Energy conserving schemes in OpenFOAM

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Introduction

Objectives

- Give the idea of **"discrete energy consistency"**
- Capability of different **schemes/solver** to satisfy **the kinetic energy conservation** property of **Navier-Stokes** equations
- Comparative study: **OpenFOAM** vs. **other CFD** solvers
- **Test Cases**:
  1. Decaying isotropic turbulence
  2. Taylor-Green flow
  3. Laminar circular cylinder
  4. DNS of supersonic channel flows

Numerical solvers

OpenFOAM

- Open Source(GPL)
- Unstructured,
  Finite Volume(FV)
- **dnsFoam**,
  incompressible DNS
  PISO algorithm
- **rhoCentralFoam**,
  compressible,
  Kurganov and
  Tadmor (TVD).

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Commercial Solver

- Proprietary
- Unstructured(FV)
- Upwinding, TVD

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Commercial Solver

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Finite Differences in-house solver

- Compressible energy-conserving
- 3D, Cartesian
- Arbitrary order of accuracy

Kinetic energy

$$\frac{d}{dt}\int\limits_V \left(\rho\,\frac{u_i\,u_i}{2}\right)\,\mathrm{d}V = -\int\limits_S \left(\rho\,\frac{u_i\,u_i}{2} + p\right) u_i\,n_i\,\mathrm{d}S + \int\limits_V \left(p\,\frac{\partial u_i}{\partial x_i}\right)\,\mathrm{d}V + \int\limits_V \frac{\partial}{\partial x_i}\left(\sigma_{ik}\,u_k\right)\,\mathrm{d}V$$

Conservation principles

Kinetic Energy

$$\frac{d}{dt}\int_V \left(\rho\,\frac{u_i\,u_i}{2}\right)\mathrm{d}V = \cancel{-\int_S \left(\rho\,\frac{u_i\,u_i}{2} + p\right) u_i\,n_i\,\mathrm{d}S} + \cancel{\int_V \left(p\,\frac{\partial u_i}{\partial x_i}\right)\mathrm{d}V} + \cancel{\int_V \frac{\partial}{\partial x_i}\left(\sigma_{ik}\,u_k\right)\mathrm{d}V}$$

It is important to satisfy the conservation of the kinetic energy in the **inviscid** and **incompressible** limit, in an **unbounded domain**

Conservation principles

Kinetic Energy

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It is important to satisfy the conservation of the kinetic energy in the **inviscid** and **incompressible** limit, in an **unbounded domain**

Discrete energy conservation:

the **nonlinear** terms in the Navier-Stokes equations:

- do **not** contribute to the **net** variation of kinetic energy
- do not **dissipate** or **inject** spurious energy

Is OpenFOAM energy conserving?

cfd-online Forum

- May 27, 2011 **longamon asked:**
  "Hey Foamers...........has anyone used ***kinetic energy conservative*** *schemes in OF?"*

Low-dissipative schemes

- Vuorinen, V., et al. **"A low-dissipative, scale-selective discretization scheme for the Navier-Stokes equations."** Computers & Fluids 70 (2012): 195-205.
- Vuorinen, V., et al. **"On the implementation of low-dissipative Runge-Kutta projection methods for time dependent flows using OpenFOAM."** Computers & Fluids 93 (2014): 153-163.

Energy-Conserving Schemes

Numerical Flux:

$$\frac{\partial(\rho\, u\, \varphi)}{\partial x} = \frac{1}{h}\left(\hat{f}_{j+1/2} - \hat{f}_{j-1/2}\right), \qquad \widehat{f}_{j+1/2} = \frac{1}{2}\left(\rho_j\, u_j\, \varphi_j + \rho_{j+1}\, u_{j+1}\, \varphi_{j+1}\right)$$

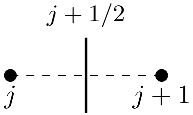
Energy-Conserving Schemes

Numerical Flux:

$$\frac{\partial(\rho\, u\, \varphi)}{\partial x} = \frac{1}{h}\left(\hat{f}_{j+1/2} - \hat{f}_{j-1/2}\right), \qquad \widehat{f}_{j+1/2} = \frac{1}{2}\left(\rho_j\, u_j\, \varphi_j + \rho_{j+1}\, u_{j+1}\, \varphi_{j+1}\right)$$

A **conservative** and **energy-consistent** scheme has been proposed by **Ducros et al. (2000)** and generalized by **Pirozzoli (2010)**:

$$\textbf{FD}: \qquad \widehat{f}_{j+1/2} = \frac{1}{8}\left(\rho_j + \rho_{j+1}\right)\left(\varphi_j + \varphi_{j+1}\right)\left(u_j + u_{j+1}\right)$$



Implementation into OpenFOAM:**rhoEnergyFoam**

Space discretization

Energy consistent **numerical fluxes** implemented in the OpenFOAM library:

- **stability** without **numerical dissipation**

Time integration

Explicit fourth-order **Runge-Kutta** time integration:

- **low-storage** implementation, suitable for LES and DNS.

Implementation into OpenFOAM:**rhoEnergyFoam**

rhoEnergyFoam:

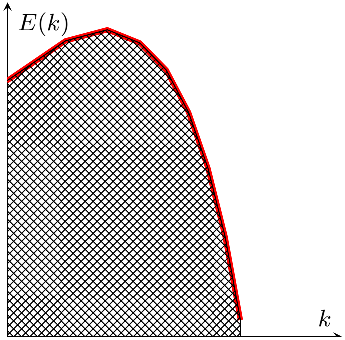
- compressible unsteady solver (subsonic and supersonic shock-free flows)
- exact kinetic energy conservation in the inviscid and incompressible limit
- 2nd order accurate in space, 4th order in time
- Integration with the OpenFOAM **thermodynamic** and **turbulence** libraries
- Suitable for DNS and LES of compressible flows.

Decaying homogeneous isotropic turbulence

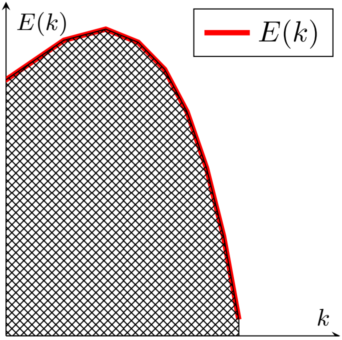
Test case proposed by **Honein and Moin (2004)** 

$$E(k) = A\left(\frac{k}{k_0}\right)^4 e^{-2\,(k/k_0)^2}$$

$$Re_\lambda \approx 100 \qquad M_{t0} = \frac{u_{rms}}{\langle a \rangle} = 0.2$$

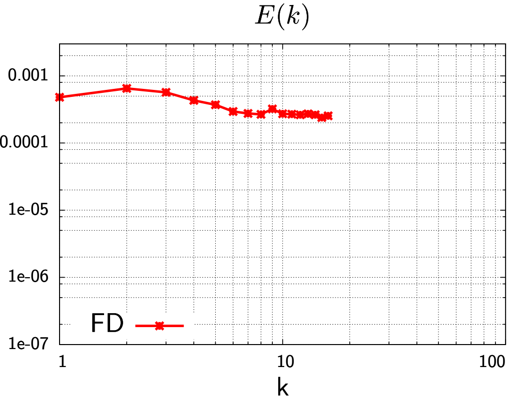


Viscous

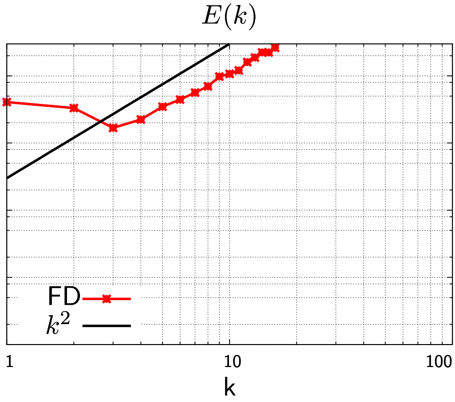


Inviscid

Mesh 32^3 $t/\tau = 5$



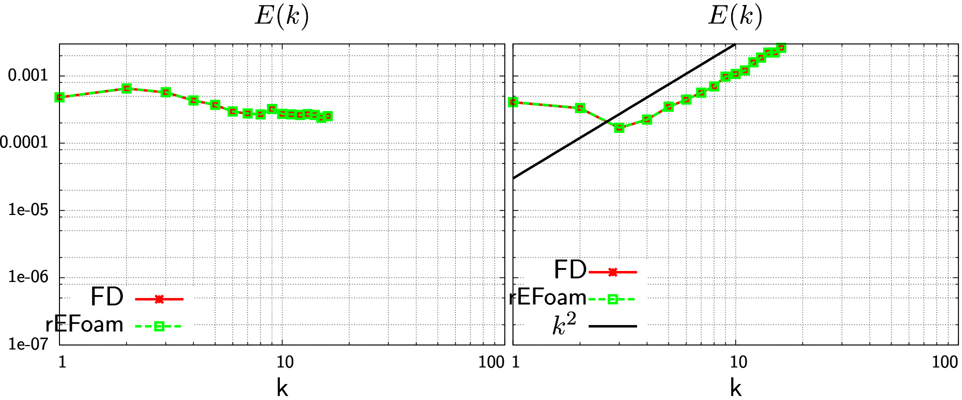
Viscous



Inviscid

Decaying homogeneous isotropic turbulence

Mesh 32^3 $\mathbf{t/\tau = 5}$



Viscous **Inviscid**

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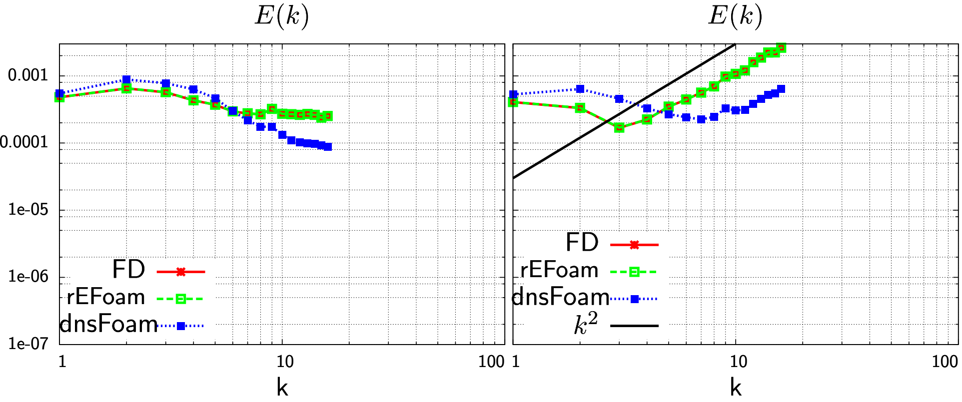
Decaying homogeneous isotropic turbulence

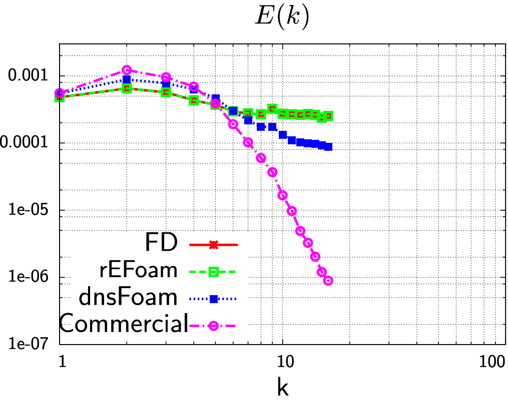
Mesh 32^3 $t/\tau = 5$

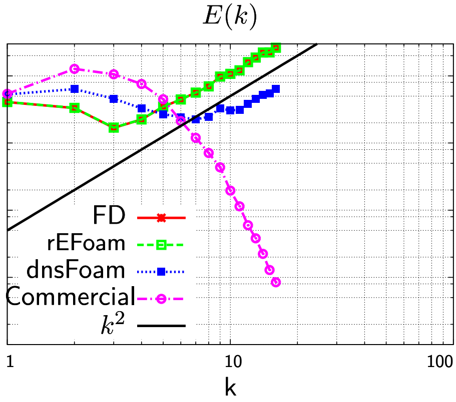


Viscous **Inviscid**

Decaying homogeneous isotropic turbulence

Mesh 32^3 $t/\tau = 5$





Viscous **Inviscid**

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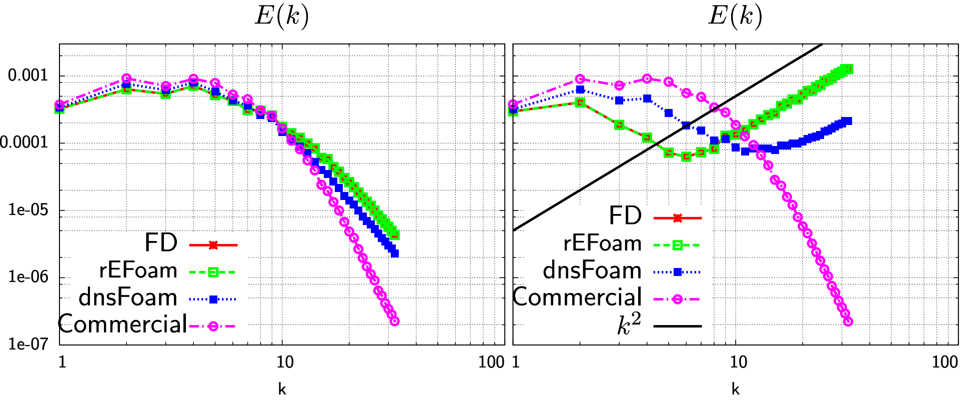
Decaying homogeneous isotropic turbulence

Mesh 64^3 $t/\tau = 5$



Viscous **Inviscid**

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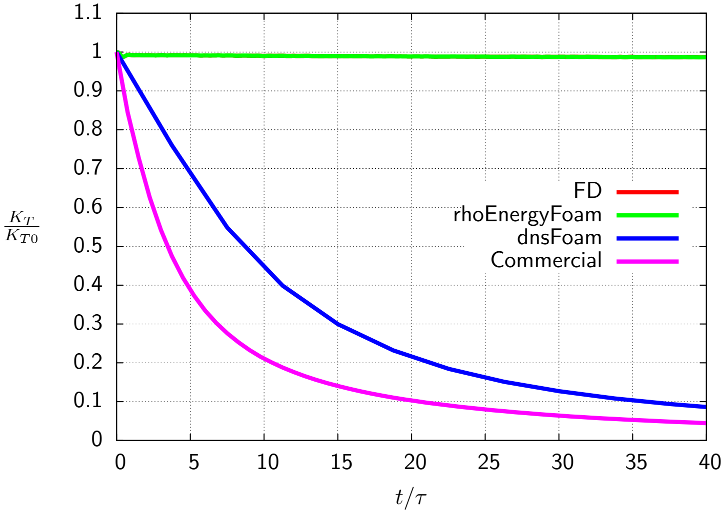
Decaying homogeneous isotropic turbulence

Mesh 32^3



Inviscid

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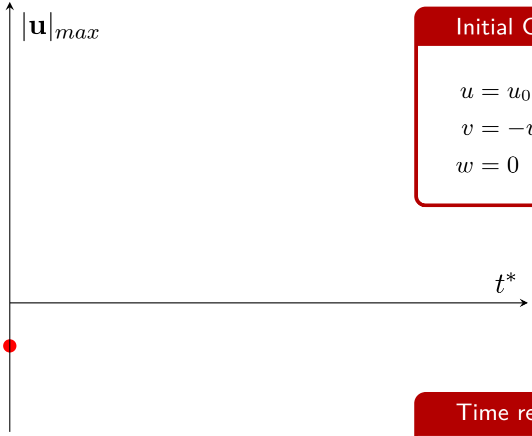
Taylor-Green flow

Test case proposed by **Duponcheel et al. (2008)**



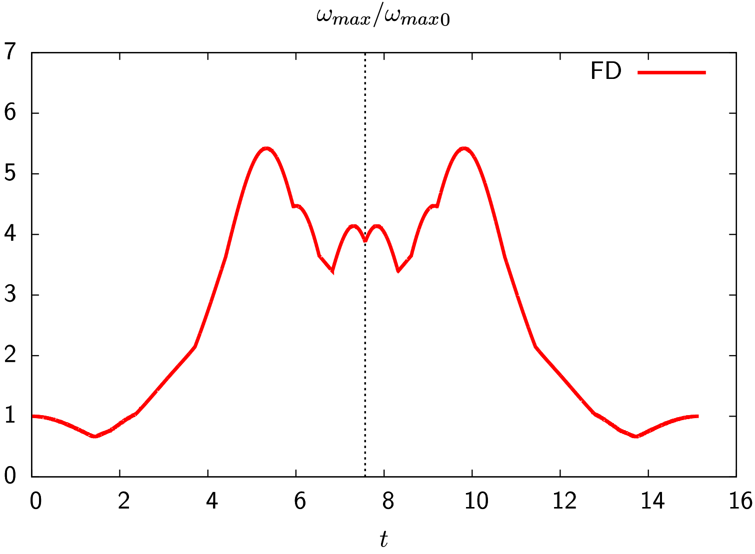
Initial Conditions

$$u = u_0 \sin(k_0\, x) \cos(k_0\, y) \cos(k_0\, z)$$
$$v = -u_0 \cos(k_0\, x) \sin(k_0\, y) \cos(k_0\, z)$$
$$w = 0$$

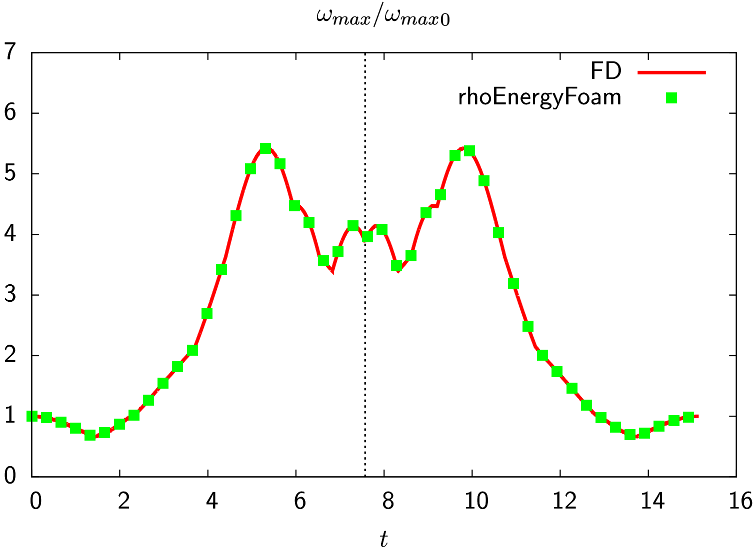
Time reversibility

Euler equations are **time reversible**, that is:
$\mathbf{u}(t, \mathbf{x}) \Rightarrow -\mathbf{u}(-t, \mathbf{x})$

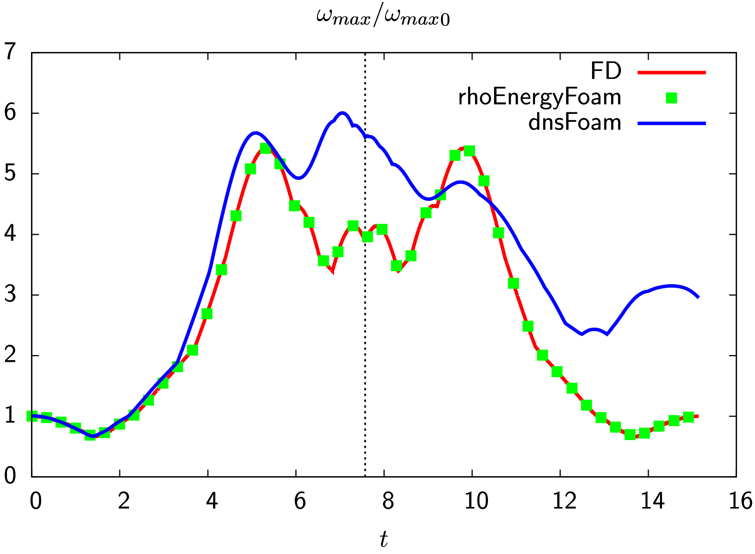
Taylor-Green flow



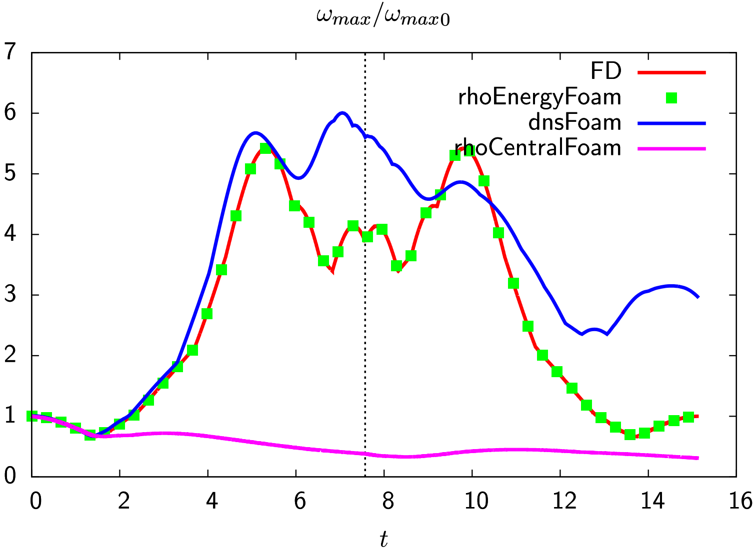
Taylor-Green flow



Taylor-Green flow



Taylor-Green flow



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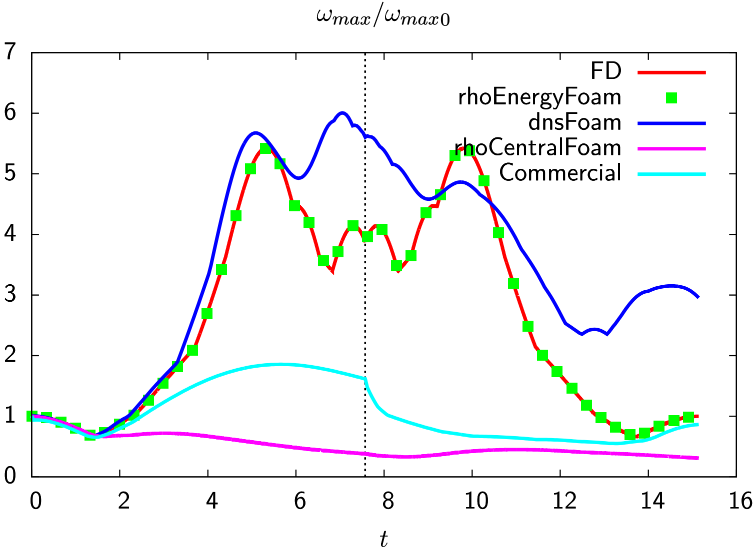
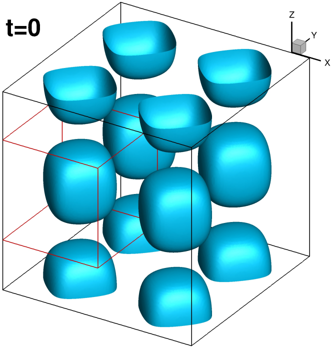
Taylor-Green flow

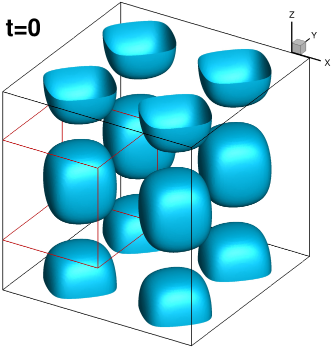


Taylor-Green flow

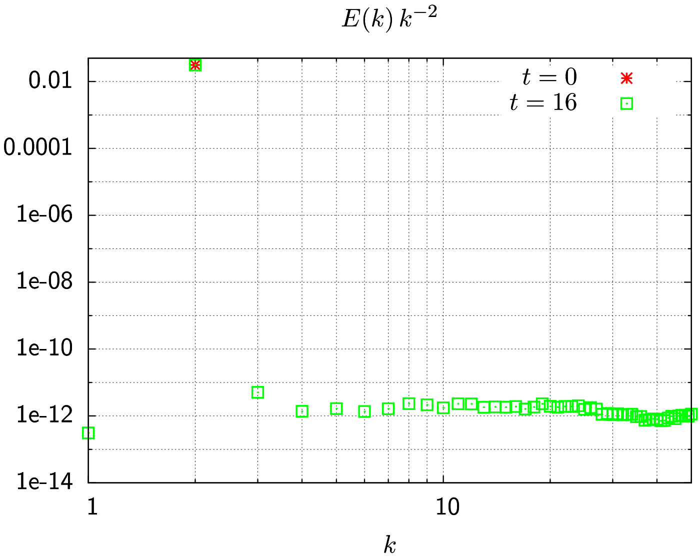
dnsFoam



rhoEnergyFoam



Taylor-Green flow: Kinetic energy spectra



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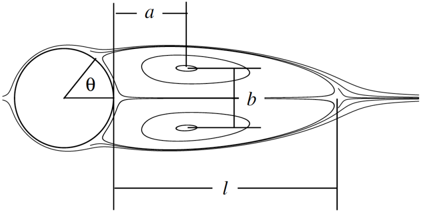
Regimes

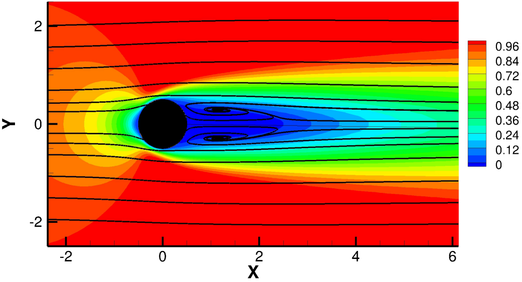
Williamson (1996)

- $\mathbf{Re} < \mathbf{49}$ stationary laminar
- $\mathbf{49} < \mathbf{Re} < \mathbf{190}$ laminar, vortex shedding
- $\mathbf{190} < \mathbf{Re} < \mathbf{260}$ transitional wake

Re=40, Mesh=64x64

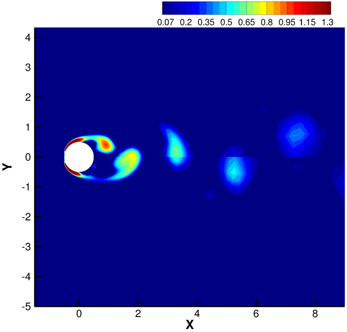
	l/d	C_D
rhoEnergyFoam	2.40	**1.54**
icoFOAM	2.36	**1.53**
rhoCentralFoam	1.19	**1.90**
Commercial	1.58	**1.63**
Taira et al. (2007)	2.30	1.56
Linnick et al. (2005)	2.28	1.54
Coutanceau et al. (1977)*	2.13	1.59





Circular Cylinder at low Reynolds number

rhoEnergyFoam



Vorticity

Re=200, Mesh=128x128

	C_D	C_L
rhoEnergyFoam	$\mathbf{1.33} \pm 0.050$	± 0.66
icoFoam	$\mathbf{1.33} \pm 0.045$	± 0.69
Commercial	$\mathbf{1.26} \pm ?$	± 0.57
rhoCentralFoam	$\mathbf{1.30} \pm ?$	± 0.40
Taira et al.	1.35 ± 0.044	± 0.69
Linnick et al.*	1.34 ± 0.044	± 0.69

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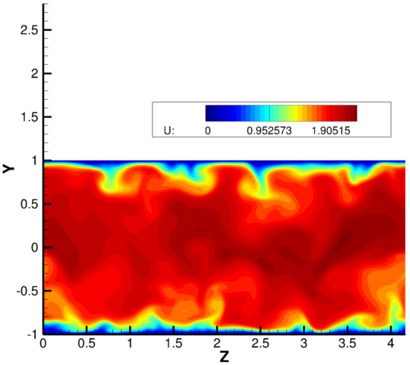
Compressible channel flow: DNS

rhoEnergyFoam

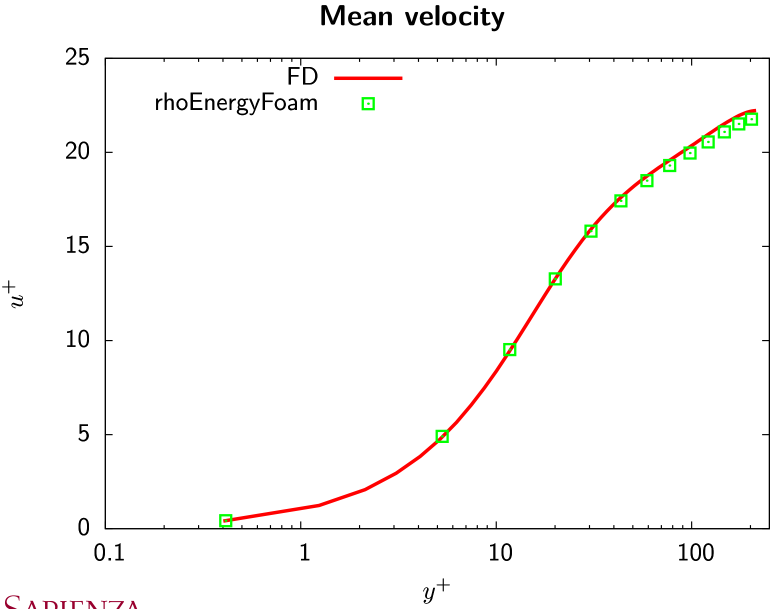


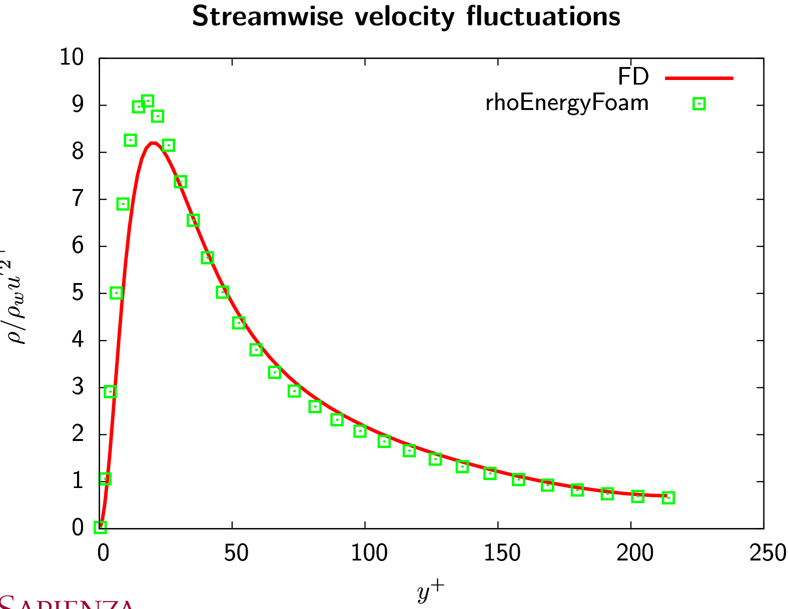
Streamwise velocity in the YZ plane

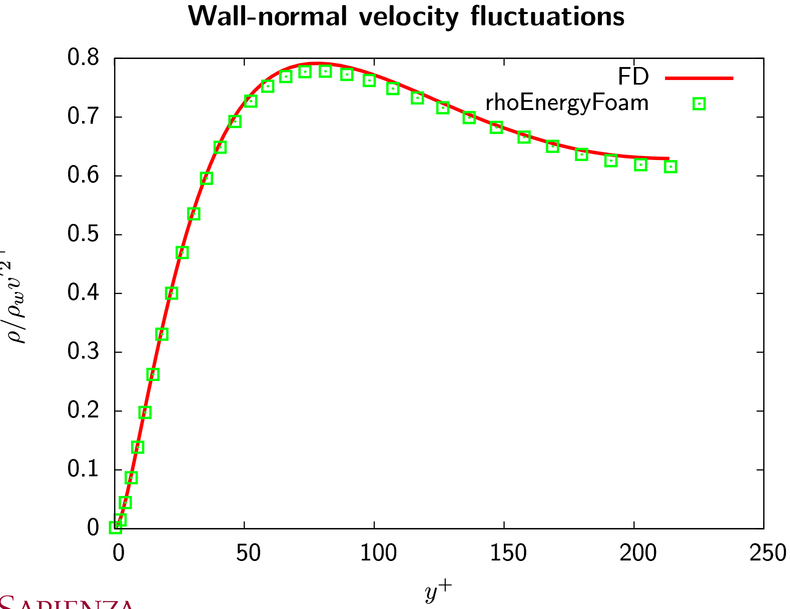
Computational set-up

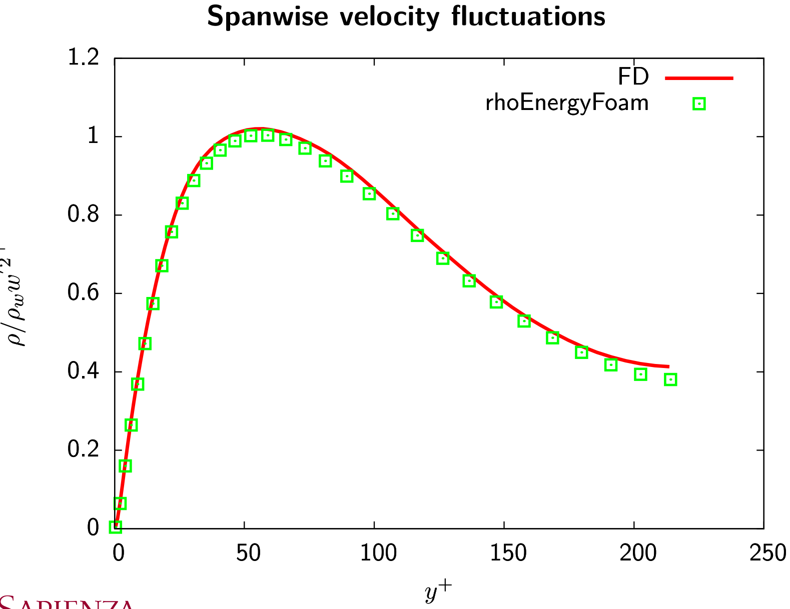
- **Parameters:** $Re_\tau (= u_\tau h/\nu) = 220$, $M_b (= u_b/a_w) = 1.5$
- **Forcing: momentum** and **total energy** equation, to maintain constant mass flow rate.
- **Box dimensions:** $L_x \times L_y \times L_z = 4\pi h \times 2h \times 4/3\pi h$
- **Mesh resolution:** $\Delta x^+ \approx 10$, $\Delta y^+ < 4$, $\Delta z^+ \approx 5$
  About 9 milions cells

Compressible channel flow: DNS

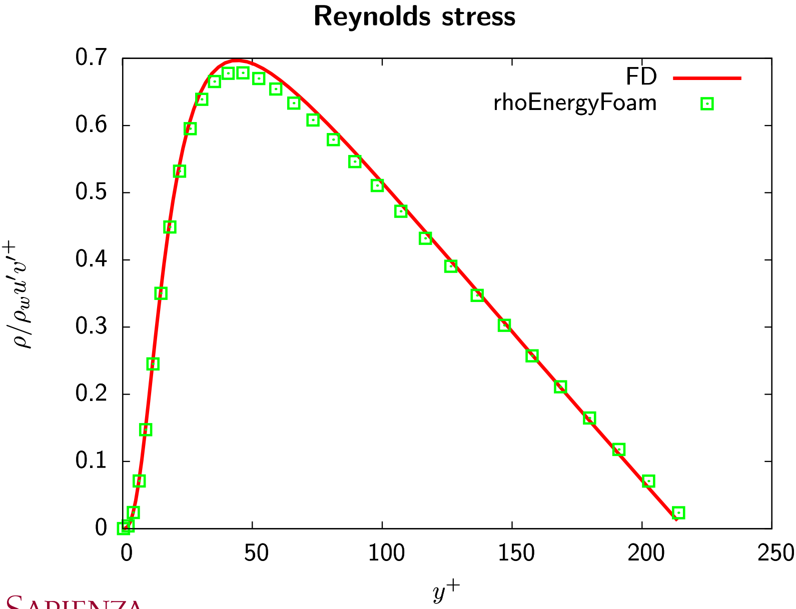



Streamwise velocity fluctuations
FD
rhoEnergyFoam
$\rho/\rho_w u'^{2+}$
y^+
0
1
2
3
4
5
6
7
8
9
10
0
50
100
150
200
250


Wall-normal velocity fluctuations
FD
rhoEnergyFoam
$\rho/\rho_w v'^{2+}$
y^+
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
50
100
150
200
250


Spanwise velocity fluctuations
FD
rhoEnergyFoam
$\rho/\rho_w {w'^2}^+$
y^+
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250

Compressible channel flow: DNS



Computational Efficiency

rhoCentralFoam gives the **same** results as the **in-house**(FD) solver.

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$$\Downarrow$$

What about the **computational efficiency** ?

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$\Downarrow$

What about the **computational efficiency** ?

Computational times

FD	**rhoEnergyFoam**	**rhoCentralFoam**	**dnsFoam**	**Commercial**
1.0	5.9	5.9	5.1	20

Scalability

Linear Scalability

- **FD:** up to 64K CPUs and more
- **standard OpenFOAM solvers:** up to 1K CPUs.
  Bottlenecks: I/O and linear solvers
- **rhoEnergyFoam: ?**

Conclusions

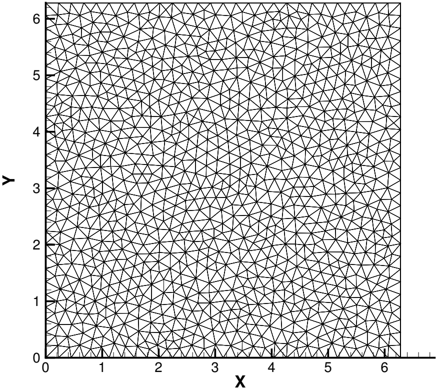
Discrete kinetic **energy conservation** guarantees:

- **stability** without addition of artificial **dissipation**
- a greater **fidelity** to the physics
- OpenFOAM/Commercial are **not energy-conserving**
- OpenFOAM can be **modified**, the proposed scheme is easy to be implemented in an existing solver

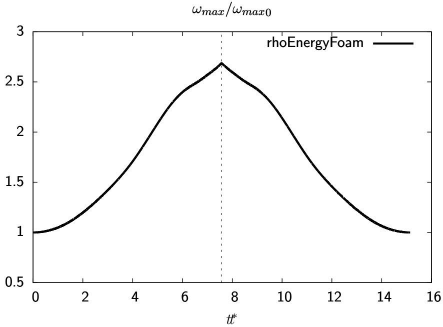
Future work

- Provide rhoEnergyFoam with shock-capturing capabilities
- Scalability analysis of rhoEnergyFoam
- RANS and LES simulations of complex geometries

Tutorial: Time reversibility on unstructured meshes

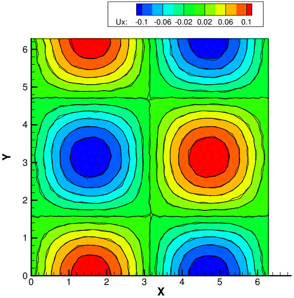


XY Plane

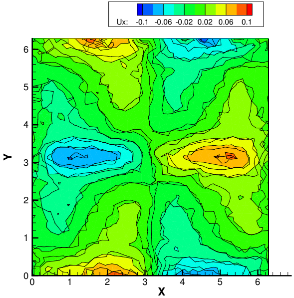


Tutorial: Time reversibility on unstructured meshes

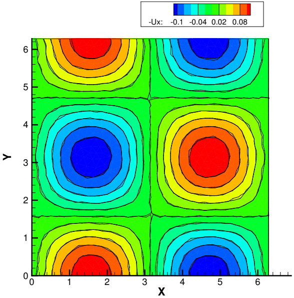
Initial Time



Reversal Time



Final Time



Tutorial: Time reversibility, extremely coarse mesh

rhoEnergyFoam

- **Time reversibility** and **energy consistency** do no depend on the mesh resolution !!!
- Let us consider a coarse mesh, 8^3.
- Kinetic energy conservation is guaranteed $\Rightarrow$ **Time reversibility**

Tutorial: Time reversibility, extremely coarse mesh

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