Energy conserving schemes in OpenFOAM

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Introduction

Objectives

▶ Give the idea of "discrete energy consistency"
▶ Capability of different schemes/solver to satisfy the kinetic energy conservation property of Navier-Stokes equations
▶ Comparative study: OpenFOAM vs. other CFD solvers
▶ Test Cases:
  1. Decaying isotropic turbulence
  2. Taylor-Green flow
  3. Laminar circular cylinder
  4. DNS of supersonic channel flows
Numerical solvers

OpenFOAM

- Open Source (GPL)
- Unstructured, Finite Volume (FV)
- `dnsFoam`, incompressible DNS PISO algorithm
- `rhoCentralFoam`, compressible, Kurganov and Tadmor (TVD).
### Numerical solvers

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#### Commercial Solver
- Proprietary
- Unstructured (FV)
- Upwinding, TVD
### Numerical solvers

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#### Commercial Solver
- Proprietary
- Unstructured (FV)
- Upwinding, TVD

#### Finite Differences in-house solver
- Compressible, energy-conserving
- 3D, Cartesian
- Arbitrary order of accuracy
Conservation principles

Kinetic energy

\[
\frac{d}{dt} \int_V \left( \rho \frac{u_i u_i}{2} \right) \, dV = -\int_S \left( \rho \frac{u_i u_i}{2} + p \right) u_i n_i \, dS + \int_V \left( p \frac{\partial u_i}{\partial x_i} \right) \, dV + \int_V \frac{\partial}{\partial x_i} \left( \sigma_{ik} u_k \right) \, dV
\]
It is important to satisfy the conservation of the kinetic energy in the **inviscid** and **incompressible** limit, in an **unbounded domain**.
Conservation principles

Kinetic Energy

\[
\frac{d}{dt} \int_V \left( \rho \frac{u_i u_i}{2} \right) \, dV = - \int_S \left( \rho \frac{u_i u_i}{2} + p \right) u_i n_i \, dS + \int_V \left( p \frac{\partial u_i}{\partial x_i} \right) \, dV + \int_V \frac{\partial}{\partial x_i} \left( \sigma_{ik} u_k \right) \, dV
\]

It is important to satisfy the conservation of the kinetic energy in the \textit{inviscid} and \textit{incompressible} limit, in an \textit{unbounded domain}.

Discrete energy conservation:

the \textbf{nonlinear} terms in the Navier-Stokes equations:

- do \textbf{not} contribute to the \textbf{net} variation of kinetic energy
- do \textbf{not} \textbf{dissipate} or \textbf{inject} spurious energy
Is OpenFOAM energy conserving?

cfd-online Forum

- May 27, 2011 longamon asked:
  "Hey Foamers..........has anyone used kinetic energy conservative schemes in OF?"

Low-dissipative schemes


Energy-Conserving Schemes

**Numerical Flux:**

\[
\frac{\partial (\rho u \varphi)}{\partial x} = \frac{1}{h} \left( \hat{f}_{j+1/2} - \hat{f}_{j-1/2} \right), \quad \hat{f}_{j+1/2} = \frac{1}{2} (\rho_j u_j \varphi_j + \rho_{j+1} u_{j+1} \varphi_{j+1})
\]
Energy-Conserving Schemes

Numerical Flux:

\[
\frac{\partial (\rho u \varphi)}{\partial x} = \frac{1}{h} \left( \hat{f}_{j+1/2} - \hat{f}_{j-1/2} \right), \quad \hat{f}_{j+1/2} = \frac{1}{2} \left( \rho_j u_j \varphi_j + \rho_{j+1} u_{j+1} \varphi_{j+1} \right)
\]

A conservative and energy-consistent scheme has been proposed by Ducros et al. (2000) and generalized by Pirozzoli (2010):

FD:

\[
\hat{f}_{j+1/2} = \frac{1}{8} \left( \rho_j + \rho_{j+1} \right) \left( \varphi_j + \varphi_{j+1} \right) \left( u_j + u_{j+1} \right)
\]
Implementation into OpenFOAM: **rhoEnergyFoam**

**Space discretization**

Energy consistent **numerical fluxes** implemented in the OpenFOAM library:
- **stability** without **numerical dissipation**

**Time integration**

Explicit fourth-order **Runge-Kutta** time integration:
- **low-storage** implementation, suitable for LES and DNS.
rhoEnergyFoam:

- compressible unsteady solver (subsonic and supersonic shock-free flows)
- exact kinetic energy conservation in the inviscid and incompressible limit
- 2nd order accurate in space, 4th order in time
- Integration with the OpenFOAM thermodynamic and turbulence libraries
- Suitable for DNS and LES of compressible flows.
Decaying homogeneous isotropic turbulence

Test case proposed by Honein and Moin (2004)

\[ E(k) = A \left( \frac{k}{k_0} \right)^4 e^{-2(k/k_0)^2} \]

\[ Re_\lambda \approx 100 \quad Mt_0 = \frac{u_{rms}}{\langle a \rangle} = 0.2 \]
Decaying homogeneous isotropic turbulence

Mesh $32^3$ $t/\tau = 5$

**Viscous**

![Viscous turbulence graph](image)

**Inviscid**

![Inviscid turbulence graph](image)
Decaying homogeneous isotropic turbulence

Mesh $32^3$ $t/\tau = 5$

$E(k)$

Viscous

Inviscid
Decaying homogeneous isotropic turbulence

Mesh $32^3$  $t/\tau = 5$

$E(k)$

Viscous

Inviscid
Decaying homogeneous isotropic turbulence

Mesh $32^3$ $t/\tau = 5$

$E(k)$

Viscous

Inviscid
Decaying homogeneous isotropic turbulence

Mesh $64^3$ $t/\tau = 5$

$E(k)$

Viscous

Inviscid

FD

rEFoam

dnsFoam

Commercial

$k^2$
Decaying homogeneous isotropic turbulence

Mesh $32^3$

$\frac{K_T}{K_{T0}}$

$t/\tau$

FD
rhoEnergyFoam
dnsFoam
Commercial

Inviscid
Taylor-Green flow

Test case proposed by Duponcheel et al. (2008)

Initial Conditions

\[
\begin{align*}
    u &= u_0 \sin (k_0 x) \cos (k_0 y) \cos (k_0 z) \\
    v &= -u_0 \cos (k_0 x) \sin (k_0 y) \cos (k_0 z) \\
    w &= 0
\end{align*}
\]

Time reversibility

**Euler** equations are time reversible, that is:

\[ u(t, x) \Rightarrow -u(-t, x) \]
Taylor-Green flow

\[ \frac{\omega_{max}}{\omega_{max0}} \]

\[ t \]

FD
Taylor-Green flow

\[ \frac{\omega_{\text{max}}}{\omega_{\text{max}0}} \]

Comparison of results from FD and rhoEnergyFoam methods over time. The graph shows oscillations in the ratio of maximum angular velocity to its initial value over time, with distinct peaks and troughs. The FD method is represented by the red line, while rhoEnergyFoam is indicated by green markers.

Time (t) is plotted on the x-axis, ranging from 0 to 16, and the ratio \( \frac{\omega_{\text{max}}}{\omega_{\text{max}0}} \) on the y-axis, ranging from 0 to 7.
Taylor-Green flow

\[ \frac{\omega_{max}}{\omega_{max0}} \]

- FD
- rhoEnergyFoam
- dnsFoam

Graph showing the ratio of \( \omega_{max} \) to \( \omega_{max0} \) over time \( t \).
Taylor-Green flow

dnsFoam

rhoEnergyFoam
Taylor-Green flow: Kinetic energy spectra

\[ E(k) \propto k^{-2} \]

- \( E(k) \) vs. \( k \) for different times:
  - \( t = 0 \)
  - \( t = 16 \)
Circular Cylinder at low Reynolds number

**Regimes**

Williamson (1996)
- $Re < 49$ stationary laminar
- $49 < Re < 190$ laminar, vortex shedding
- $190 < Re < 260$ transitional wake

<table>
<thead>
<tr>
<th>$Re=40$, Mesh=64x64</th>
<th>$l/d$</th>
<th>$C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rhoEnergyFoam</td>
<td>2.40</td>
<td>1.54</td>
</tr>
<tr>
<td>icoFOAM</td>
<td>2.36</td>
<td>1.53</td>
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<td>rhoCentralFoam</td>
<td>1.19</td>
<td>1.90</td>
</tr>
<tr>
<td>Commercial</td>
<td>1.58</td>
<td>1.63</td>
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<td>Taira et al. (2007)</td>
<td>2.30</td>
<td>1.56</td>
</tr>
<tr>
<td>Linnick et al. (2005)</td>
<td>2.28</td>
<td>1.54</td>
</tr>
<tr>
<td>Coutanceau et al. (1977)*</td>
<td>2.13</td>
<td>1.59</td>
</tr>
</tbody>
</table>
Circular Cylinder at low Reynolds number

rhoEnergyFoam

Re=200, Mesh=128x128

<table>
<thead>
<tr>
<th></th>
<th>$C_D$</th>
<th>$C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rhoEnergyFoam</td>
<td>1.33 ± 0.050</td>
<td>±0.66</td>
</tr>
<tr>
<td>icoFoam</td>
<td>1.33 ± 0.045</td>
<td>±0.69</td>
</tr>
<tr>
<td>Commercial</td>
<td>1.26±?</td>
<td>±0.57</td>
</tr>
<tr>
<td>rhoCentralFoam</td>
<td>1.30±?</td>
<td>±0.40</td>
</tr>
<tr>
<td>Taira et al.</td>
<td>1.35 ± 0.044</td>
<td>±0.69</td>
</tr>
<tr>
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</tbody>
</table>

Vorticity
Compressible channel flow: DNS

rhoEnergyFoam

Computational set-up

- **Parameters:**
  \(Re_\tau(= u_\tau h/\nu) = 220,\)
  \(M_b(= u_b/a_w) = 1.5\)

- **Forcing:** momentum and total energy equation, to maintain constant mass flow rate.

- **Box dimensions:**
  \(L_x \times L_y \times L_z = 4\pi h \times 2h \times 4/3\pi h\)

- **Mesh resolution:** \(\Delta x^+ \approx 10, \Delta y^+ < 4, \Delta z^+ \approx 5\)
  
  About 9 milions cells

Streamwise velocity in the YZ plane
Compressible channel flow: DNS

Mean velocity

\[ u^+ \]

FD
rhoEnergyFoam

\[ y^+ \]
Compressible channel flow: DNS

Streamwise velocity fluctuations

\[ \frac{\rho}{\rho_w u'^2} \]

\[ y^+ \]

FD

rhoEnergyFoam

0 50 100 150 200 250

0 1 2 3 4 5 6 7 8 9 10
Compressible channel flow: DNS

Wall-normal velocity fluctuations

\( \frac{\rho}{\rho w' v'^2} \) vs. \( y^+ \)

FD
rhoEnergyFoam

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Compressible channel flow: DNS

Reynolds stress

\[ \frac{\rho \omega u' v'}{u' v'} \] vs. \[ y^+ \]

FD
rhoEnergyFoam
Computational Efficiency

\texttt{rhoCentralFoam} gives the \textbf{same} results as the \textbf{in-house}(FD) solver.
rhoCentralFoam gives the same results as the in-house (FD) solver.

What about the computational efficiency?
**Computational Efficiency**

\[ \text{rhoCentralFoam gives the same results as the in-house (FD) solver.} \]

\[ \text{\[\uparrow\]} \]

What about the **computational efficiency**?

<table>
<thead>
<tr>
<th>Computational times</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>
Scalability

Linear Scalability

» FD: up to 64K CPUs and more
» standard OpenFOAM solvers: up to 1K CPUs.
  Bottlenecks: I/O and linear solvers
» rhoEnergyFoam: ?
Conclusions

**Discrete kinetic energy conservation guarantees:**
- **stability** without addition of artificial **dissipation**
- a greater **fidelity** to the physics
- OpenFOAM/Commercial are **not energy-conserving**
- OpenFOAM can be **modified**, the proposed scheme is easy to be implemented in an existing solver

**Future work**
- Provide rhoEnergyFoam with shock-capturing capabilities
- Scalability analysis of rhoEnergyFoam
- RANS and LES simulations of complex geometries
Tutorial: Time reversibility on unstructured meshes

XY Plane
Tutorial: Time reversibility on unstructured meshes

Initial Time

Reversal Time

Final Time
Time reversibility and energy consistency do no depend on the mesh resolution !!!

Let us consider a coarse mesh, $8^3$.

Kinetic energy conservation is guaranteed $\Rightarrow$ Time reversibility
Tutorial: Time reversibility, extremely coarse mesh