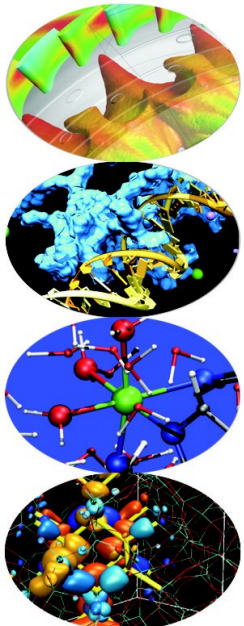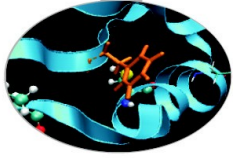# HPC Numerical Libraries

Nicola Spallanzani – n.spallanzani@cineca.it
SuperComputing Applications and Innovation Department

Algorithms and Libraries

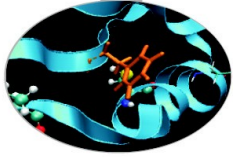Many numerical algorithms are well known and largely available. See for instance:

http://www.nr.com

So why should I use libraries?
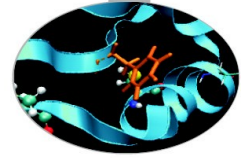- They are available on many platforms
- … and they are usually optimized by vendors
- In case vendor libraries are not installed:

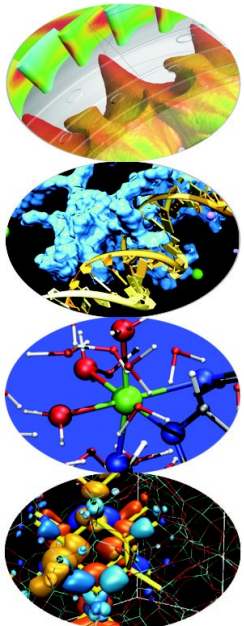http://www.netlib.org

Numerical Libraries

- Scalable Linear Algebra

- Fast Fourier Transform

- Adaptive Mesh Refinement (AMR)

- Input/Output

Scalable Linear Algebra

Nicola Spallanzani – n.spallanzani@cineca.it
SuperComputing Applications and Innovation Department

Standard Linear Algebra Libraries
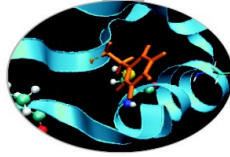
BLACS

ACML

PETSc

LAPACK

MKL

PLASMA

PSBLAS

ATLAS

MAGMA

BLAS

… but not only

SLEPc

TRILINOS

ESSL

ARPACK

SCALAPACK

SCOTCH

CUBLAS

which library should I use?

http://www.netlib.org/utk/people/JackDongarra/la-sw.html

(**P**arallel**)** **B**asic **L**inear **A**lgebra **S**ubprograms (BLAS and PBLAS)
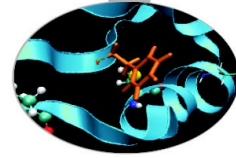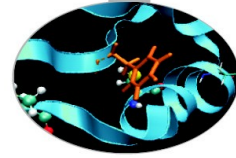
- **Level 1) : Vector - Vector operations**

- **Level 2) : Vector - Matrix operations**

- **Level 3) : Matrix - Matrix operations**

Linear systems, Eigenvalue equations (LAPACK)

3) M x M products

2) M x V products

1) V x V products

BLAS subroutines

matrix multiplication: C = A * B (level 3)

DGEMM(TRANSA, TRANSB, M, N, L, ALPHA, A, LDA, B, LDB, BETA, C, LDC)

 'N' or 'T'   max(1,M)

matrix times vector: Y = A * X   (level 2)

DGEMV(TRANS, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY)

              1.0d0   0.0d0

vector swap: X ⇔ Y   (level 1)

DSWAP(N, X, INCX, Y, INCY)

scalar product: p = X'·Y   (level 1)

p = DDOT(N, X, INCX, Y, INCY)

          Function   Increment for elements

Quick Reference Guide to the BLAS

http://www.netlib.org/lapack/lug/node145.html

CBLAS subroutines
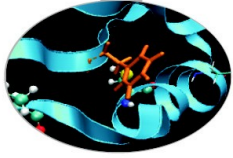
Instead of calling BLAS routines from a C-language program, you can use the CBLAS interface.

CBLAS is a C-style interface to the BLAS routines. You can call CBLAS routines using regular C-style calls. Use the **cblas.h** header file with the CBLAS interface. The header file specifies enumerated values and prototypes of all the functions.

```
matrix multiplication: C = A * B  (level 3)

cblas_dgemm(ORDER, TRANSA, TRANSB, M, N, L, ALPHA, A, LDA, B, LDB, BETA, C, LDC)

CblasRowMajor                      CblasNoTrans

matrix times vector: Y =  A * X   (level 2)

cblas_dgemv(ORDER, TRANS, M, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCX)
```
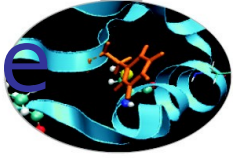
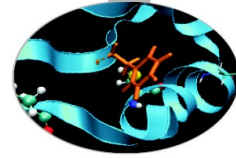# (**Sca**lable**)** **L**inear **A**lgebra **PACK**age (LAPACK and ScaLAPACK)

- **Matrix Decomposition**

- **Linear Equation Systems**

- **Eigenvalue Equations**

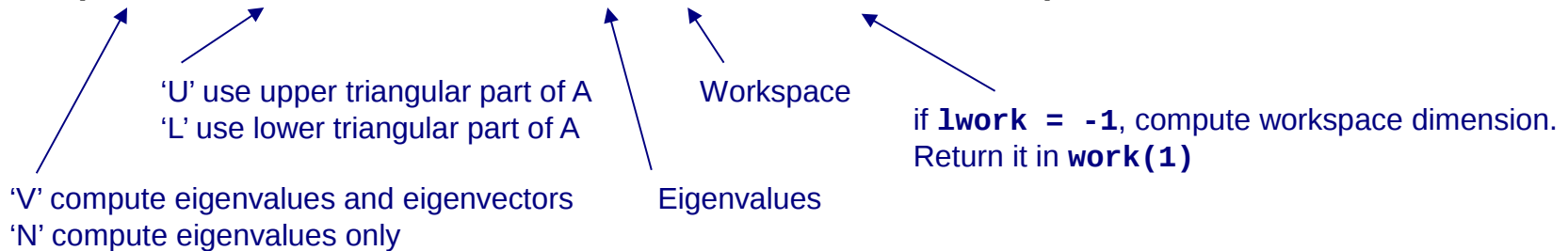- **Linear Least Square Equations**

Designed for:

- **dense, banded, triangular matrices**

- **real and complex matrices**

LAPACK subroutines

Eigenvalues and, optionally, eigenvectors of a real symmetric matrix:

`DSYEV( JOBZ, UPLO, N, A, LDA, W, WORK, LWORK, INFO )`

'U' use upper triangular part of A
'L' use lower triangular part of A

Workspace

if **lwork = -1**, compute workspace dimension.
Return it in **work(1)**

'V' compute eigenvalues and eigenvectors
'N' compute eigenvalues only

Eigenvalues

```
void dsyev_( const char* jobz, const char* uplo, const MKL_INT* n,
             double* a, const MKL_INT* lda, double* w, double* work,
             const MKL_INT* lwork, MKL_INT* info );
```
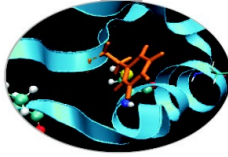
Index of Driver and Computational Routines:

http://www.netlib.org/lapack/lug/node142.html

Index of Auxiliary Routines:

http://www.netlib.org/lapack/lug/node144.html

LAPACKE subroutines

Two-level C interface to LAPACK, consisting of a high-level interface and a middle-level interface.

Use the **lapacke.***h* header file with the LAPACKE interface. The header file specifies enumerated values and prototypes of all the functions.

```
system of linear equations:

lapack_int LAPACKE_dgesv (int matrix_layout, lapack_int n, lapack_int nrhs,
                          double *a, lapack_int lda, lapack_int *ipiv,
                          double *b, lapack_int ldb);


        LAPACK_ROW_MAJOR
        LAPACK_COL_MAJOR

lapack_int LAPACKE_dgesv_work (int matrix_layout, lapack_int n, lapack_int nrhs,
                          double *a, lapack_int lda, lapack_int *ipiv,
                          double *b, lapack_int ldb);
```
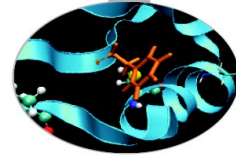
How To Compile

 # load these modules on Galileo

```
module load gnu/4.9.2

module load blas/3.5.0--gnu--4.9.2

module load lapack/3.5.0--gnu--4.9.2
```

#FORTRAN:

```
LALIB="-L${LAPACK_LIB} -llapack -L${BLAS_LIB} -lblas"

gfortran -o program.x program.f90 ${LALIB}
```
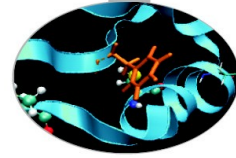
*#C: remember to include the header files **"cblas.h"** and/or **"lapacke.h"***

```
LALIB="-I${BLAS_INC} -I${LAPACK_INC} \

        -L${LAPACK_LIB} -llapacke -llapack \

        -L${BLAS_LIB} -lcblas -lblas -lgfortran -lm"

gcc -o program.x program.f90 ${LALIB}
```

How To Compile

```
module load intel/pe-xe-2016--binary
module load mkl/11.3.0--binary


LALIB="-I${MKL_INC} -L${MKL_LIB} -lmkl_intel_lp64 \
      -lmkl_sequential -lmkl_core -lpthread -lm -ldl"
```
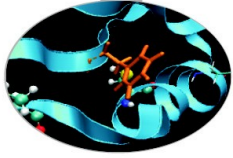
*#C: remember to include the header file **"mkl.h"***
```
icc -o program.x program.c ${LALIB}
icc -mkl -o program.x program.c -lpthread -lm -ldl
```

#FORTRAN:
```
ifort -o program.x program.f90 ${LALIB}
ifort -mkl -o program.x program.f90 -lpthread -lm -ldl
```

```
http://software.intel.com/en-us/articles/intel-mkl-link-line-advisor
```

# **A**utomatically **T**uned **L**inear **A**lgebra **S**oftware

- ATLAS is both a research project and a software package.

- ATLAS's purpose is to provide portably optimal linear algebra software (recommended as a way to automatically generate an optimized BLAS library).

- The current version provides a complete BLAS API (for both C and Fortran77), and a very small subset of the LAPACK API.

(**P**arallel) **AR**noldi **PACK**age
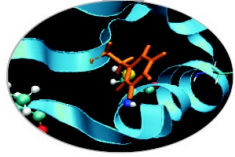
- ARPACK is a collection of Fortran77 subroutines designed to solve large scale eigenvalue problems.

- It is most appropriate for large sparse or structured matrices. (Where structured means that a matrix-vector product requires order n rather than the usual order n^2 floating point operations.)

- ARPACK is dependent upon a number of subroutines from LAPACK and the BLAS.

- Main feature: reverse communication interface.

- A parallel version of the ARPACK library is availible. The message passing layers currently supported are BLACS and MPI .
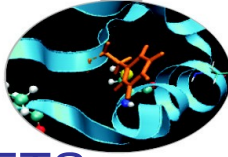
PETSc

The Portable, Extensible Toolkit for Scientific Computation (PETSc, pronounced PET-see; the S is silent), is a suite of data structures and routines developed by Argonne National Laboratory for the scalable (parallel) solution of scientific applications modeled by partial differential equations. It employs the Message Passing Interface (MPI) standard for all message-passing communication. PETSc includes a large suite of parallel linear and nonlinear equation solvers that are easily used in application codes written in C, C++, Fortran and now Python.

Particularly well suited for Sparse Linear Algebra.

Language: C, C++, Fortran, Python
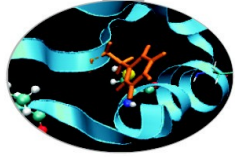
Availability: open source

Developers: various

Distributors: Mathematics and Computer Science Division, Argonne National Laboratory

Ref.: Argonne National Laboratory

TRILINOS

The Trilinos Project is an effort to develop algorithms and enabling technologies within an object-oriented software framework (written in C++) for the solution of large-scale, complex multi-physics engineering and scientific problems. A unique design feature of Trilinos is its focus on packages.

It is organized in Capability Areas:

- User Experience

- Parallel Programming Environments

- Framework & Tools

- Software Engineering Technologies and Integration

- I/O Support

- Meshes, Geometry, & Load Balancing

- Discretizations

- Scalable Linear Algebra

- Linear & Eigen Solvers

- Embedded Nonlinear Analysis Tools

MAGMA
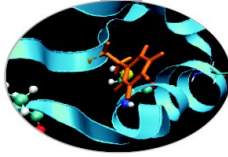
Matrix **A**lgebra for **G**PU and **M**ulticore **A**rchitecture
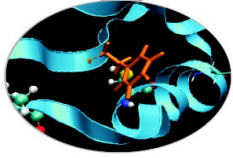
http://icl.cs.utk.edu/magma/

The MAGMA project aims to develop a dense linear algebra library similar to LAPACK but for heterogeneous/hybrid architectures, starting with current "Multicore+GPU" systems.

Methodology: CPU and GPU overlapping

MAGMA uses HYBRIDIZATION methodology based on

- Representing linear algebra algorithms as collections of TASKS and DATA DEPENDENCIES among them
- Properly SCHEDULING tasks' execution over multicore and GPU hardware components

MAGMA

CPU versus GPU interfaces
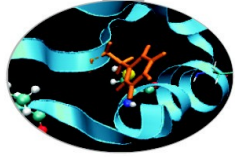
Why two different interfaces?

If data is already on the GPU

- pointer to GPU memory
- (some) additional memory allocation on CPU side

If data is already on the CPU

- no changes on the prototype
- internal overlap communication/computation (it uses pinned)
- (some) additional memory allocation on GPU side

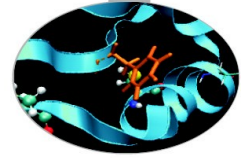# MKL on Intel Xeon Phi (MIC)

Intel released a version for Xeon Phi of the MKL mathematical libraries
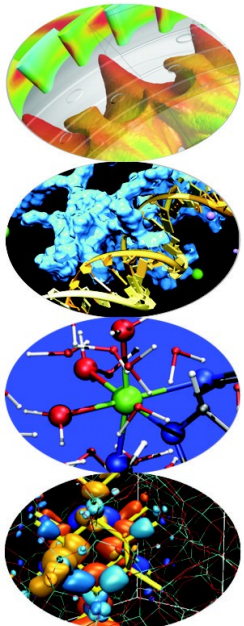
MKL have three different usage models:

- Automatic offload (AO)
- Compiler assisted offload (CAO)
- Native execution

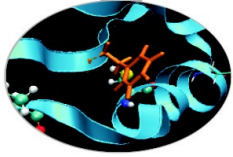https://wiki.u-gov.it/confluence/display/SCAIUS/Guide+for+Intel+Xeon+Phi+%28MIC%29+Usage

Fast Fourier Transform

Nicola Spallanzani – n.spallanzani@cineca.it
SuperComputing Applications and Innovation Department
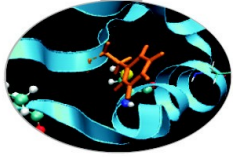
Fast Fourier Transform

A fast Fourier transform (FFT) algorithm computes the discrete Fourier transform (DFT) of a sequence, or its inverse. Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.

An FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors.

As a result, it manages to reduce the complexity of computing the DFT from O(n^2), which arises if one simply applies the definition of DFT, to O(n log n), where n is the data size.

FFTPACK

FFTPACK is a package of Fortran subroutines for the fast Fourier transform. It includes complex, real, sine, cosine, and quarter-wave transforms. It is included in the general-purpose mathematical library SLATEC.
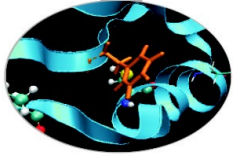
Language: FORTRAN

Availability: public domain

Developers: Paul N. Swarztrauber, National Center for Atmospheric Research, Boulder, CO

Distributors: NETLIB

Ref.: The University of Tennessee at Knoxville and Bell Laboratories

Fast Fourier Transform

FFTW

(Fastest Fourier Transform in the West)

FFTW is a C subroutine library for computing the discrete Fourier transform (DFT) in one or more dimensions, of arbitrary input size, and of both real and complex data (as well as of even/odd data, i.e. the discrete cosine/sine transforms or DCT/DST).

The currently most used free library to compute DFTs.

It uses both MPI and OpenMP.
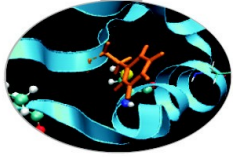
Language: C

Availability: public domain

Developers: Matteo Frigo and Steven G. Johnson.

Distributors: FFTW

Ref.: MIT

Fast Fourier Transform

FFTW

 (Fastest Fourier Transform in the West)

- Written in C (FORTRAN 77/90/2003 wrappers are included)
- FFTW adapt itself to your system and your problem: It check both total memory, size of your array, cache size register, CPU type, …
- … and it chose the best algorithm for you!
- 2 setps is required:
- Plan creation
- Execution
- It supports FFTs for any different array size (< total memory)
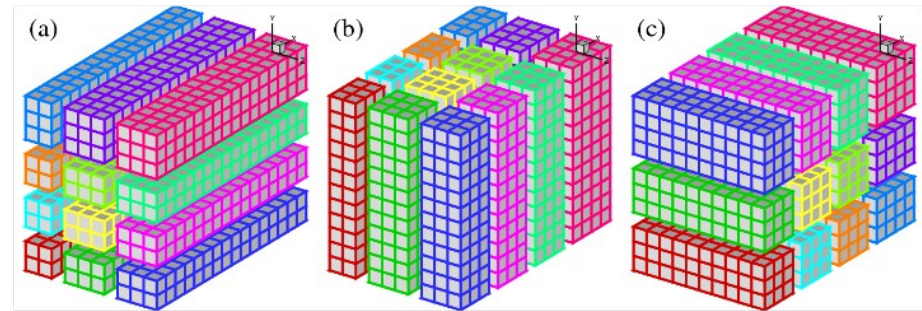
Fast Fourier Transform
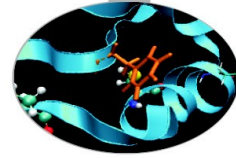
2Decomp&FFT

The 2DECOMP&FFT library is a software framework in Fortran to build large-scale parallel applications. It is designed for applications using three-dimensional structured mesh and spatially implicit numerical algorithms. At the foundation it implements a general-purpose 2D pencil decomposition for data distribution on distributed-memory platforms. On top it provides a highly scalable and efficient interface to perform three-dimensional distributed FFTs. The library is optimised for supercomputers and scales well to hundreds of thousands of cores. It relies on MPI but provides a user-friendly programming interface that hides communication details from application developers.
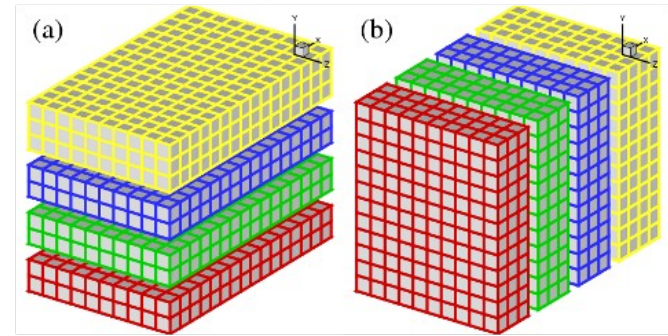
VS

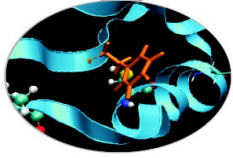Language: FORTRAN

Availability: public domain

Developers: Ning LI.

Distributors: 2decomp.org

Ref.: HECToR Distributed Computational Science and Engineering (CSE) Service operated by NAG Ltd.

http://www.2decomp.org/

P3DFFT

3DFFT is a scalable software library implementing three-dimensional spectral transforms. It has been used in a variety of codes from many areas of computational science. It has been tested and used on many high-end computational system. It uses two-dimensional domain decomposition in order to overcome a scaling bottleneck of one-dimensional decomposition. This allows the programs with this library to scale well on a large number of cores, consistent with bisection bandwidth scaling of interconnect of the underlying hardware system.
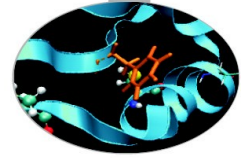
Language: C

Availability: public domain
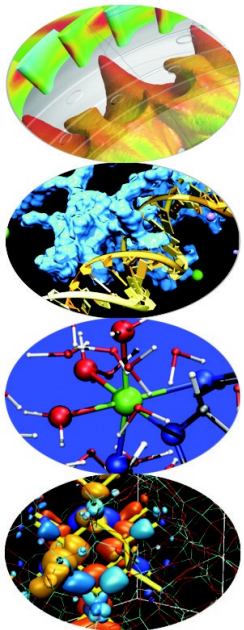
Developers: Dmitry Pekurovsky.

Distributors: 2decomp.org

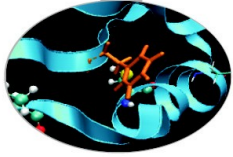Ref.: San Diego Supercomputer Center (SDSC) at UC San Diego.

https://code.google.com/p/p3dfft/

Adaptive Mesh Refinement (AMR)

Nicola Spallanzani – n.spallanzani@cineca.it
SuperComputing Applications and Innovation Department

AMR libraries

Many problems in numerical analysis, do not require a uniform precision in the numerical grids used for computational simulation.

Adaptive mesh refinement provides a dynamic programming environment for adapting the precision of the numerical computation based on the requirements of a computation problem in specific areas.
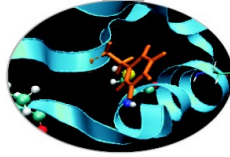
METIS

METIS is a software package for graph partitioning that implements various multilevel algorithms. It was written in C, and it can generate, manipulate and partition, graphs grids and matrices. Particularly well suited for partitioning regular grids on a parallel distributed memory system.
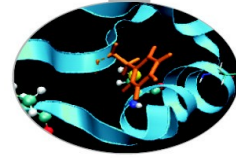
Language: C

Availability: free for academy and research

Developers: George Karypis Lab

Distributors: George Karypis Lab

Ref.: Department of Computer Science & Engineering, University of Minnesota

ParMETIS

ParMETIS is an MPI-based parallel library that implements a variety of algorithms for partitioning unstructured graphs, meshes, and for computing fill-reducing orderings of sparse matrices. ParMETIS extends the functionality provided by METIS and includes routines that are especially suited for parallel AMR computations and large scale numerical simulations
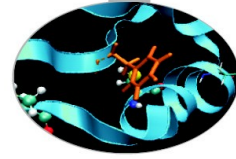
Language: C

Availability: free for academy and research

Developers: George Karypis Lab

Distributors: George Karypis Lab

Ref.: Department of Computer Science & Engineering, University of Minnesota

http://glaros.dtc.umn.edu/gkhome/views/metis

AMR libraries

PARAMESH

PARAMESH is a package of Fortran 90 subroutines designed to provide an application developer with an easy route to extend an existing serial code which uses a logically Cartesian structured mesh into a parallel code with adaptive mesh refinement(AMR).
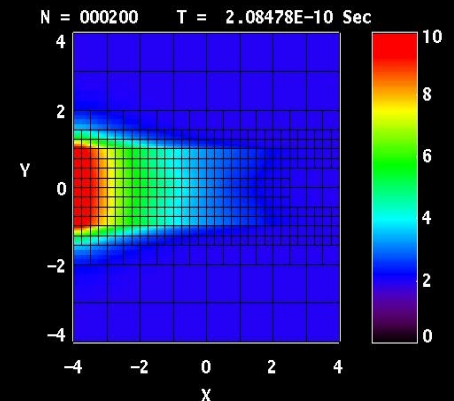
Language: FORTRAN

Availability: free for academy and research

Developers: Peter MacNeice, Kevin M. Olson et al.
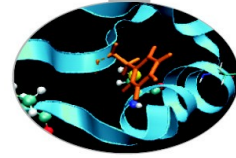
Distributors: Kevin M. Olson

Ref.: NASA e Drexel University

http://sourceforge.net/projects/paramesh/

AMR libraries

CHOMBO

Chombo provides a set of tools for implementing finite difference and finite volume methods for the solution of partial differential equations on block-structured adaptively refined rectangular grids.
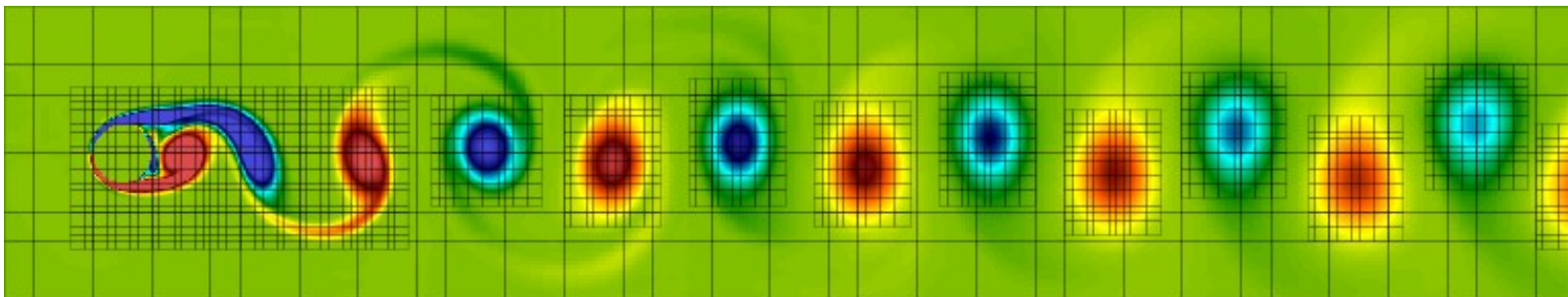
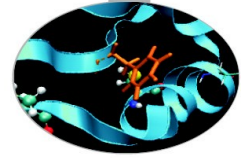Language: C

Availability: free for academy and research

Developers: P. Colella, D. T. Graves, J. N. Johnson et al.

Distributors: D. T. Graves

Ref.: Applied Numerical Algorithms Group - Computational Research Division - Lawrence Berkeley National Laboratory- Berkeley, CA, USA
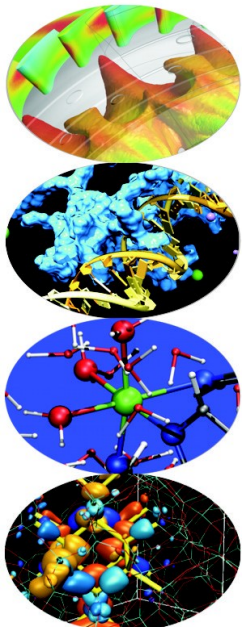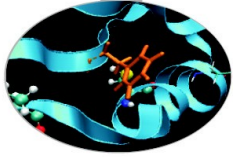
https://commons.lbl.gov/display/chombo/

I/O Libraries

Nicola Spallanzani – n.spallanzani@cineca.it
SuperComputing Applications and Innovation Department

HDF5

HDF5 is a data model, library, and file format for storing and managing data. It supports an unlimited variety of datatypes, and is designed for flexible and efficient I/O and for high volume and complex data. HDF5 is portable and is extensible, allowing applications to evolve in their use of HDF5. The HDF5 Technology suite includes tools and applications for managing, manipulating, viewing, and analyzing data in the HDF5 format.
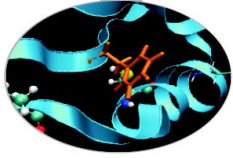
Language: Fortran/C/C++/Java

Availability: open source.

Developers: NCSA at the University of Illinois

Distributors: HDF Group

Ref.: http://www.hdfgroup.org

I/O libraries

NetCDF

NetCDF is a set of software libraries and self-describing, machine-independent data formats that support the creation, access, and sharing of array-oriented scientific data.

The NetCDF source-code is hosted at GitHub, and may be found directly at http://github.com/Unidata/netcdf-c.

NetCDF-4 (and pNetCDF) work with MPI version 2 (MPI-2).

Language: Fortran/C/C++/Java

Availability: open source.

Developers: Unidata

Distributors: Unidata

Ref.: http://www.unidata.ucar.edu/netcdf