Simulation of turbulent convective flow with conjugate heat transfer

Carlo Cintolesi

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CINECA Scai & Prace
Workshop HPC enabling of OpenFOAM for CFD applications
Casalecchio di Reno, 25 March 2015

Outline

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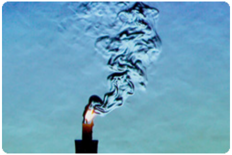
Convective flow

▷ Convective flows start when a temperature gradients give rise to a **buoyancy force** that drives the fluid and triggers turbulence generation.

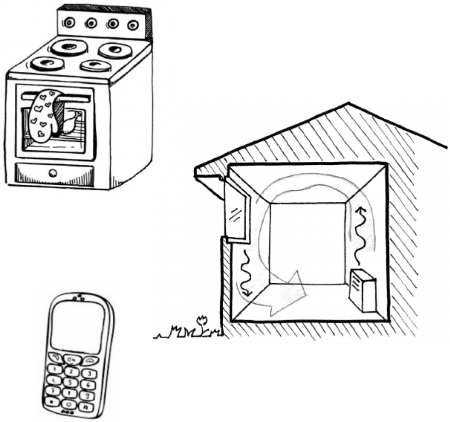


▷ Thermal energy is **transported** by the flow and transfered to the solid boundaries of the system. The interaction between solid/fluid starts.

▷ The **heat transfer** changes the temperature profile of the solids and perturbs the fluid flow.

Applications

- ▷ optimization of home appliances
- ▷ study the efficiency of heating and ventilation systems
- ▷ develop cooling systems for electronic devices



Outline

A simplified case: the closed cavity

Rectangular cavity filled with air,

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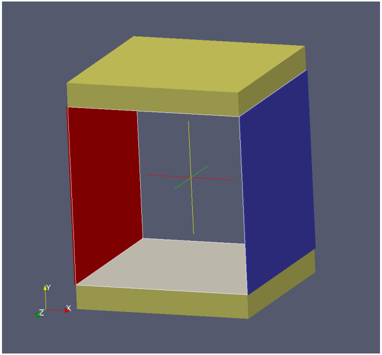

A simplified case: the closed cavity

Rectangular cavity filled with air, with two differently heated vertical walls

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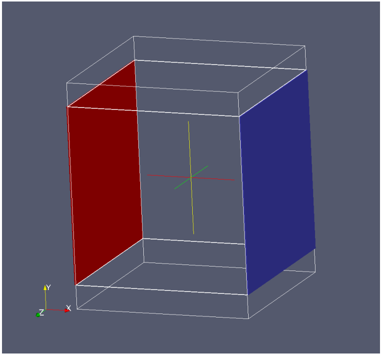

A simplified case: the closed cavity

Rectangular cavity filled with air,
with two differently heated vertical walls
and conductive horizontal boundaries

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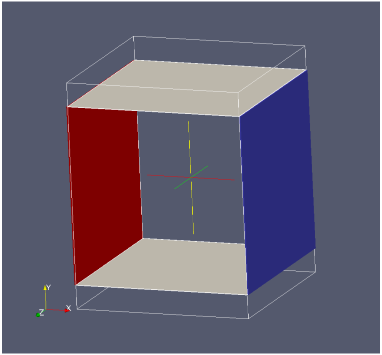
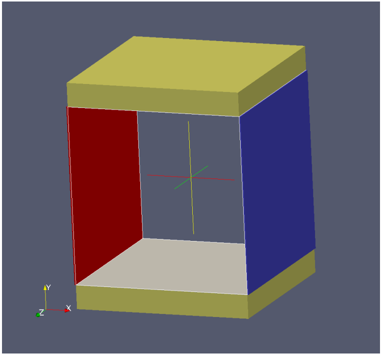


A simplified case: the closed cavity

Rectangular cavity filled with air,
with two differently heated vertical walls
and conductive horizontal boundaries
isolated from outside with an insulator.



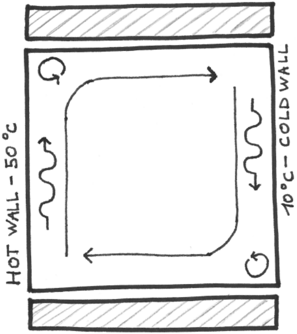
Flow sketch

Imposing a difference of temperature $\Delta T = 40^o C$, the resulting flow is characterised by low and localised turbulence:

$$Re = \frac{UL}{\nu} = 5.0 \times 10^4 \,,$$

and heat transfer is dominated by convection instead of conduction:

$$Ra = \frac{g\beta}{\nu k} \Delta T L^3 = 1.58 \times 10^9 \,.$$



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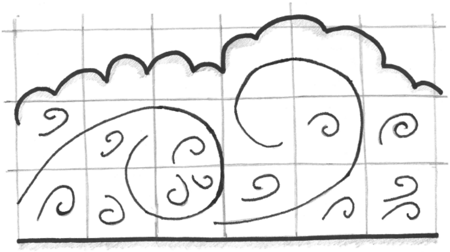
Simulation methodology

Large-eddy simulation technique: direct computation of large scale of motion and modelisation of the effects of the not-computed small scale.



Filtering operation:

$$\overline{u_i}(\mathbf{x}, t) = \int u_i(\mathbf{x}', t)\mathcal{G}(\mathbf{x}, \mathbf{x}')d\mathbf{x}'$$

Mathematical model: turbulence

▷ **Eddy viscosity** turbulence model: the contribution of small scale motion is modelled by an increasing of fluid viscosity

$$
\begin{aligned}
\frac{\partial \overline{u}_i}{\partial x_i} &= 0, \\
\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} &= -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x_i} - g\frac{\Delta\rho}{\rho_0}\delta_{i2} + \nu\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\tilde{\tau}_{ij},
\end{aligned}
$$

where the deviatoric part of stress tensor is modelled by

$$\tau_{ij} = -2c_s\overline{\Delta}^2|\overline{S}|\overline{S}_{ij},$$

with c_s the Smagorinsky constant. This is computed with the **Lagrangian dynamic** model, using information from the big scale of motion (Meneveau *et al.* 1995).

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Mathematical model: temperature and buoyancy

▷ The Boussinesq approximation is used for buoyancy force:

$$\frac{\Delta\rho}{\rho_0} = -\beta\Delta T.$$

▷ Temperature diffusion in **fluid domain** follows

$$\frac{\partial \overline{T_f}}{\partial t} + \frac{\partial \overline{T_f}\,\overline{u}_j}{\partial x_j} = \alpha_f \frac{\partial^2 \overline{T_f}}{\partial x_j \partial x_j} - \frac{\partial \lambda_j}{\partial x_j},$$

where the turbulent thermal flux $\lambda_j = \overline{u_j T_f} - \overline{u}_j \overline{T}$ is modelled using the Lagrangian dynamic method, adapted to scalar quantities (Armenio & Sarkar, 2002).

▷ Temperature diffusion in **solid domain** follows the classical law

$$\frac{\partial T_s}{\partial t} = \alpha_s \nabla^2 \cdot T_s$$

where $\alpha_{s/f}$ is the thermal diffusivity of the solid/fluid.

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Mathematical model: conjugate heat transfer

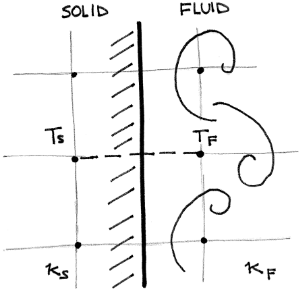
▷ **Thermal coupling** is obtained enforcing the continuity of temperature at the solid/fluid interface

$$T_{s,w} = T_{f,w} \,,$$

and imposing the balance of heat fluxes

$$k_s \left(\frac{\partial T_s}{\partial n} \right) = k_f \left(\frac{\partial T_f}{\partial n} \right)$$

where $k_{s/f}$ is thermal conductivity, and n is the normal to the interface.



Implementation in OpenFOAM

Solver is based on PISO algorithm.

Main loop steps:

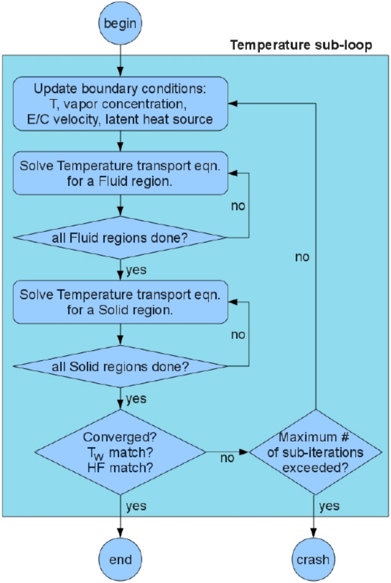
1. initialisation, load parameters
2. solve temperature for fluid/solid domains: coupling sub-loop

$$|T_{s,w} - T_{f,w}| < T_{err}$$

$$\left| k_s \left(\frac{\partial T_s}{\partial n} \right) - k_f \left(\frac{\partial T_f}{\partial n} \right) \right| < HF_{err}$$

3. solve the fluid motion equations

for more details
P. Sosnowski, PhD Thesis (2013).



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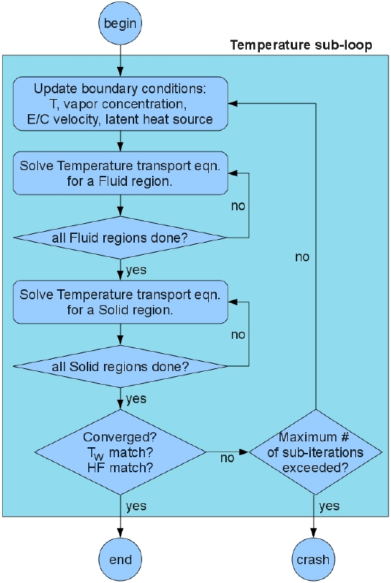

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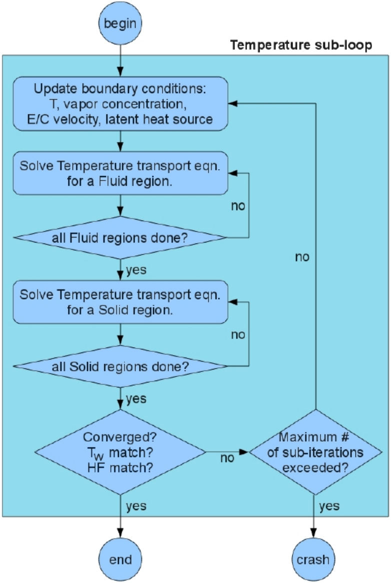

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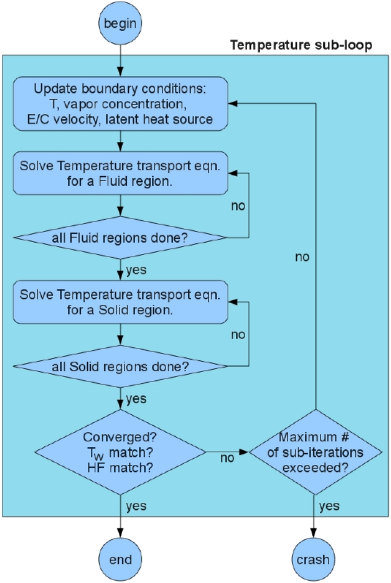

Outline

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Validation

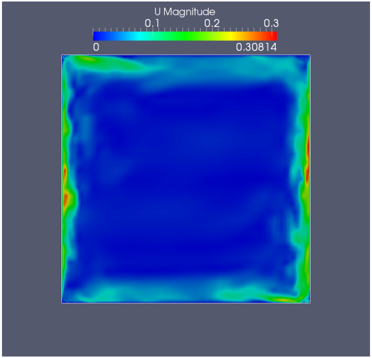
Validation of numerical solver is made against experimental and simulation results:

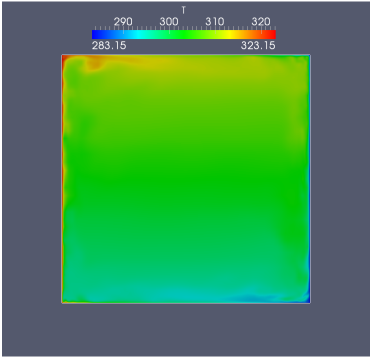
▷ Y.S. **Tian**, T.G. Karayiannis. *Low turbulence natural convection in an air filled square cavity. Part I and II*. Int. J. Heat and Mass Transfer 43, pp. 849–866 (2000)

▷ S.H. **Peng**, L. Davidson. *Large eddy simulation for turbulent buoyant flow in a confined cavity*. Int. J. Heat and Mass Transfer 22, pp. 323–331 (2001)

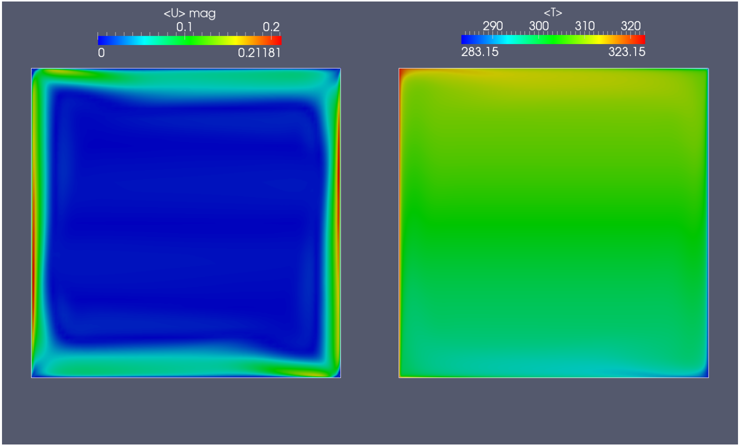
▷ F. **Ampofo**, T.G. Karayiannis. *Experimental benchmark data for turbulent natural convection in air filled square cavity*. Int. J. Heat and Mass Transfer 46, pp. 3551–3572 (2003)

Fluid flow



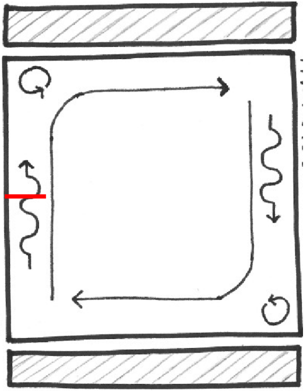


Fluid flow averaged

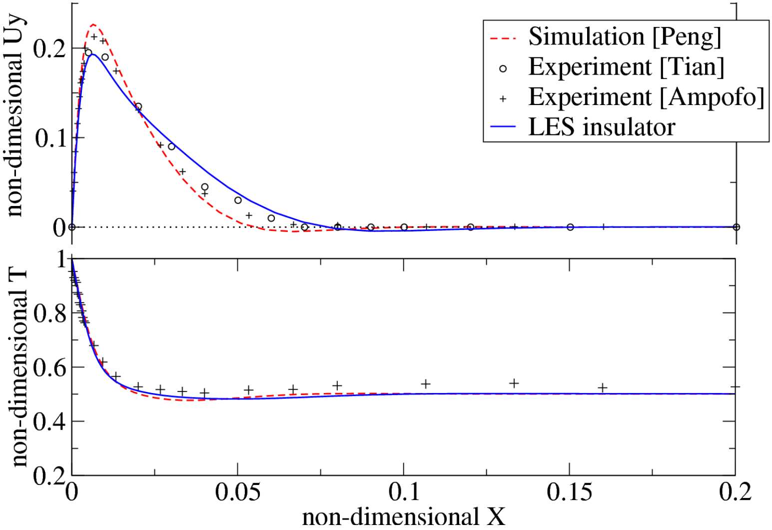


Validation of fluid solver: mean quantities

The following plots are take on a line close to the hot wall, at half high of the cavity.



Validation of fluid solver: first order statistics



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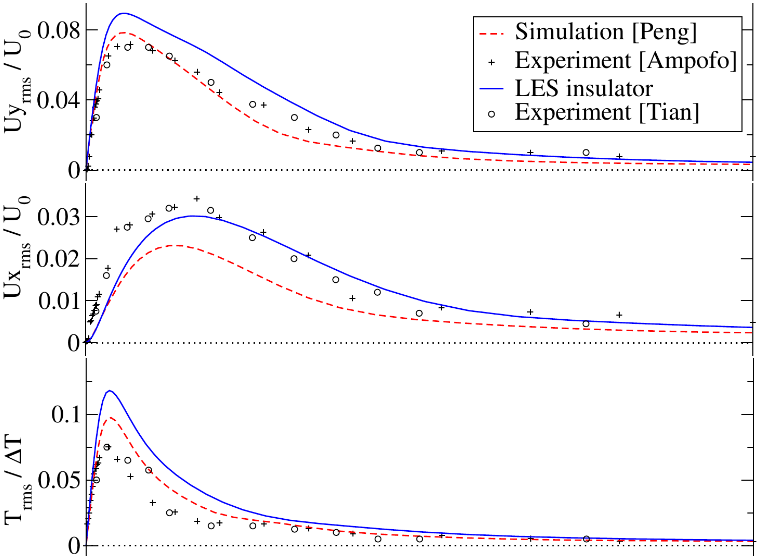
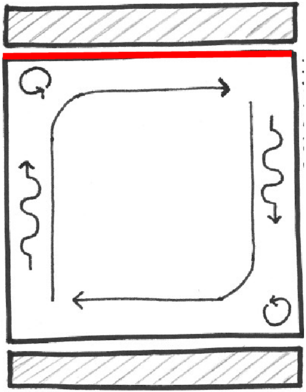


Validation of heat transfer: conductor's temperature

The following plots show the temperature profile on the top conductive plate.



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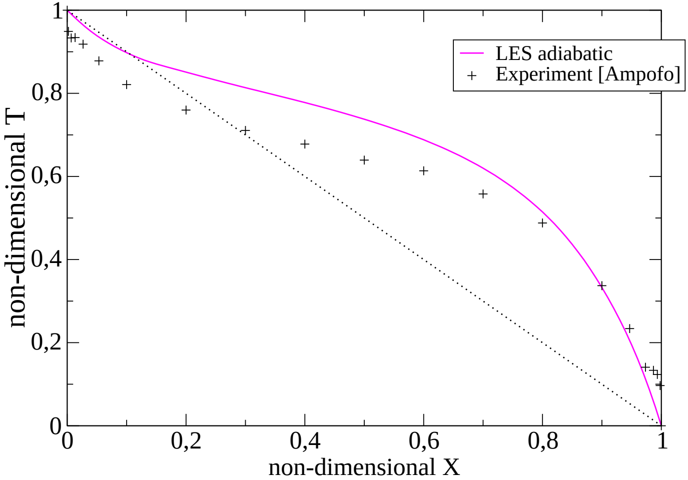

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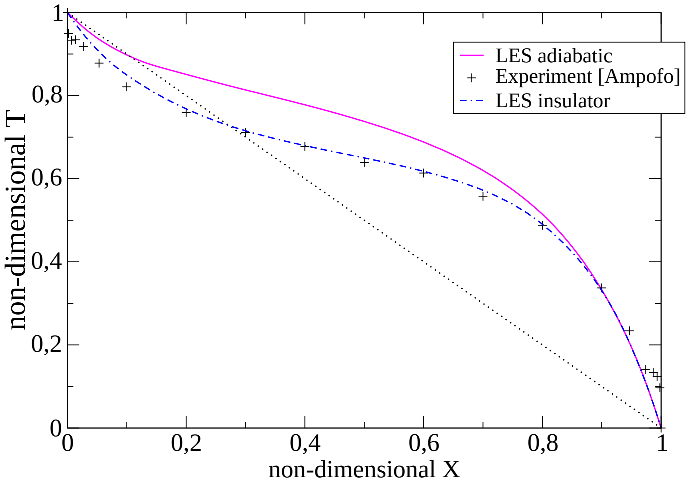
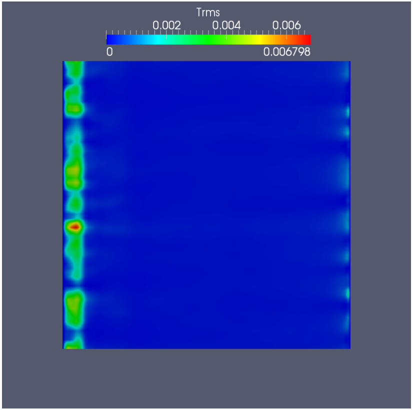


T_{RMS} on the top solid/fluid interface



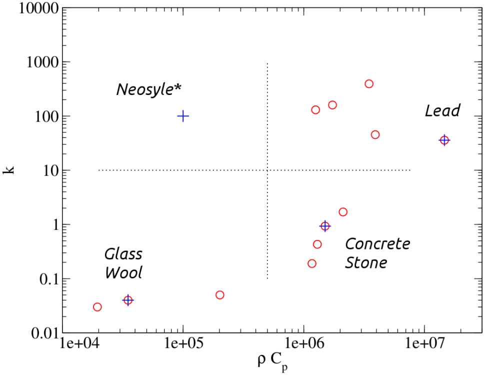
Outline

New materials for horizontal sheets

Two cases presented, representatives of insulator and conductor materials:

▷ Perfect Insulator → adiabatic;

▷ Neosyle → good conductor.



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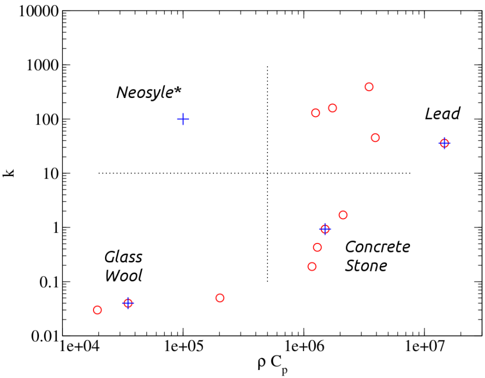
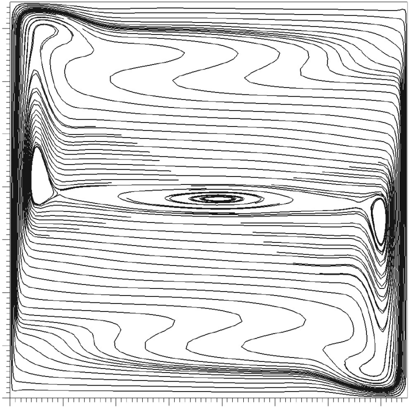
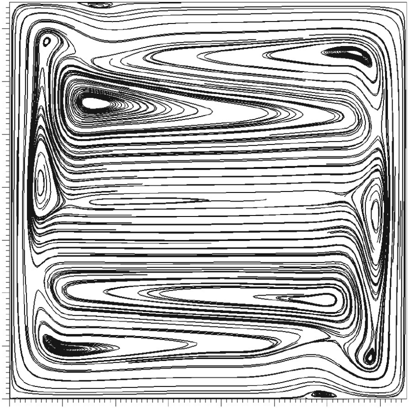


Streamlines

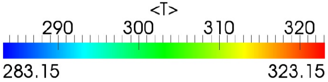


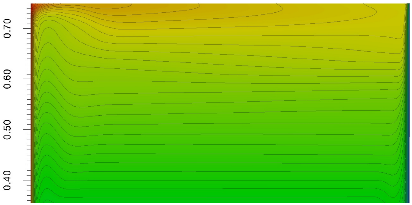
perfect insulator



neosyle

Temperature contour plots





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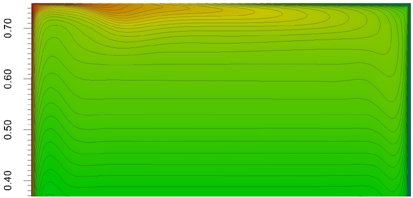

neosyle

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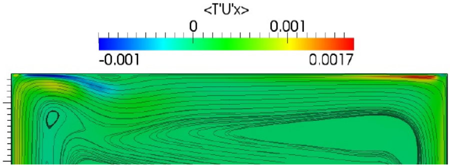
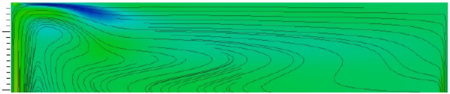
Turbulent thermal fluxes: horizontal and vertical

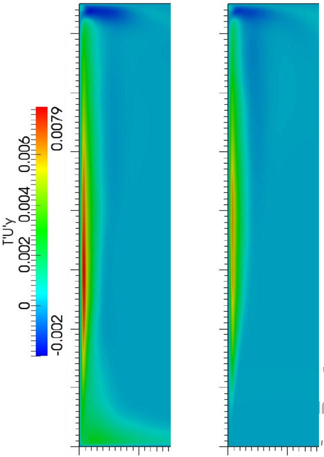


neosyle



perfect insulator

neosyle p. insulator



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Conclusions

Numerical model and implementation are successfully validated:

- Accurate turbulent model, suitable for anisotropic and localised turbulence;
- Thermal coupling implementation able to reproduce the heat transfer mechanism.

Strong influence of conductive solid boundaries on the fluid flow:

- Rise of recirculation bubble near the horizontal walls;
- Higher turbulence in the top and bottom region;
- Weaker temperature stratification in the core region.

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THANK YOU

Extra-contents

Mathematical model: Lagrangian dynamic model

▷ **Dynamic Lagrangian** model: c_s is dynamically computed using information from the big scale of motion: $\widehat{\Delta} = 2\overline{\Delta}$

The differences between the two scales can be quantified by

$$\mathcal{T}_{ij} - \widehat{\tau}_{ij} = \widehat{\overline{u}_i\overline{u}_j} - \widehat{\overline{u}}_i\widehat{\overline{u}}_j \stackrel{def}{=} L_{ij},$$

and the analogous turbulent model approximation

$$\mathcal{T}_{ij} - \widehat{\tau}_{ij} \cong c_s\, 2\overline{\Delta}^2 \left(\widehat{|\overline{S}|\overline{S}_{ij}} - 4|\widehat{\overline{S}}|\widehat{\overline{S}}_{ij} \right) \stackrel{def}{=} c_s M_{ij}.$$

Imposing the equivalence, we obtain an over-determinate system in the variable c_s:

$$L_{ij} = c_s M_{ij}.$$

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$$\mathcal{T}_{ij} - \widehat{\tau}_{ij} = \widehat{\overline{u}_i \overline{u}_j} - \widehat{\overline{u}}_i \widehat{\overline{u}}_j \overset{def}{=} L_{ij},$$

and the analogous expression of Smagorinsky turbulent model

$$\mathcal{T}_{ij} - \widehat{\tau}_{ij} \cong c_s \, 2\overline{\Delta}^2 \left(\widehat{|\overline{S}|\overline{S}_{ij}} - 4|\widehat{\overline{S}}|\widehat{\overline{S}}_{ij} \right) \overset{def}{=} c_s M_{ij}.$$

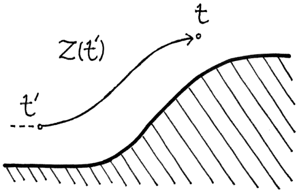
An unique solution can be found minimising the error of turbulent approximation

$$e_{ij} = L_{ij} - c_s M_{ij}.$$

Mathematical model: Lagrangian dynamic model

- following (Meneveau *et al.*, 1995), error is minimised in a last-square sense, along the particle trajectories cover in a time T:

$$\frac{\partial}{\partial c_s} \int_{-\infty}^{t} e_{ij}(z,t')e_{ij}(z,t') \; \frac{1}{\mathrm{T}} \, e^{\frac{-(t-t')}{\mathrm{T}}} \; dt' = 0$$



Mathematical model: Lagrangian dynamic model

The coefficient is computer using

$$c_s = \frac{\mathcal{I}_{LM}}{\mathcal{I}_{MM}},$$

where the $\mathcal{I}$ are integrals that can be computed via PDE

$$\begin{aligned} \frac{\partial \mathcal{I}_{LM}}{\partial t} + \overline{\mathbf{u}} \cdot \nabla \mathcal{I}_{LM} &= \frac{1}{\mathrm{T}} \left(L_{ij} M_{ij} - \mathcal{I}_{LM} \right), \\ \frac{\partial \mathcal{I}_{MM}}{\partial t} + \overline{\mathbf{u}} \cdot \nabla \mathcal{I}_{MM} &= \frac{1}{\mathrm{T}} \left(M_{ij} M_{ij} - \mathcal{I}_{MM} \right), \end{aligned}$$

where the characteristic time is

$$\mathrm{T} = \theta \overline{\Delta} (\mathcal{I}_{LM} \mathcal{I}_{MM})^{-1/8}, \quad \theta = 1.5\,.$$

Computation of dynamic Lagrangian turbulent model

The variable c_s can be also computer via a sequence specify by recursion

$$\begin{cases} \mathcal{I}_{LM}^{n+1}(\mathbf{x}) = \epsilon[L_{ij}M_{ij}]^{n+1}(\mathbf{x}) + (1-\epsilon)\cdot\mathcal{I}_{LM}^{n}(\mathbf{x}-\overline{\mathbf{u}}^n\Delta t) \\ \mathcal{I}_{LM}^{0}(\mathbf{x}) = c_{s,0}[M_{ij}M_{ij}]^0(\mathbf{x}) \end{cases} \tag{1}$$

where $c_{s,0} = 0.0256$ is a classical value for the Smagorinsky constant, and

$$\begin{cases} \mathcal{I}_{MM}^{n+1}(\mathbf{x}) = \epsilon[M_{ij}M_{ij}]^{n+1}(\mathbf{x}) + (1-\epsilon)\cdot\mathcal{I}_{MM}^{n}(\mathbf{x}-\overline{\mathbf{u}}^n\Delta t) \\ \mathcal{I}_{MM}^{0}(\mathbf{x}) = [M_{ij}M_{ij}]^0(\mathbf{x}) \end{cases} \tag{2}$$

with

$$\epsilon = \frac{\Delta t/^n}{1+\Delta t/\mathrm{T}^n}, \quad \mathrm{T}^n = 1.5\,\overline{\Delta}(\mathcal{I}_{LM}^n\mathcal{I}_{MM}^n)^{-1/8}.$$

Characteristic time for temperature

Non-dimensional analysis shows that the thermal equilibrium a time is

$$\tau = \frac{L^2}{\alpha} = \frac{\rho C_p L^2}{k} .$$

For the cases considered we have:

	$\rho C_p < 10^6$	$\rho C_p > 10^6$
$k > 10$	**Neosyle**: $\tau = 5.6 \cdot 10^2$,	**Lead**: $\tau = 2.3 \cdot 10^5$,
$k < 10$	**Glass wool**: $\tau = 3.65 \cdot 10^5$,	**Concrete**: $\tau = 0.9 \cdot 10^6$,