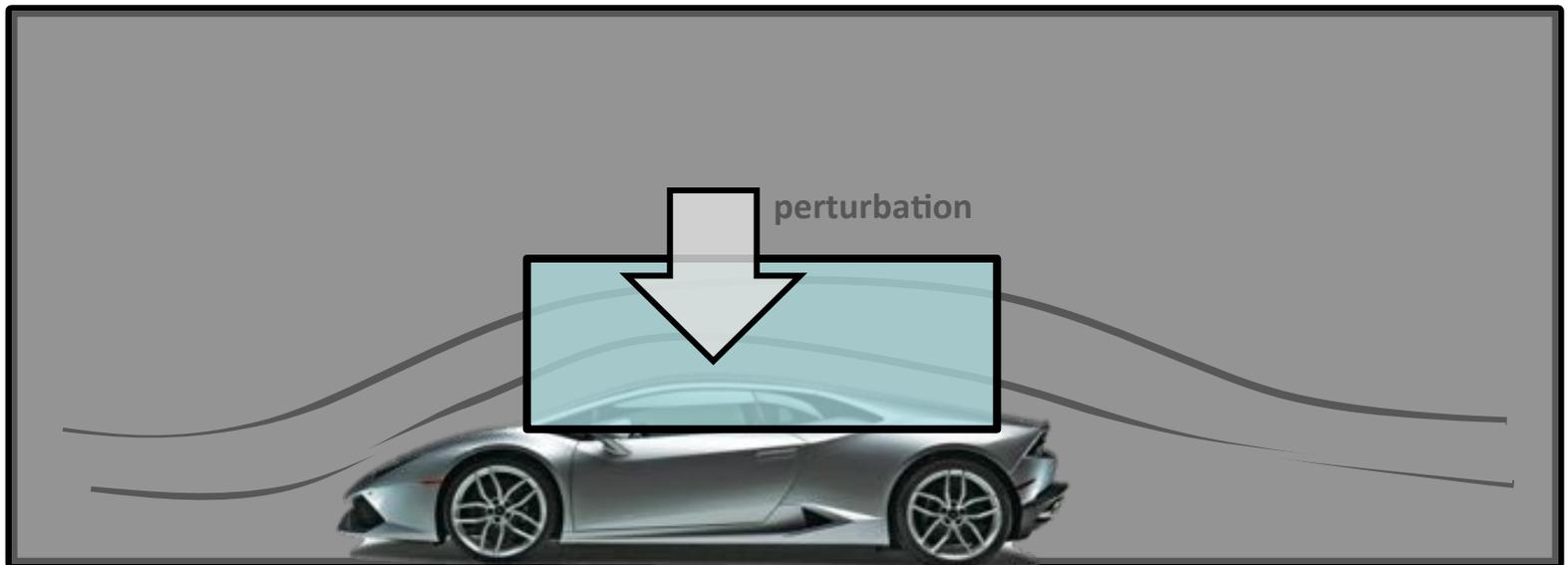
- database of loose solutions
- parametric geometrical model ($a_0 \dots a_{N-1}$)
- feed your CFD with information from DB to reduce cost

podFOAM

inner zone: use non-linear CFD

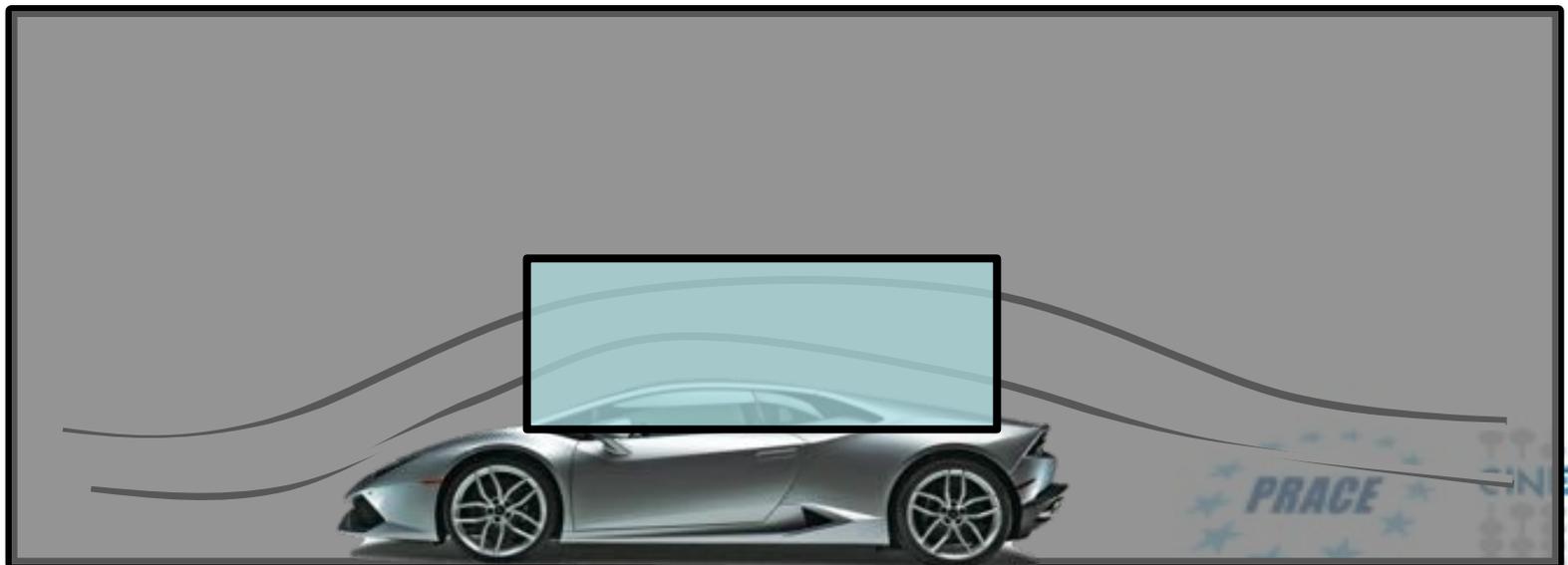
outer zone: use simplified model to impose BC to inner zone

Far Field BC



podFOAM

- Our assumptions are that
 - in the outer zone, the perturbation becomes linear
 - the **information needed** for describing the flow field of outer zone, is **already available** in the data stored on your HD
- **Represent the green zone by Proper Orthogonal Decomposition**
- **Couple to CFD in blue zone** through a Least-Squares Problem on the data at the interface



- **Proper Orthogonal Decomposition**

- representation of a solution as $\mathbf{u}_i^j(\mathbf{x}) = \sum \mathbf{a}_i^j \Phi_i(\mathbf{x})$ for $i=0 \dots N-1$
- $\Phi_i(\mathbf{x})$ | sorthogonal POD basis, which can be found by solving the eigen-problem of the **snapshot correlation matrix**
- **no series converges faster** than POD; identification of **coherent structures**; **very few modes** to capture 99% of the energy

Sampling strategy

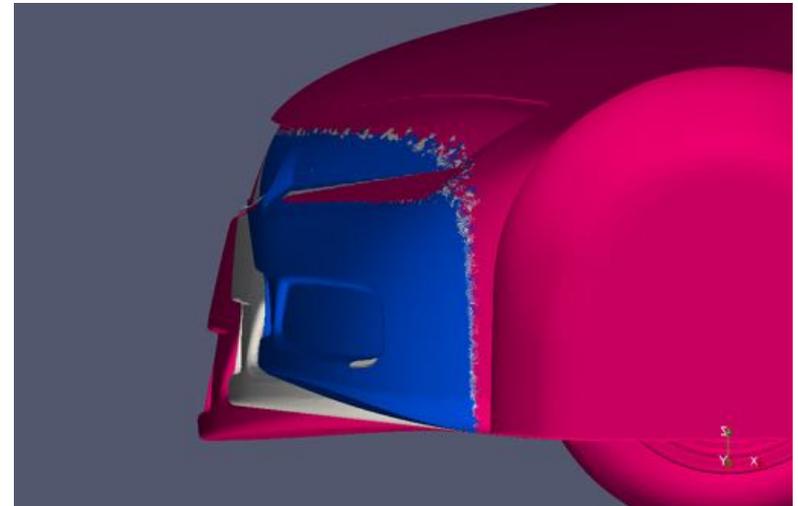
- both ROMs zero error at solution of DB
- the ROM should be reliable in entire parameter space
- if *a priori* error available, additional snapshots in critical zones
- Leave-One-Out strategy to determine pseudo error
foreach solution of DB
 - remove solution from DB
 - recalculate ROM
 - evaluate ROM at solution point
 - calculate error = $\|U_{HF} - U_{ROM}\|$end foreach

add new snapshot where indicator is high & far from points

DrivAer model

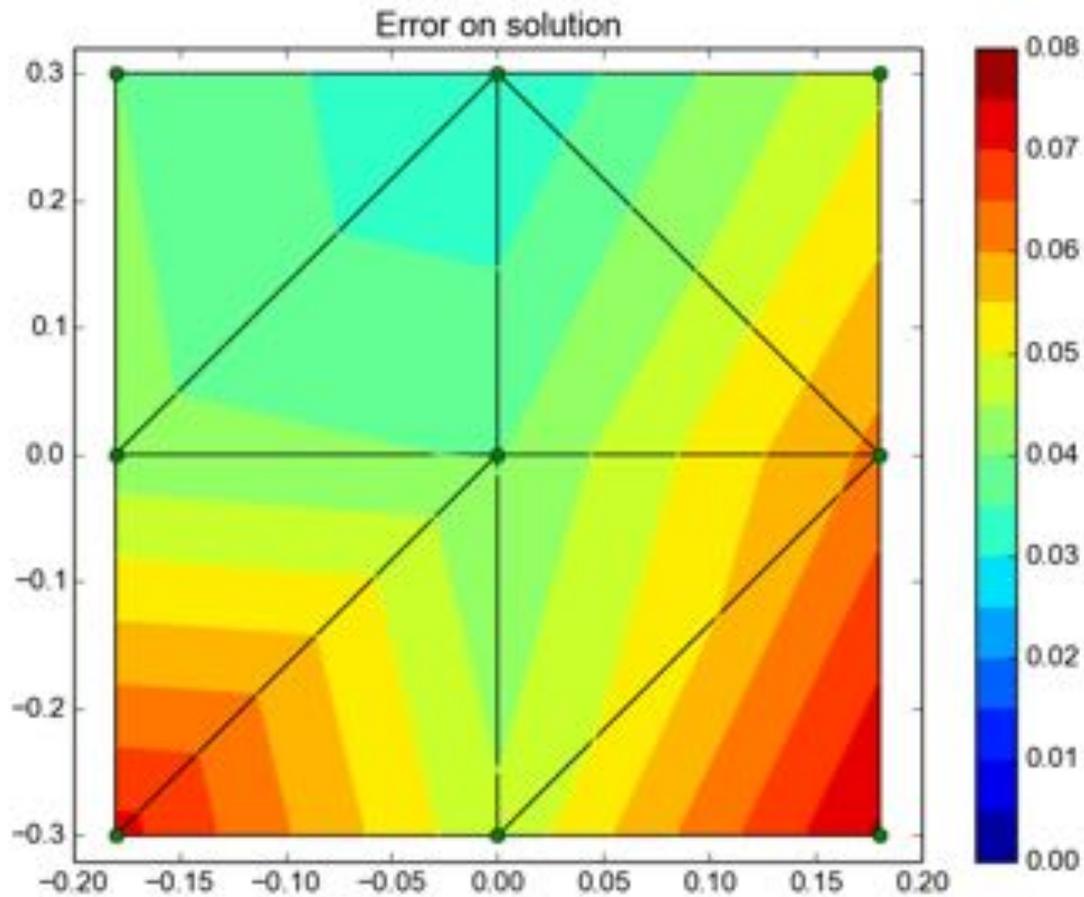
Free model by TU Munich in collaboration with Audi & BMW

- Clean symmetric model: 14M cells
- 2 control parameters
- $S_{\text{forces}} = 0.1$



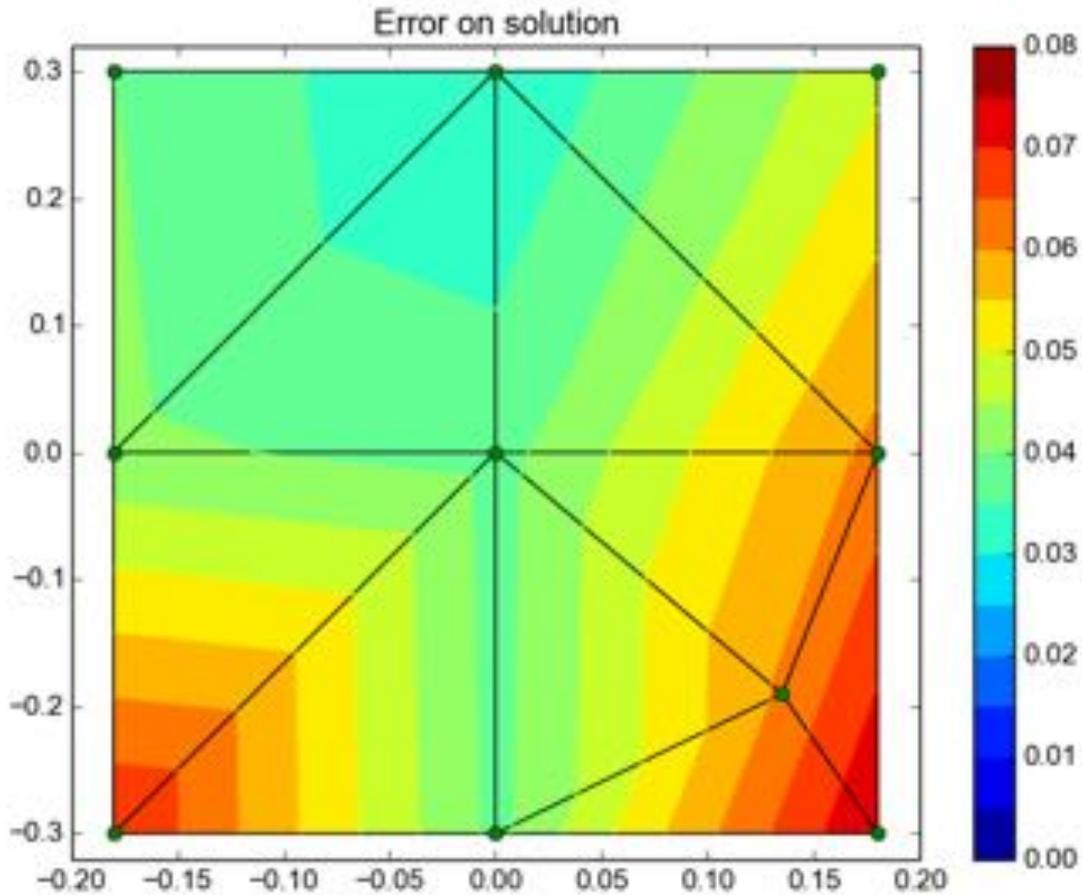
Sampling strategy

Iteration 0



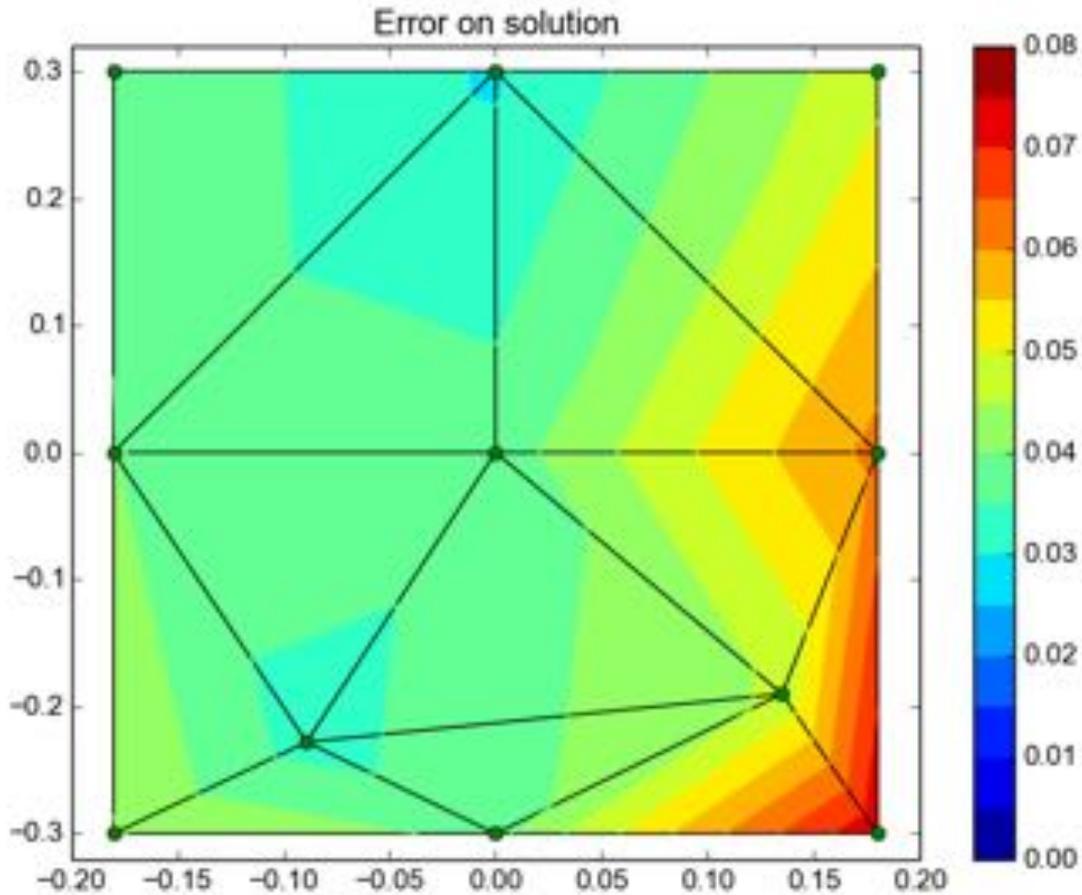
Sampling strategy

Iteration 1



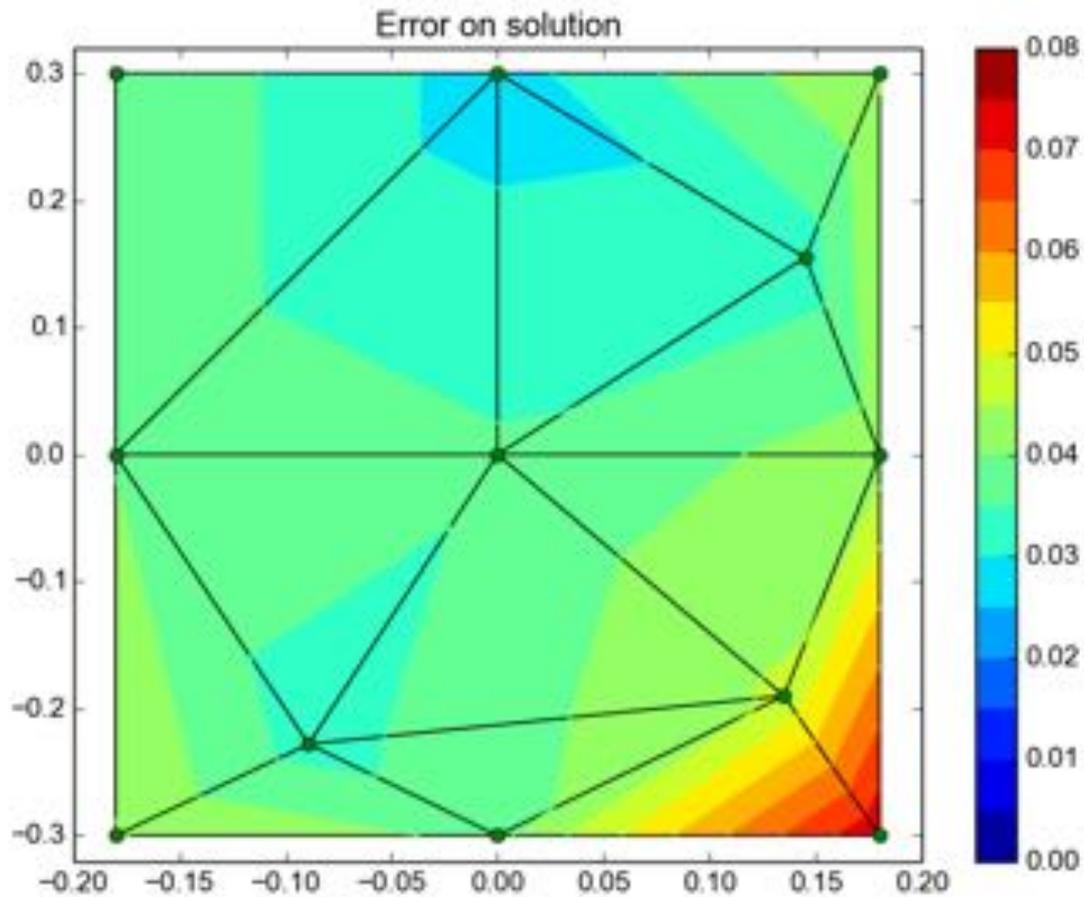
Sampling strategy

Iteration 2



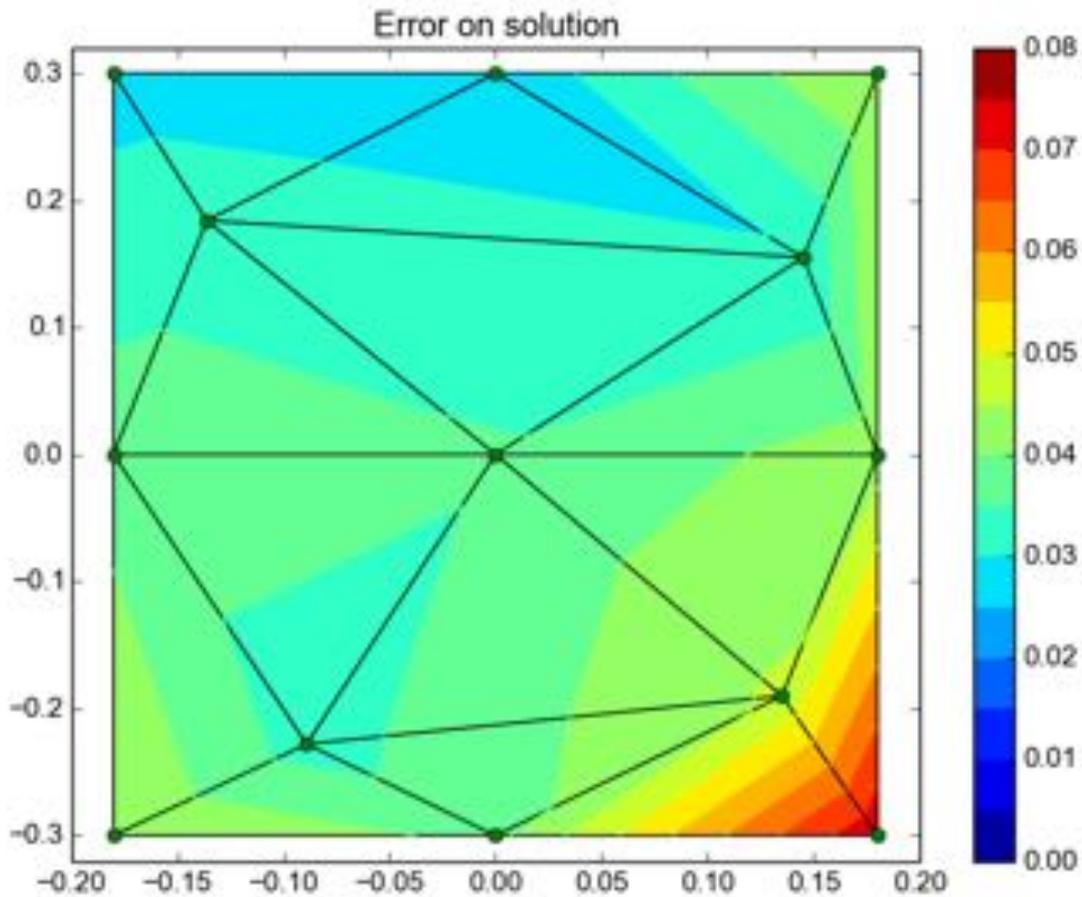
Sampling strategy

Iteration 3



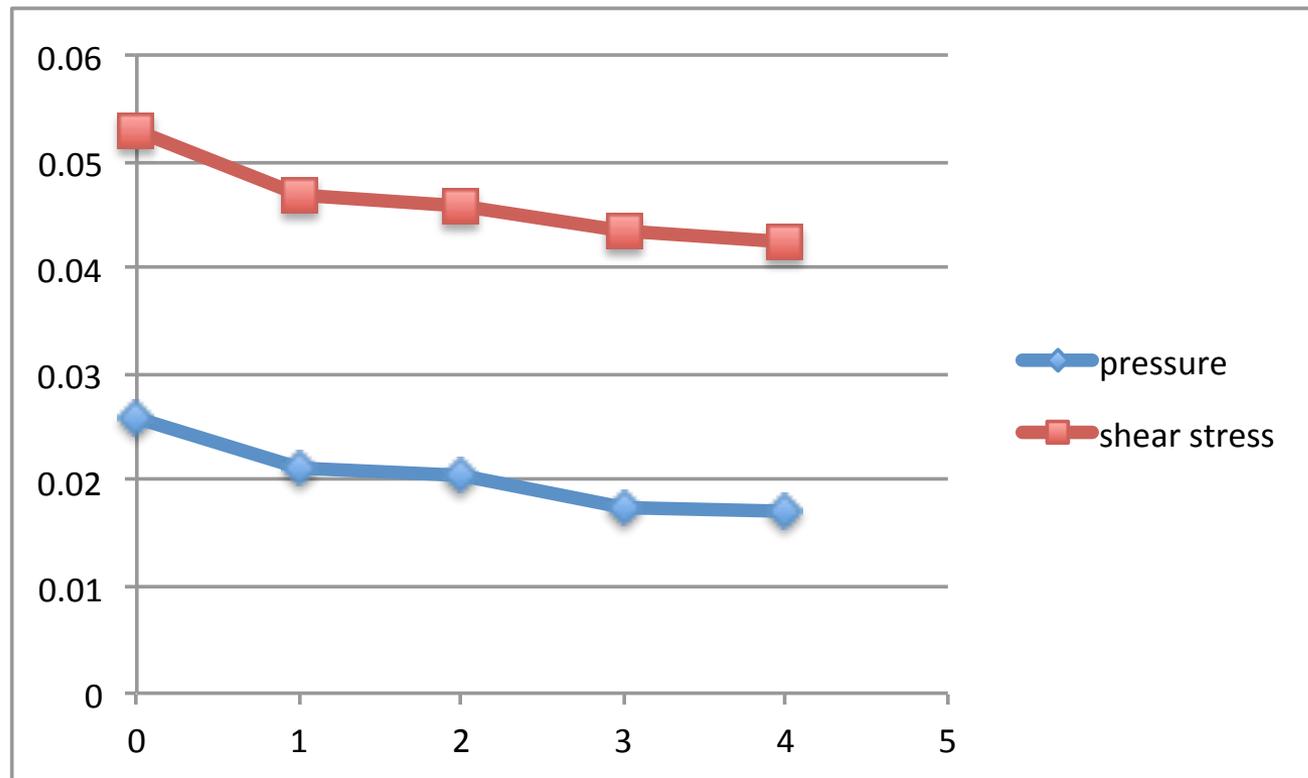
Sampling strategy

Iteration 4

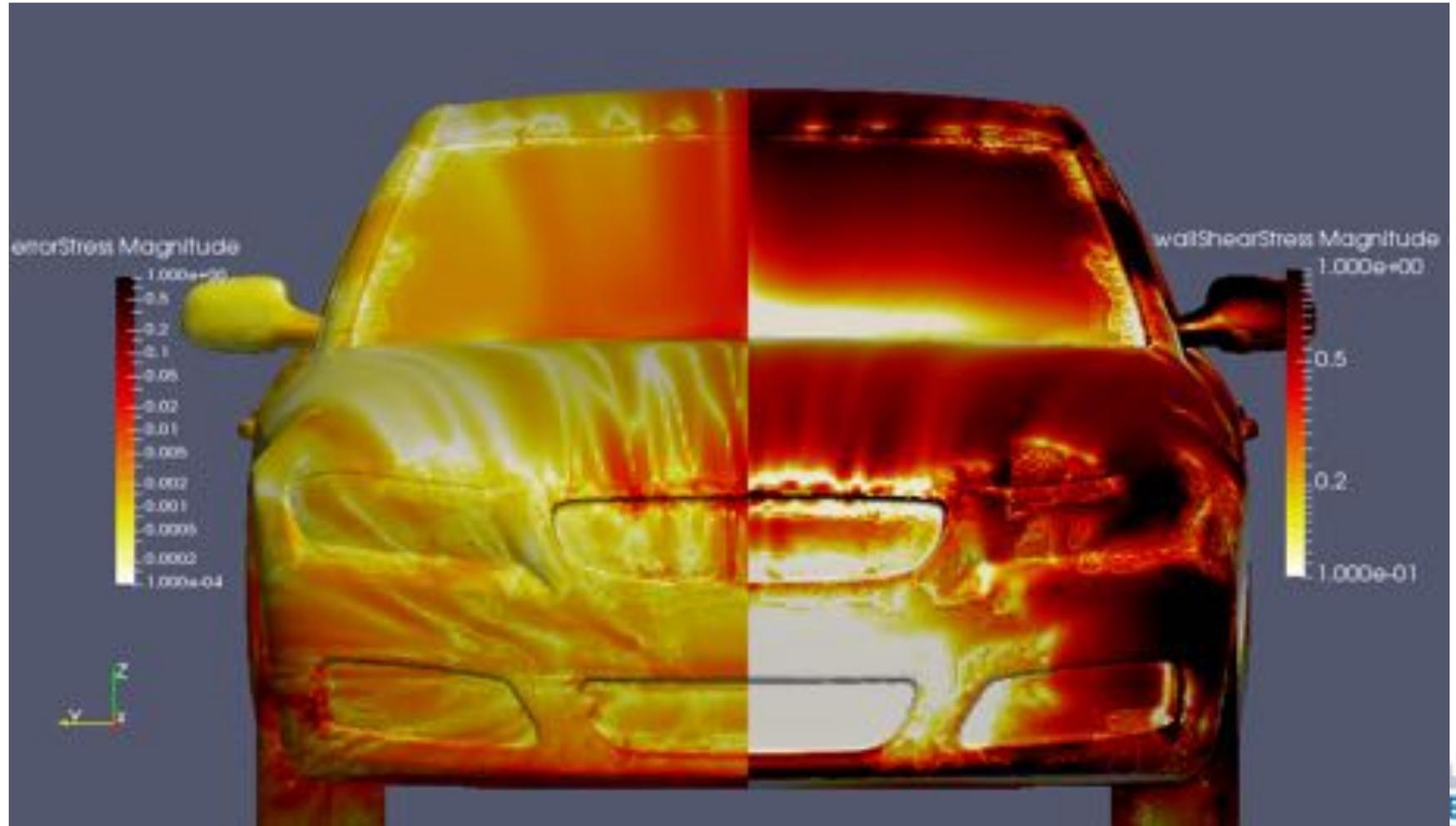


ezRB

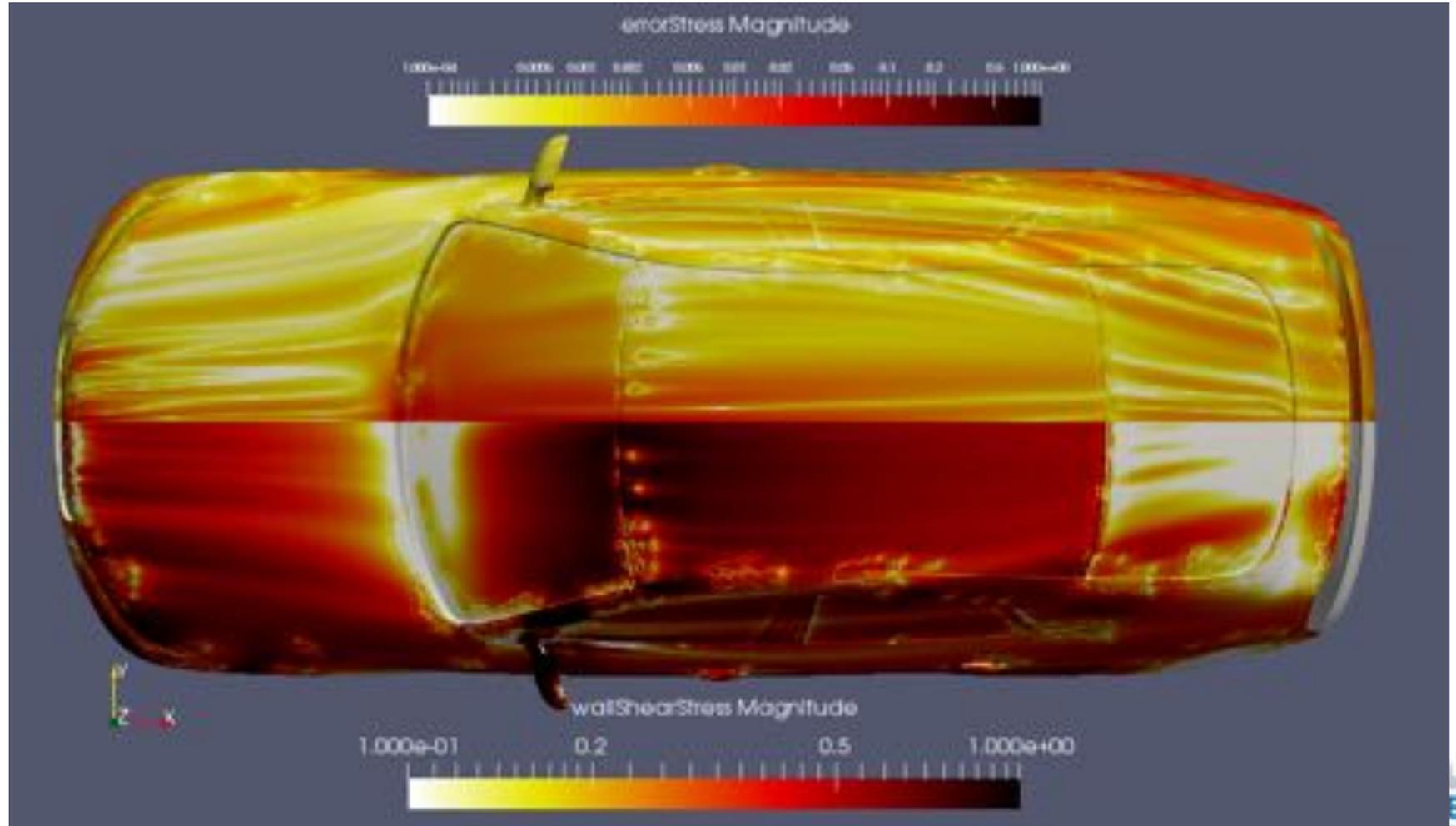
- reconstruction of surface pressure and shear stress
- mean error over 4 random configurations out-of-DB
- Cost $O(s)$



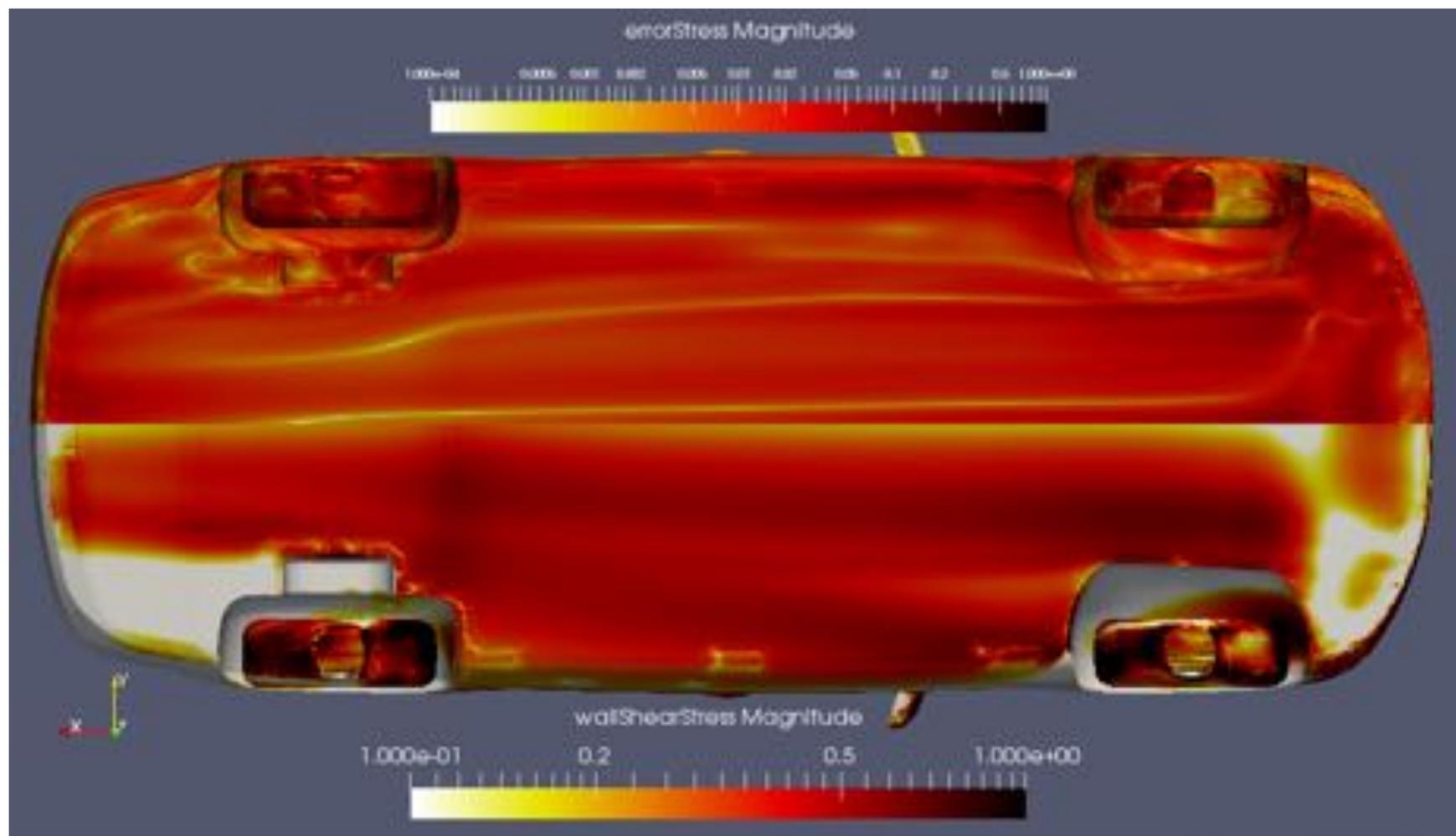
ezRB



ezRB

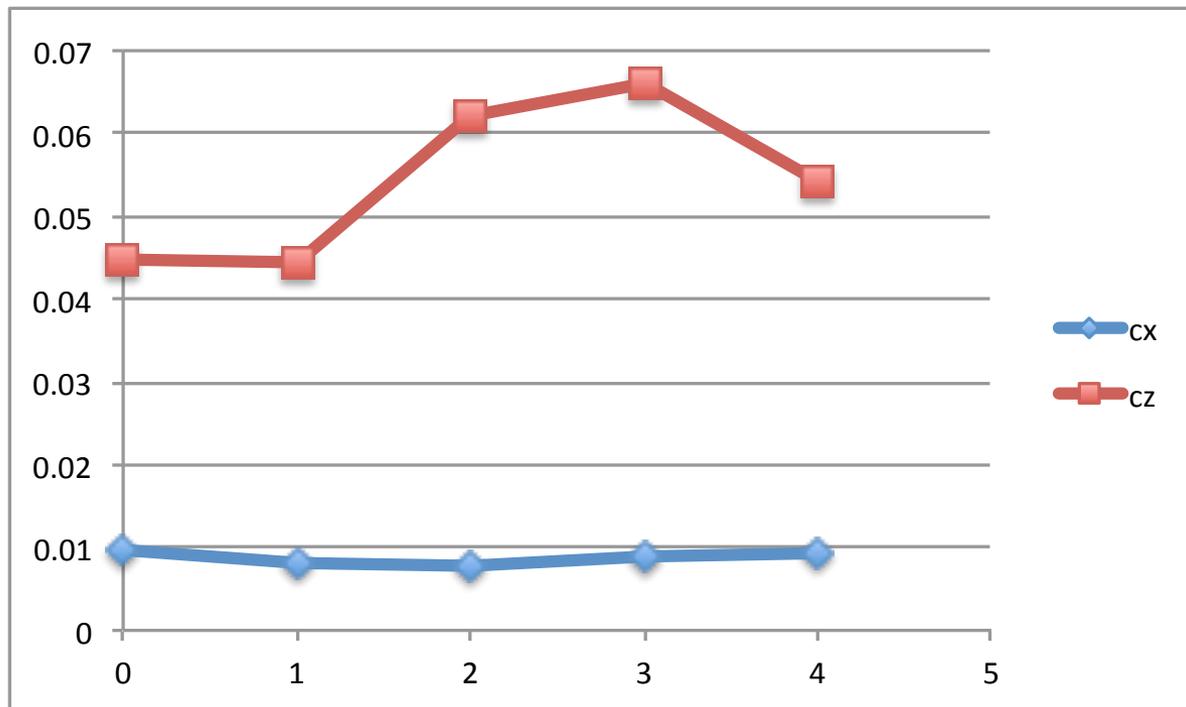


ezRB

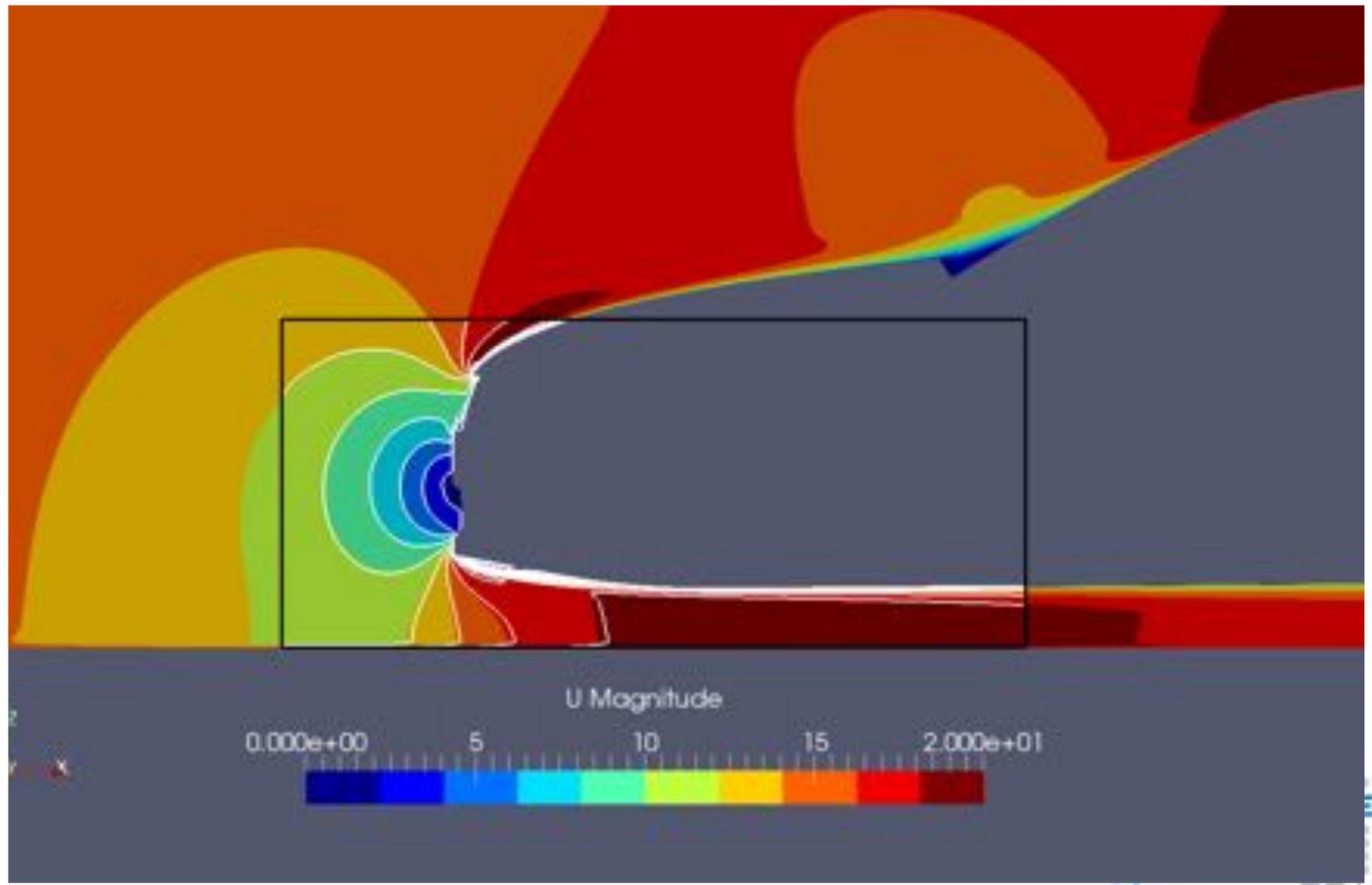


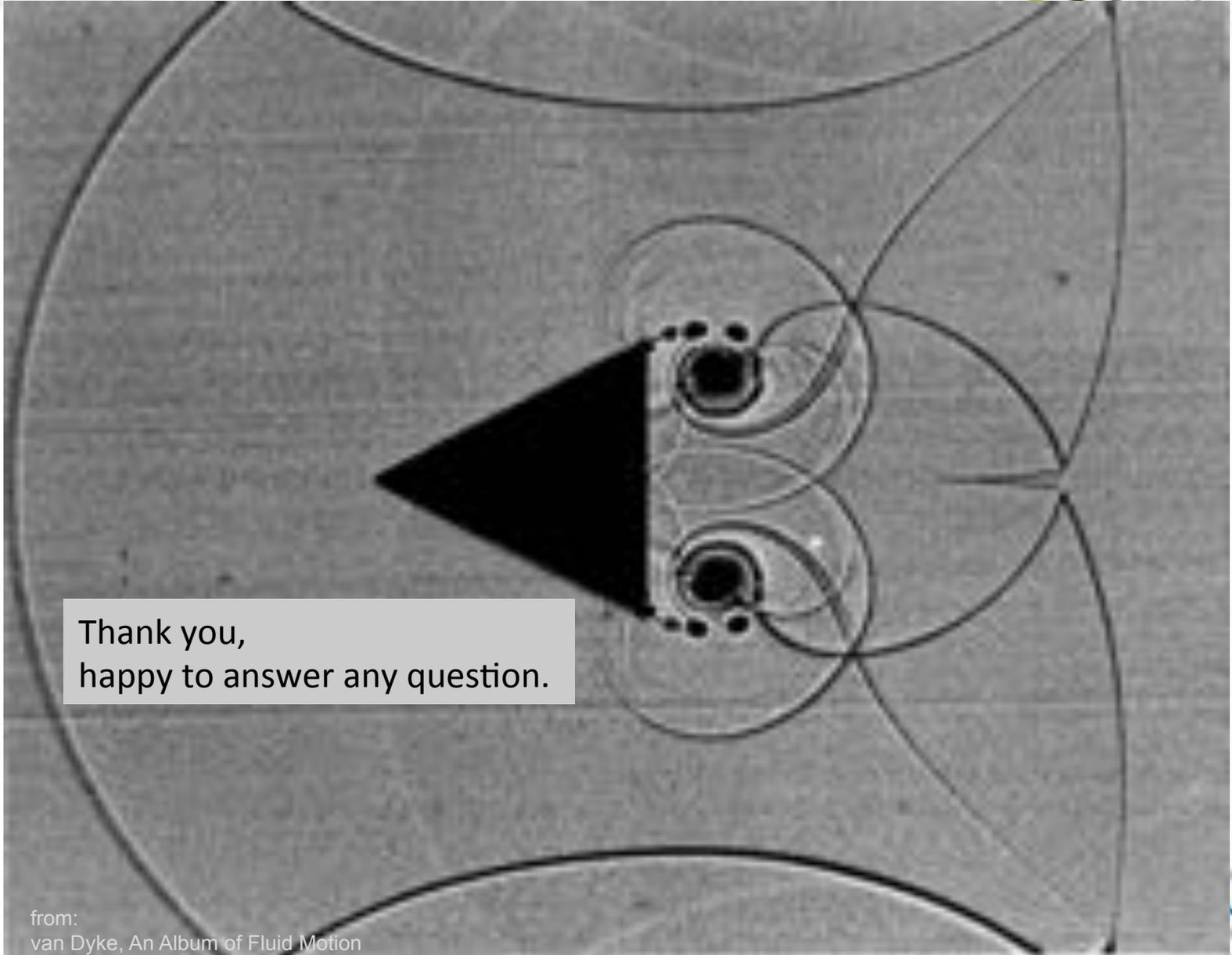
podFOAM

- recalculation of inner & outer flow field
- mean error over 4 random configurations out-of-DB
- speed-up $O(50)$



podFOAM





Thank you,
happy to answer any question.

from:
van Dyke, An Album of Fluid Motion